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On the Boundary Behaviour, Including Second Order Effects, of Solutions to Singular Elliptic Problems

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摘要

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Abstract For $\gamma\ge 1$ we consider the solution $u=u(x)$ of the Dirichlet boundary value problem $\Delta u+u^{-\gamma}=0$ in Ω, $u=0$ on $\partial\Omega$. For $\gamma=1$ we find the estimate $$u(x)=p(\delta(x))\Bigl[1+A(x)\Bigl(\log\frac{1}{\delta(x)}\Bigr)^{-\epsilon} \Bigr],$$ where $p(r)\approx r\, \sqrt{2\, \log(1/r)}$ near $r=0$, $\delta(x)$ denotes the distance from x to $\partial\Omega$, $0<\epsilon<1/2$, and $A(x)$ is a bounded function. For $1<\gamma<3$ we find $$u(x)=\Bigl(\frac{\gamma+1}{\sqrt{ 2(\gamma-1)}}\delta(x)\Bigr)^{\frac{2}{\gamma+1}}\Bigl[1+A(x)(\delta(x))^{2\frac{\gamma-1}{\gamma+1}}\Bigr].$$ For $\gamma=3$ we prove that $$u(x)=(2\delta(x))^{\frac{1}{2}}\Bigl[1+A(x)\delta(x)\log\frac{1}{\delta(x)}\Bigr].$$

Key words elliptic problems singular equations boundary behaviour

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