Acta Mathematica Sinica, English Series 2006 22 (4): 1175-1182 ISSN: 0583-1431 CN: 11-1914/R

22(4)

HILBERT SPACE OF PROBABILITY DENSITY FUNCTIONS BASED ON AITCHISON GEOMETRY

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收稿日期 2004-3-18 修回日期 网络版发布日期 2006-7-15 接受日期 2004-10-18

摘要 The set of probability functions is a convex subset of L^1 and it does not have a linear space structure when using ordinary sum and multiplication by real constants. Moreover, difficulties arise when dealing with distances between densities. The crucial point is that usual distances are not invariant under relevant transformations of densities. To overcome these limitations, Aitchison's ideas on compositional data analysis are used, generalizing perturbation and power transformation, as well as the Aitchison inner product, to operations on probability density functions with support on a finite interval. With these operations at hand, it is shown that the set of bounded probability density functions on finite intervals is a pre-Hilbert space. A Hilbert space of densities, whose logarithm is square-integrable, is obtained as the natural completion of the pre-Hilbert space.

关键词 Bayes' theorem Fourier coefficients Haar basis Aitchison distance Simplex Least squares

分类号 46C15

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Key words Bayes' theorem Fourier coefficients Haar basis Aitchison distance Simplex Least squares approximation

DOI: 10.1007/s10114-005-0678-2

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