Mathematics > Commutative Algebra

Castelnuovo Mumford Regularity with respect to multigraded ideals

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In this article we extend a previous definition of Castelnuovo-Mumford regularity for modules over an algebra graded by a finitely generated abelian group.

Our notion of regularity is based on Maclagan and Smith's definition, and is extended first by working over any commutative base ring, and second by considering local cohomology with support in an arbitrary finitely generated graded ideal B, obtaining, for each B, a B-regularity region. The first extension provides a natural approach for working with families of sheaves or of graded modules, while the second opens new applications.

We provide tools to transfer knowledge in two directions. First to deduce some information on the graded Betti numbers from the knowledge of regions where the local cohomology with support in a given graded ideal vanishes. This is one of our main results. Conversely, vanishing of local cohomology with support in any graded ideal is deduced from the shifts in a free resolution and the local cohomology of the polynomial ring. Furthermore, the flexibility of treating local cohomology with respect to any B open new possibilities for passing information.

We provide new persistence results for the vanishing of local cohomology that extend the fact that weakly regular implies regular in the classical case, and we give sharp estimates for the regularity of a truncation of a module.

In the last part, we present a result on Hilbert functions for multigraded polynomial rings, that in particular provides a simple proof of the Grothendieck-Serre formula.

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