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Integrable GL(2) Geometry and Hydrodynamic Partial Differential Equations

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This article is a local analysis of integrable GL(2)-structures of degree 4. A GL(2)-structure of degree n corresponds to a distribution of rational normal cones over a manifold M of dimension (n+1). Integrability corresponds to the existence of many submanifolds that are spanned by lines in the cones. These GL(2)-structures are important because they naturally arise from a certain family of second-order hyperbolic PDEs in three variables that are integrable via hydrodynamic reduction. Familiar examples include the wave equation, the first flow of the dKP equation, and the Boyer--Finley equation.

The main results are a structure theorem for integrable GL(2)-structures, a classification for connected integrable GL(2)-structures, and an equivalence between local integrable GL(2)-structures and Hessian hydrodynamic hyperbolic PDEs in three variables.

This yields natural geometric characterizations of the wave equation, the first flow of the dKP equation, and several others. It also provides an intrinsic, coordinate-free infrastructure to describe a large class of hydrodynamic integrable systems in three variables.

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