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A matrix approach to the statistics of longevity in heterogeneous frailty models

By Hal Caswell

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Abstract

Background: The gamma-Gompertz model is a fixed frailty model in which baseline mortality increases exponentially with age, frailty has a proportional effect on mortality, and frailty at birth follows a gamma distribution. Mortality selects against the more frail, so the marginal mortality rate decelerates, eventually reaching an asymptote. The gamma-Gompertz is one of a wider class of frailty models, characterized by the choice of baseline mortality, effects of frailty, distributions of frailty, and assumptions about the dynamics of frailty.

Objective: To develop a matrix model to compute all the statistical properties of longevity from the gamma-Gompertz and related models.

Methods: I use the vec-permutation matrix formulation to develop a model in which individuals are jointly classified by age and frailty. The matrix is used to project the age and frailty dynamics of a cohort and the fundamental matrix is used to obtain the statistics of longevity.

Results: The model permits calculation of the mean, variance, coefficient of variation, skewness and all moments of longevity, the marginal mortality and survivorship functions, the dynamics of the frailty distribution, and other quantities. The matrix formulation extends naturally to other frailty models. I apply the analysis to the gamma-Gompertz model (for humans and laboratory animals), the gamma-Makeham model, and the gamma-Siler model, and to a hypothetical dynamic frailty model characterized by diffusion of frailty with reflecting boundaries. The matrix model permits partitioning the variance in longevity into components due to heterogeneity and to individual stochasticity. In several published human data sets, heterogeneity accounts for less than 10% of the variance in longevity. In laboratory populations of five invertebrate animal species, heterogeneity accounts for 46% to 83% of the total variance in longevity.

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