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Unobserved population heterogeneity: A review of formal relationships

By [James W. Vaupel](#), [Trifon I. Missov](#)

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Updated Items: The paper was updated at the authors' request. The authors deleted section 6E in the original version and moved it to section 5G in the updated version. In these sections, they deleted (52*) in the old version and replaced it with (39*) in the new version.

Abstract

Background: Survival models accounting for unobserved heterogeneity (frailty models) play an important role in mortality research, yet there is no article that concisely summarizes useful relationships.

Objective: We present a list of important mathematical relationships that govern populations in which individuals differ from each other in unobserved ways. For some relationships we present proofs that, albeit formal, tend to be simple and intuitive.

Methods: We organize the article in a progression, starting with general relationships and then turning to models with stronger and stronger assumptions.

Results: We start with the general case, in which we do not assume any structure of the underlying baseline hazard, the frailty distribution, or their link to one another. Then we sequentially assume, first, a relative-risk model; second, a gamma distribution for frailty; and, finally, a Gompertz and Gompertz-Makeham specification for baseline mortality.

Comments: The article might serve as a handy overall reference to frailty models, especially for mortality research.

Author's Affiliation

[James W. Vaupel](#) - Syddansk Universitet, Denmark [[Email](#)]

[Trifon I. Missov](#) - Max-Planck-Institut für Demografische Forschung, Germany [[Email](#)]

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