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Liouville-Arnold integrability of the pentagram map on closed polygons

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The pentagram map is a discrete dynamical system defined on the moduli space of polygons in the projective plane. This map has recently attracted a considerable interest, mostly because its connection to a number of different domains, such as: classical projective geometry, algebraic combinatorics, moduli spaces, cluster algebras and integrable systems. Integrability of the pentagram map was conjectured by R. Schwartz and later proved by V. Ovsienko, R. Schwartz and S. Tabachnikov for a larger space of twisted polygons. In this paper, we prove the initial conjecture that the pentagram map is completely integrable on the moduli space of closed polygons. In the case of convex polygons in the real projective plane, this result implies the existence of a toric foliation on the moduli space. The leaves of the foliation carry affine structure and the dynamics of the pentagram map is quasi-periodic. Our proof is based on an invariant Poisson structure on the space of twisted polygons. We prove that the Hamiltonian vector fields corresponding to the monodoromy invariants preserve the space of closed polygons and define an invariant affine structure on the level surfaces of the monodromy invariants.

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