On the concordance orders of knots

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(Submitted on 4 Jun 2012)

This thesis develops some general calculational techniques for finding the orders of knots in the topological concordance group C. The techniques currently available in the literature are either too theoretical, applying to only a small number of knots, or are designed to only deal with a specific knot. The thesis builds on the results of Herald, Kirk and Livingston [HKL10] and Tamulis [Tam02] to give a series of criteria, using twisted Alexander polynomials, for determining whether a knot is of infinite order in C.

There are two immediate applications of these theorems. The first is to give the structure of the subgroups of the concordance group C and the algebraic concordance group G generated by the prime knots of 9 or fewer crossings. This should be of practical value to the knot-theoretic community, but more importantly it provides interesting examples of phenomena both in the algebraic and geometric concordance groups. The second application is to find the concordance orders of all prime knots with up to 12 crossings. At the time of writing of this thesis, there are 325 such knots listed as having unknown concordance order. The thesis includes the computation of the orders of all except two of these.

In addition to using twisted Alexander polynomials to determine the concordance order of a knot, a theorem of Cochran, Orr and Teichner [COT03] is applied to prove that the n-twisted doubles of the unknot are not slice for n not 0 or 2. This technique involves analysing the `second-order' invariants of a knot; that is, slice invariants (in this case, signatures) of a set of metabolising curves on a Seifert surface for the knot. The thesis extends the result to provide a set of criteria for the n-twisted double of a general knot K to be slice; that is, of order 0 in C.

Comments:	146 pages, full PhD thesis
Subjects:	**Geometric Topology (math.GT)**
MSC classes:	57M25, 57Q60
Cite as:	**arXiv:1206.0669 [math.GT]**
	(or **arXiv:1206.0669v1 [math.GT]** for this version)

Submission history

From: Julia Collins [view email]
[v1] Mon, 4 Jun 2012 16:33:59 GMT (4567kb,D)

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