

## Mathematical Physics

# The Continuum Limit of Toda Lattices for Random Matrices with Odd Weights

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This paper is concerned with the asymptotic behavior of the free energy for a class of Hermitean random matrix models, with odd degree polynomial potential, in the large  $N$  limit. It continues an investigation initiated and developed in a sequence of prior works whose ultimate aim is to reveal and understand, in a rigorous way, the deep connections between correlation functions for eigenvalues of these random matrix ensembles on the one hand and the enumerative interpretations of their matrix moments in terms of map combinatorics (a branch of graph theory) on the other. In doing this we make essential use of the link between the asymptotics of the random matrix partition function and orthogonal polynomials with exponential weight equal to the random matrix potential. Along the way we develop and analyze the continuum limits of both the hierarchy of Toda lattice equations and the hierarchy of difference string equations associated to these orthogonal polynomials. The former are found to have the structure of a hierarchy of near-conservation laws universal in the potential; the latter are a novel semi-classical extension of the traditional string equations. Our methods apply to regular maps of both even and odd valence, however we focus on the latter since that is the relevant case for this paper. These methods enable us to rigorously determine closed form expressions for the generating functions that enumerate trivalent maps, in general implicitly, but also explicitly in a number of cases.

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