Competition between Vertically Integrated Networks: a Generalized Model[*]

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Abstract

Using the Laffont, Rey and Tirole (1998) framework, a model of competition between *vertically integrated telecommunications networks* in a deregulated environment is developed. Two local operators compete in linear and non linear tariffs (i.e. two-part tariffs) in the subscribers market. In addition, they are integrated downstream in a potentially competitive sector (i.e. long distance sector) where they face competition of other firms which require (one way) access to local networks as an "essential facility". The purpose of the paper is to introduce a "downstream" competition in the usual framework of network competition and to focus on how the one way access charges are set in an oligopolistic market. In a mature phase of the industry, the presence of competition in both local and long distance sectors leads to lower local and long distance tariffs. The strategic role of the two-way and one-way access charges is pointed out, with particular reference to the effect that the reciprocal (two-way) access charge has on competition in the complementary sector. Finally, in case of competition in two-part tariffs, the paper investigates: 1) the asymmetric case in which only one network is integrated; 2) the entry process when the two local networks have different coverage. The results show how the level of the two-way and one-way access charges affects the "level playing field" between networks.

Key words: *Telecommunications; Interconnection; Integration*; *Competition Policy*.

JEL numbers: D4, K21, L41, L51, L96.

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1. Introduction

In the past, it was generally accepted that the telecommunications industry was characterized by the existence of one essential part of the production process which couldn't be reasonably duplicated (the *local loop*). Since all the competitive services (i.e. long distance calls, mobile calls, value added services, Internet services, …) had to pass through the same bottleneck, the local network controlled by a monopolistic firm, regulation was therefore necessary. Recently, the forces of technological development and the convergence of wireline telecommunications, video distribution and computer technologies are creating opportunities for new suppliers in the local market long dominated by monopolistic operators. The development of more sophisticated technologies and the difficulty in regulating access to a single bottleneck is pushing many governments to promote competition among networks in the local area. However, since customers of different operators want to get in touch with each other, network operators need to be interconnected, so they need to subscribe an agreement on mutual provision of access. We thus face with a two way access problem in which each competitor owns a bottleneck facility, its subscriber base.

In this context, can competition perfectly replace regulation? In other words, is competition a better substitute for regulation? In general, this statement is clearly true when markets approximate some ideal conditions, but careful discussion is needed in the presence of imperfectly competitive firms. In fact, it seems unlikely that free entry and price-taking behavior represent the way network utilities will be working in the near future; on the contrary it is rather more realistic to think of oligopolies made by a limited number of firms. In a two way bottleneck problem, since network externalities require interconnection between local operators, the provision of mutual access asks for some form of cooperation among competitors, who need to reach an agreement on modes of operation and on the access price. The presence of a limited number of firms raises the natural question of whether, in a mature phase of the industry, established networks could use their interconnection agreements to enforce collusive behaviour.

Recently, economic literature has focused on this problem (Armstrong, 1998; Laffont, Rey and Tirole, 1998). However, in these models of "pure network competition", the problem of undestanding how the (one way) access charge to local network for firms competing on complementary segment (i.e. long distance, mobile sectors, Internet, etc…) is set, is completely neglected. In fact, the traditional two way access models are characterised by an horizontal competition between two local operators. Nevertheless, they completely neglect the effects deriving from the activities carried out by network firms in other potentially competitive sectors, which are vertically related to the local segment. In fact, due to the several acquisitions and mergers which are emerging worldwide, many local operators – now competing in the subscribers market - are also vertically integrated in other potentially competitive sectors.

The aim of this paper is to introduce "downstream" competition in the usual framework of network competition and to focus on how the one way access charges are set in an oligopolistic market. Using the LRT framework, we develop a model of competition between *vertically integrated telecommunications networks* in a deregulated environment. Two local operators compete both in linear and non linear tariffs (i.e. two-part tariffs) in the subscribers market. In addition, they are integrated downstream in a potentially competitive sector – i.e. long distance market – where they face competition of other firms which require (one way) access to local networks as an "essential facility". Thus, a combination between a two way and a one way access problem is analysed here. Customers may only subscribe to one of the two networks after having observed local tariffs supplied by the two network operators, but they cannot change their subscription decision after having observed prices for long distance calls. As a result, after their subscription decision to the local network customers are "captive". For this reason, our model can be considered a mix between the LRT model and the one suggested by Economides *et al*. (1996).

This paper focuses on the effect of competition in both local and long distance markets in terms of retail tariffs in a mature phase of the industry. In addition, the strategic role of the two-way and one-way access charges is pointed out. In particular, the effect of the reciprocal (two-way) access charge on the level of the downstream (one-way) access charge is shown. Finally, in case of local competition in two-part tariffs, the paper investigates two special cases: first, the asymmetric case in which only one network is integrated; second, the entry process when the two local networks have different coverage. It is shown that in this asymmetric context the level of the two-way and one-way access charges affects the "level playing field" between networks. Thus, while in the mature phase of the industry competition between networks (especially in two-part tariff) leads to "true competition", in the entry process regulation is still necessary to protect new entrants.

The model presented here is based on an our previous paper (Cambini, 1999) wherewe propose a simplified version of this model with some stronger assumptions on the functional form of the demand function in the local market. Here, the analysis is more general and, in particular, more suitable to be compared with the results obtained in Laffont *et al*. (1998).

