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Quantitative Stratification and the Regularity of Harmonic Maps and Minimal Currents

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We introduce techniques for turning estimates on the infinitesimal behavior of solutions to nonlinear equations (statements concerning tangent cones and blow ups) into more effective control. In the present paper, we focus on proving regularity theorems for stationary and minimizing harmonic maps and minimal currents. There are several aspects to our improvements of known estimates. First, we replace known estimates on the Hausdorff dimension of singular sets by estimates on their Minkowski r-content, or equivalently, on the volumes of their r-tubular neighborhoods. Second, we give improved regularity control with respect to the number of derivatives bounded and/or on the norm in which the derivatives are bounded. As an example of the former, our results for minimizing harmonic maps f : M->N between riemannian manifolds include a priori bounds in W^{1,p}\cap W^{2,p} for all p < 3. These are the first such bounds involving second derivatives in general dimensions. Finally, the quantity we control is actually provides much stronger information than follows from a bound on the L^p norm of derivatives. Namely, we obtain L^p bounds for the inverse of the regularity scale r_f(x) =max{r:sup_{B_r(x)}(r|\nabla f|+r^2|\nabla^2 f|)\leq 1}. Applications to minimal hypersufaces include a priori L^p bounds for the second fundamental form A for all p < 7. Previously known bounds were for p < 2+\epsilon(n). Again, the full theorem is much stronger and yields Lp bounds for the corresponding regularity scale r_A(x)=max{r:sup_{B_r(x)}rA\leq 1}. In outline, our discussion follows that of an earlier paper in which we proved analogous estimates in the context of noncollapsed riemannian manifolds with a lower bound on Ricci curvature. These were applied to Einstein manifolds. A key point in all of these arguments is to establish the relevant quantitative differentiation theorem.

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