Mathematics > Operator Algebras

Index theory for locally compact noncommutative geometries

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Spectral triples for nonunital algebras model locally compact spaces in noncommutative geometry. In the present text, we prove the local index formula for spectral triples over nonunital algebras, without the assumption of local units in our algebra. This formula provides a practical way to calculate index pairings and this utility is one of the main justifications for using spectral triples as geometric models. We apply the formula in the commutative setting to prove an analogue of the Gromov-Lawson relative index formula (for Dirac type operators) for even dimensional manifolds with bounded geometry, without invoking compact supports. For odd dimensional manifolds our index formula appears to be completely new. As we prove our local index formula in the framework of semifinite noncommutative geometry we are also able to prove, for manifolds of bounded geometry, a version of Atiyah's L^2-index Theorem for covering spaces. We also demonstrate the effectiveness of our methods in two noncommutative examples. In the course of proving the local index formula we clarify some aspects of index theory for nonunital algebras, and develop a suitable integration theory which is compatible with a refinement of the existing pseudodifferential calculus for spectral triples.

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