The rest of the paper is organized as follows. In section 2 defines the "network competition" model between vertically integrated operators. Section 3 analyses the subgame concerning competition in the downstream market, which is supposed to be the long distance market. Section 4 investigates competition in linear tariffs between integrated, and then separated, network firms. In Section 5 analyses competition in two-part tariffs between network operators and we compare the results obtained under the two different market structures (integration and separation). The asymmetric cases in which only one network is integrated and networks have different coverage are also analysed. Section 6 concludes.

2. The model

Consider a telecommunications industry structurally divided in two vertically related sectors: a local sector – the upstream segment of the industry – where two network operators compete with each other for customers (as in LRT), and a complementary sector – the downstream segment – where *n* firms, which need the access to the network as an essential input, offer telecommunications services in a competitive environment. To simplify, we consider the complementary sector as the long distance market, but the analysis can also be extended to other sectors where the access to local network is an "essential input".

In the local sector, as in LRT, the two telecommunications operators have an identical cost structure and are horizontally differentiated à la Hotelling. We consider a mature phase of the industry in which each operator control a full coverage network which is interconnected to the other so that subscribing to one operator allows to call subscribers from the rival network. The two operators compete for customers but at the same time they have to agree on a mutual (*two way*) access condition for cross-networks communications. In addition, suppose that the two network operators are vertically integrated with the downstream sector (directly or through a subsidiary) so that they can provide not only local calls but also long distance calls to the customers of its own network and of the rival network.

In the long distance sector there are also some independent operators that can supply long distance calls to all customers. They have no local infrastructures so that they have to pay a *one way* access charge for having access to customers as an essential input. This one way access is set freely and non-cooperatively by each network. Competition in this sector is à la Cournot.[1] The possibility of bypassing the local access bottleneck in supplying long distance calls is excluded. Each customer, whatever the local network she chooses, represents a "bottleneck" for long distance operators. Then, the long distance market may be considered *as if* it was ideally made by two distinct submarkets, one for customers connected with network *i* and the other for customers connected with network *j*.

As in LRT, the model is based on two main assumptions, that we will maintain throughout the following sections: reciprocal access pricing for the two-way interconnection and balanced calling pattern.[2]

[1] We imagine a situation in which long distance firms, or other telecommunications firms operating in complementary sectors (for instance, mobile operators), could face some capacity limitations due to the amount of fiber cables or to the available spectrum of radio frequencies; then, quantity competition can be reasonably assumed. However, since the development of more sophisticated technologies allows a more efficient use of the existing resources, in the future price competition should be closer to reality. See, for example, Valletti (1999).

[2] Balanced calling pattern requires that, for equal prices, the percentage of "on-net calls" (calls originating on a network and completed on the same network) is equal to the fraction of customers subscribing to this network. If customers are homogeneous in their taste, this implies that flows in and out of the network are always balanced. However, this is not necessarily true if customers are heterogeneous. For more details, see Dessein (1999b).

Cost structure and retail tariffs. The cost structure in the local area is the same of LRT. Both networks have full coverage. Coverage is defined as the fraction of customers who can but are not necessarily served by the network. Serving a customer involves a fixed cost f of connection and billing, a marginal cost c_0 per call at the originating and terminating ends of a local call and a marginal cost c_1 in between. The total marginal cost for a local call is thus $c^{loc} = 2c_0 + c_1$. Networks pay each other an exogenous negotiated two-way access charge, denoted by a, for terminating each others calls.

Providing long distance services involves no fixed costs and a constant marginal cost, besides the one way access charge to connect to the network to which the consumer has subscribed to. We assume that the long distance technology used by the network operators and its long distance rivals may be different. Let c and $\tilde{c}$ be the marginal cost of transporting the call and terminating it for the n-2 long distance operators and the network operators i and j, respectively. Let $\Delta c = \tilde{c} - c$ be the difference in this marginal cost. The n long distance operators compete in quantity simultaneously. Because of the integration with the local sector, the network's subsidiary pays only the marginal cost c_0 of access while its rivals must pay the entire one way access charge. Denote with $\tilde{a}_i$ (resp. $\tilde{a}_j$) the non cooperative one-way access charge set by network i (resp. j). I consider first the case when networks compete in linear tariffs. Customers of network i pay a usage fee p_i^{loc} per minute for local calls and a usage fee p_i^{ld} per minute for long distance calls.

Demand structure. As in LRT, in the local sector the two networks are differentiated à la Hotelling. Consumers are uniformly located on the segment [0,1] while the network operators are located at the two extremities. Consumers have a constant elasticity demand for local calls, $q = (p^{loc})^{-h}$ where h is the demand elasticity of local calls. The variable gross surplus for local calls, denoted by $u(q)$, is given by $u(q) = \frac{q^{1-1/h}}{1-1/h}$. Thus, under uniform pricing, the consumers' variable net surplus is:

$$v(p^{loc}) = \max_q \{u(q) - p^{loc}q\} = \frac{p^{loc-(h-1)}}{h-1}$$

In the downstream sector, the total quantity of long distance calls provided to a customers of network i is denoted by Q_i. We assume that customers have a quadratic utility function defined on long distance minutes, $u^{ld}(Q_i) \equiv \frac{b}{2} