Mathematics > Representation Theory

Gluing derived equivalences together

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The Grothendieck construction of a diagram X of categories can be seen as a process to construct a single category $\Gr(X)$ by gluing categories in the diagram together. Here we formulate diagrams of categories as colax functors from a small category I to the 2-category $\kCat$ of small $\k$-categories for a fixed commutative ring $\k$. In our previous paper we defined derived equivalences of those colax functors. Roughly speaking two colax functors $X, X' \colon I \to \kCat$ are derived equivalent if there is a derived equivalence from $X(i)$ to $X'(i)$ for all objects i in I satisfying some "I-equivariance" conditions. In this paper we glue the derived equivalences between $X(i)$ and $X'(i)$ together to obtain a derived equivalence between Grothendieck constructions $\Gr(X)$ and $\Gr(X')$, which shows that if colax functors are derived equivalent, then so are their Grothendieck constructions. This generalizes and well formulates the fact that if two $\k$-categories with a G-action for a group G are "G-equivariantly" derived equivalent, then their orbit categories are derived equivalent. As an easy application we see by a unified proof that if two $\Bbbk$-algebras A and A' are derived equivalent, then so are the path categories AQ and $A'Q$ for any quiver Q; so are the incidence categories AS and $A'S$ for any poset S; and so are the monoid algebras AG and $A'G$ for any monoid G. Also we will give examples of gluing of many smaller derived equivalences together to have a larger derived equivalence.

Comments:	28 pages. 2nd version: many changes with oplax --> colax. 3rd version: minor changes including "The k-flatness assumption was added to apply Keller's theorem on derived equivalences of categories."
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