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# Knot state asymptotics I, AJ Conjecture and abelian representations

Laurent Charles, Julien Marche

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Consider the Chern-Simons topological quantum field theory with gauge group  $SU(2)$  and level  $k$ . Given a knot in the 3-sphere, this theory associates to the knot exterior an element in a vector space. We call this vector the knot state and study its asymptotic properties when the level is large. The latter vector space being isomorphic to the geometric quantization of the  $SU(2)$ -character variety of the peripheral torus, the knot state may be viewed as a section defined over this character variety. We first conjecture that the knot state concentrates in the large level limit to the character variety of the knot. This statement may be viewed as a real and smooth version of the AJ conjecture. Our second conjecture says that the knot state in the neighborhood of abelian representations is a Lagrangian state. Using microlocal techniques, we prove these conjectures for the figure eight and torus knots. The proof is based on  $q$ -difference relations for the colored Jones polynomial. We also provide a new proof for the asymptotics of the Witten-Reshetikhin-Turaev invariant of the lens spaces and a derivation of the Melvin-Morton-Rozansky theorem from the two conjectures.

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