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A New Graded Algebra Structure on Differential Polynomials: Level Grading and its Application to the Classification of Scalar Evolution Equations in 1+1 Dimension

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We define a new grading, that we call the "level grading", on the algebra of polynomials generated by the derivatives $u_{k+i}=\partial^{k+i}u/\partial x^{k+i}$ over the ring $K^{(k)}$ of C^{∞} functions of $u,u_1,...,u_k$. This grading has the property that the total derivative and the integration by parts with respect to x are filtered algebra maps. In addition, if u satisfies an evolution equation $u_t=F[u]$ and F is a level homogeneous differential polynomial, then the total derivative with respect to t, D_t, is also a filtered algebra map. Furthermore if ρ is level homogeneous over $K^{(k)}$, then the top level part of $D_t\rho$ depends on u_k only. This property allows to determine the dependency of $F[u]$ on u_k from the top level part of the conserved density conditions. We apply this structure to the classification of "level homogeneous" scalar evolution equations and we obtain the top level parts of integrable evolution equations of "KdV-type", admitting an unbroken sequence of conserved densities at orders $m=5,7,9,11,13,15$.

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