Mathematical Physics

Barrier methods for critical exponent problems in geometric analysis and mathematical physics

Jennifer Erway, Michael Holst

(Submitted on 2 Jul 2011)

We consider the design and analysis of numerical methods for approximating positive solutions to nonlinear geometric elliptic partial differential equations containing critical exponents. This class of problems includes the Yamabe problem and the Einstein constraint equations, which simultaneously contain several challenging features: high spatial dimension n >= 3, varying (potentially non-smooth) coefficients, critical (even super-critical) nonlinearity, non-monotone nonlinearity (arising from a non-convex energy), and spatial domains that are typically Riemannian manifolds rather than simply open sets in Rn. These problems may exhibit multiple solutions, although only positive solutions typically have meaning. This creates additional complexities in both the theory and numerical treatment of such problems, as this feature introduces both non-uniqueness as well as the need to incorporate an inequality constraint into the formulation. In this work, we consider numerical methods based on Galerkin-type discretization, covering any standard bases construction (finite element, spectral, or wavelet), and the combination of a barrier method for nonconvex optimization and global inexact Newton-type methods for dealing with nonconvexity and the presence of inequality constraints. We first give an overview of barrier methods in non-convex optimization, and then develop and analyze both a primal barrier energy method for this class of problems. We then consider a sequence of numerical experiments using this type of barrier method, based on a particular Galerkin method, namely the piecewise linear finite element method, leverage the FETK modeling package. We illustrate the behavior of the primal barrier energy method for several examples, including the Yamabe problem and the Hamiltonian constraint.

Subjects: **Mathematical Physics (math-ph)**; General Relativity and Quantum Cosmology (gr-qc); Analysis of PDEs (math.AP); Numerical Analysis (math.NA); Optimization and Control (math.OC); Computational Physics (physics.comp-ph)

Cite as: **arXiv:1107.0360 [math-ph]**
(or **arXiv:1107.0360v1 [math-ph]** for this version)

Submission history

From: Jennifer Erway [view email]
[v1] Sat, 2 Jul 2011 04:41:52 GMT (63kb,D)