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On the Relationship between the Baum-Katz-Spitzer Complete Convergence Theorem and the Law of the Iterated Logarithm

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摘要

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Abstract For a sequence of i.i.d. Banach space-valued random variables $\{X_{n}; ~n \geq 1 \}$ and a sequence of positive constants $\{a_{n}; ~n \geq 1 \}$, the relationship between the Baum--Katz--Spitzer complete convergence theorem and the law of the iterated logarithm is investigated. Sets of conditions are provided under which {\rm (i)}\; $ \limsup_{n \rightarrow \infty} \frac{\|S_{n}\|}{a_{n}} < \infty$~~a.s. and \[\sum_{n=1}^{\infty} \frac{1}{n} P \left (\frac{\|S_{n}\|}{a_{n}} \geq \varepsilon \right) < \infty ~~\mbox{for all} ~\varepsilon > \lambda ~~\mbox{for some constant}~\lambda \in [0, \infty) \] are equivalent; {\rm (ii)}\; For all constants $\lambda \in [0, \infty)$, \[\limsup_{n \rightarrow \infty}\frac{\|S_{n}\|}{a_{n}} = \lambda ~~\mbox{a.s.} \] and \[\sum_{n=1}^{\infty} \frac{1}{n} P \left (\frac{\|S_{n}\|}{a_{n}} \geq \varepsilon \right) \left \{ \begin{array}{l} < \infty, ~~\mbox{if}~ \varepsilon > \lambda \= \infty, ~~\mbox{if}~ \varepsilon < \lambda \end{array} \right. \] are equivalent. In general, no geometric conditions are imposed on the underlying Banach space. Corollaries are presented and new results are obtained even in the case of real-valued random variables.

Key words partial sums of i.i.d. Banach space-valued random variables Baum--Katz--Spitzer complete convergence theorem law of the iterated logarithm almost sure convergence

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