Mathematics > Commutative Algebra

Decompositions of commutative monoid congruences and binomial ideals

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We demonstrate how primary decomposition of commutative monoid congruences fails to capture the essence of primary decomposition in commutative rings by exhibiting a more sensitive theory of mesoprimary decomposition of congruences, complete with witnesses, associated prime objects, and an analogue of irreducible decomposition called coprincipal decomposition. We lift the combinatorial theory of mesoprimary decomposition to binomial ideals in monoid algebras. The resulting binomial mesoprimary decomposition is a new type of intersection decomposition for binomial ideals that enjoys computational efficiency and independence from ground field hypotheses. Furthermore, binomial primary decomposition is easily recovered from mesoprimary decomposition, as is binomial irreducible decomposition -- which was previously not known to exist -- from binomial coprincipal decomposition.

Comments:	64 pages, 7 figures, v2: small improvements over v1, v3: added Problem 17.7, Corollary 4.15 and other small refinements
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