

31. $\frac{1}{2} \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{y^2-(x-1)^2}}^{\sqrt{y^2-(x-1)^2}} \int_{-\sqrt{z^2-y^2-(x-1)^2}}^{\sqrt{z^2-y^2-(x-1)^2}} dz dy dx$
 $= \frac{1}{2} \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-\sqrt{y^2-(x-1)^2}}^{\sqrt{y^2-(x-1)^2}} dz dy dx$
 $= \frac{1}{2} \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \pi y^2 dy dx = \frac{\pi}{2} \int_{-1}^1 x^2 dx = \frac{\pi}{2} \cdot \frac{2}{3} = \frac{\pi}{3}$