数学物理学报 2009, 29(4) 833-842 DOI: ISSN: 1003-3998 CN: 42-1226/0

论文

由分数次导数所确定的$L_2(T)$中的一元周期函数类在$L_q(T)$中的相对宽度

(1. 北京师范大学 数学科学学院, 数学与复杂系统教育部重点实验室, 北京 100875, 2. 北方工业大学 理学院, 北京 100144)

摘要:

作者研究了相对宽度$K_n(W_2^{\alpha}(T), MW_2^{\beta}(T), L_2(T))$, $T=[0,2\pi]$, 确定了使等式$K_n(W_2^{\alpha}(T), MW_2^{\beta}(T), L_2(T))=d_n(W_2^{\alpha}(T), L_2(T))$成立的最小$M$值, 得到了相对宽度$K_n(W_2^{\alpha}(T), W_2^{\alpha}(T), L_q(T))$的渐近阶, 其中$\alpha\geq\beta>0$, $1\leq q\leq\infty$, $K_n(.,.,L_q(T))$ 和 $d_n(.,L_q(T))$分别表示Kolmogorov意义下$L_q(T)$尺度下的相对宽度和宽度, $MW_p^{\alpha}(T)$, $1\leq p\leq\infty$, 表示有如下卷积表达式的2π 周期函数类, $f(t)=c+(B_\alpha * g)(t)$, $c\in R$, $B_\alpha * g$ 表示 B_α 和g 的卷积, $g\in L_p(T)$ 满足$\int_0^{2\pi}g(\tau)d\tau=0$ 和$||g||_p\leq M$, $B_\alpha\in L_1(T)$ 有如下Fourier展开: $B_\alpha(t)=1/2\pi\sum'_{k\in Z}(ik)^{-\alpha}e^{ikt}$, $\sum'$表示去掉 $k=0$的项.

关键词: 相对宽度 n-K宽度 分数次导数

分类号:

41A46; 41A65; 46E35

Relative Widths of Function Classes of $L_2(T)$ Determined by Fractional Order Derivatives in $L_q(T)$

(1. School of Mathematical Sciences, Beijing Normal University, Laboratory of Mathematics and Complex Systems, Ministry of Education, Beijing 100875 2. College of Sciences, North China University of Technology, |Beijing 100144)

Abstract:

The relative widths $K_n(W_2^{\alpha}(T), MW_2^{\beta}(T), L_2(T))$, $T=[0, 2\pi]$, is studied and the smallest number M which makes the equality $K_n(W_2^{\alpha}(T), MW_2^{\beta}(T), L_2(T))=d_n(W_2^{\alpha}(T), L_2(T))$ valid is obtained, and the asymptotic order of relative widths $K_n(W_2^{\alpha}(T), W_2^{\alpha}(T), L_q(T))$ is obtained, where $\alpha\geq\beta>0$, $1\leq q\leq\infty$, $K_n(.,.,L_q(T))$ and $d_n(.,L_q(T))$ denote respectively the relative widths and the widths in the sense of Kolmogorov in $L_q(T)$, and $MW_p^{\alpha}(T)$, $1\leq p\leq\infty$, denotes the collection of 2π-periodic and continuous functions f representable as a convolution $f(t)=c+(B_\alpha * g)(t)$, where $B_\alpha * g$ denotes the convolution of B_α and g, for $g\in L_p(T)$ satisfying $\int_0^{2\pi}g(\tau)d\tau=0$ and $||g||_p\leq M$. Here B_α is in $L_1(T)$ with the Fourier expansion $B_\alpha(t))=1/2\pi\sum'_{k\in Z}(ik)^{-\alpha}e^{ikt}$, where $\sum'$ means that the term is omitted when $k=0$.

Keywords: Relative widths n-K widths Derivatives of fractional order

收稿日期 2007-10-29 修回日期 2009-03-25 网络版发布日期 2009-08-25

DOI:

基金项目:

国家自然科学基金(10771016)、北京师范大学``985项目"和北京自然科学基金(1062004)资助

通讯作者:

作者简介:

参考文献:

[1] Konovalov V N. Estimates of diameters of Kolmogorov type for classes of differentiable periodic functions. Mat Zametki, 1984, 35(3): 369--380

[2] Butzer P L, Westphal U. An Access to Fractional Differentiation via Fractional Difference Quotients//Lecture Notes in Math. Berlin: Springer, 1975

[3] Butzer P L, Dyckhoff H, Görlich E, Stens R L. Best trigonometric approximation, fractional order derivatives and
Lipschitz classes. Canad J Math, 1977, 29(4): 781--793

[4] Zygmund A. Trigonometric Series I-II. New York: Cambridge University Press, 1959

[5] Tikhomirov V M. Widths of sets in functional spaces and theory of best approximations. Uspekhi Mat Nauk, 1960, 15(3): 81--120

[6] Tikhomirov V M. Best methods for approximation and interpolation of differentiable functions in the space *C*
[-1, 1]. Mat Sb, 1969, 80(2): 290--304

[7] Tikhomirov V M. Some remarks on relative diameters. Banach Center Publ, 1989, 22: 471--474

[8] Babenko V F. On the relative widths of classes of functions with bounded mixed derivative. East J Approx, 1996, 2(3): 319--330

[9] Konovalov V N. Approximation of Sobolev classes by their sections of finite dimensional. Ukain Math J, 2002, 54(5): 795--805

[10] Konovalov V N. Approximation of Sobolev classes by their finite-dimensional sections. Mat Zametki, 2002, 72(3): 370--382

[11] Liu Yongping, Yang Lianhong. Relative width of smooth classes of multivariate periodic functions with restrictions on iterated Laplace derivatives in the L_2-metric. Acta Math Scientia, 2006, 26B(4): 720--728

[12] Liu Yongping, Yang Lianhong. Relative widths of smooth functions determined by fractional order derivatives. J Complexity, 2008, 24: 259--282

[13] Subbotin Y N, Telyakovskii S A. Relative widths of classes of differentiable functions in the L_2 metric. Uspekhi Mat Nauk, 2001, 56(4): 159--160

[14] 肖维维, 刘永平. L_2尺度下可微函数类的相对宽度. 北京师范大学学报(自然科学版), 2007, 43(5): 493--495

[15] Liu Yongping, Xiao Weiwei. Relative average width of Sobolev space in $L_2(R($d). Analysis Mathematica, 2008, 34(1): 71--82

[16] Lorentz G G, Golitschek M V, Makovoz Y. Constructive Approximation. Berlin: Springer, 1996

[17] 孙永生. 函数逼近论 (上册). 北京:北京师范大学出版社, 1989

[18] Pinkus A. *n*-widths in Approximation Theory. Berlin: Springer, 1985

[19] Nikol'skii S M. Approximation of Functions of Several Variables and Imbedding Theorems. New York: Springer-Verlag, 1975

[20] Tikhomirov V M. Some Problems in Approximation Theory (in Russian). Moscow: Moscow University, 1976

[21] Dzyadyk V K. Introduction to the Theory of Uniform Polynomial Approximation of Functions (in Russian). Moscow: Nauka, 1977