

Toric Stacks I: The Theory of Stacky Fans

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The purpose of this paper and its sequel (Toric Stacks II) is to introduce and develop a theory of toric stacks which encompasses and extends the notions of toric stacks defined in [Laf02, BCS05, FMN10, Iwa09, Sat12, Tyo12], as well as classical toric varieties.

In this paper, we define a *toric stack* as a quotient of a toric variety by a subgroup of its torus (we also define a generically stacky version). Any toric stack arises from a combinatorial gadget called a *stacky fan*. We develop a dictionary between the combinatorics of stacky fans and the geometry of toric stacks, stressing stacky phenomena such as canonical stacks and good moduli space morphisms.

We also show that smooth toric stacks carry a moduli interpretation extending the usual moduli interpretations of \mathbb{P}^n and $\mathbb{A}^1/\mathbb{G}_m$. Indeed, smooth toric stacks precisely solve moduli problems specified by (generalized) effective Cartier divisors with given linear relations and given intersection relations. Smooth toric stacks therefore form a natural closure to the class of moduli problems introduced for smooth toric varieties and smooth toric DM stacks in [Cox95] and [Per08], respectively.

We include a plethora of examples to illustrate the general theory. We hope that this theory of toric stacks can serve as a companion to an introduction to stacks, in much the same way that toric varieties can serve as a companion to an introduction to schemes.

Comments: 39 pages, 25 examples (update to match version to be published)

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