中国科学院研究生院学报 2011, 28(5) 583-590 DOI: ISSN: CN:

论文

一个含临界指数的拟线性椭圆型方程的注记

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摘要:

研究了如下的拟线性椭圆型方程:

$$\Delta_p u + u^q + \lambda u^{p^*-1} = 0,\ u \in W_0^{1,p}(\Omega),\ (1_\lambda)$$

其中, Ω是R^N 中具有光滑边界的有界区域,$\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$, $N \geqslant 3$, $2 \leqslant p < N$, $0 < q < 1$, $p^* = (Np)/(N-p)$. 设$\lambda^*(\Omega,p,q)$ 是拟线性椭圆型方程(1_λ) 可解的参数集的上确界.运用变分方法,在不要求具有对称性质的一般区域Ω上得到了$\lambda^*(\Omega, p, q)$ 的一个可以精确计算的下界.

关键词: 拟线性椭圆型方程 临界指数 Ekeland变分原理 参数计算

Some remarks on a quasilinear elliptic equation with critical exponent

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Abstract:

We investigate the following quasilinear elliptic equation:

$$\Delta_p u + u^q + \lambda u^{p^*-1} = 0\ ,\ u \in W^{1,p}_0(\Omega),\ (1_\lambda)$$

where Ω is a bounded domain in R^N with smooth boundary, $\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$, $N \geqslant 3$, $2 \leqslant p < N$, $0 < q < 1$, and $p^* = (Np)/(N-p)$. By using variational methods, we obtain a lower bound of the extremal value $\lambda^*(\Omega,p,q)$ for equation (1_λ), which can be explicitly calculated.

Keywords: quasilinear elliptic equation critical exponent Ekeland's variational principle extremal value

收稿日期 2010-10-09 **修回日期** 2010-11-08 **网络版发布日期**

DOI:

基金项目:

Supported by the Presidential Foundation of GUCAS

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参考文献：

[1] Garca Azorero J, Peral Alonso I. Multiplicity of solutions for elliptic problems with critical exponent or with a non-symmetric term
[J]. Trans Amer Math Soc, 1991, 323(2): 877- 895.

[2] Garcia Azorero J, Peral Alonso I. Some results about the existence of a second positive solution in a quasilinear critical problem
[J]. Indiana Univ Math J, 1994, 43(3): 941-957.

[3] Huang Y X. Positive solutions of certain elliptic equations involving critical Sobolev exponents
[J]. Nonlinear Analysis TMA, 1998, 33(6): 617- 636.

[4] Tan Z, Yao Z G. The existence of multiple solutions of p-Laplacian elliptic equation
[J]. Acta Mathematica Scientia, 2001, 21B(2): 203-212.

[5] Gazzola F, Malchiodi A. Some remarks on the equation $-\Delta u=\lambda(1+u)^p$ for varying λ, p and varying domains
[J]. Comm Partial Differential Equations, 2002, 27(4): 809- 845.

[6] Sun Y J, Li S J. A nonlinear elliptic equation with critical exponent: Estimates for extremal values
[J]. Nonlinear Analysis TMA, 2008, 69(5): 1856-1869.

[7] Sun Y J, Li S J. Some remarks on a superlinear-singular problem: Estimates of λ^*
[J]. Nonlinear Analysis TMA, 2008, 69(8): 2636-2650.

[8] Talenti G. Best constant in sobolev inequality
[J]. Ann Math Pure Appl, 1976, 110(1): 353-372.

[9] Chang K C. Methods in nonlinear analysis
[M]. Heidelberg: Springer-Verlag, 2005.

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2．郭千桥, 崔学伟.具有Sobolev临界指数的半线性椭圆方程的正解[J]. 中国科学院研究生院学报, 2009,26(2): 158-166