Mathematics > Analysis of PDEs

Surface Gap Soliton Ground States for the Nonlinear Schrödinger Equation

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(Submitted on 12 Nov 2010)

We consider the nonlinear Schr\"{o}dinger equation $(-\Delta +V(x))u = \Gamma(x) |u|^{p-1}u$, $x\in \R^n$ with $V(x) = V_1(x) \chi_{\{x_1>0\}}(x) +V_2(x) \chi_{\{x_1<0\}}(x)$ and $\Gamma(x) = \Gamma_1(x) \chi_{\{x_1>0\}}(x)+\Gamma_2(x) \chi_{\{x_1<0\}}(x)$ and with $V_1, V_2, \Gamma_1, \Gamma_2$ periodic in each coordinate direction. This problem describes the interface of two periodic media, e.g. photonic crystals. We study the existence of ground state H^1 solutions (surface gap soliton ground states) for $0<\min \sigma(-\Delta +V)$. Using a concentration compactness argument, we provide an abstract criterion for the existence based on ground state energies of each periodic problem (with $V\equiv V_1, \Gamma\equiv \Gamma_1$ and $V\equiv V_2, \Gamma\equiv \Gamma_2$) as well as a more practical criterion based on ground states themselves. Examples of interfaces satisfying these criteria are provided. In 1D it is shown that, surprisingly, the criteria can be reduced to conditions on the linear Bloch waves of the operators $-\tfrac{d^2}{dx^2} +V_1(x)$ and $-\tfrac{d^2}{dx^2} +V_2(x)$.

Comments:	32 pages, 4 figures
Subjects:	**Analysis of PDEs (math.AP)**; Pattern Formation and Solitons (nlin.PS)
MSC classes:	35Q55, 78M30, 35J20, 35Q60
Cite as:	**arXiv:1011.2886v1 [math.AP]**

Submission history

From: Tomas Dohnal [view email]

[v1] Fri, 12 Nov 2010 11:39:34 GMT (502kb)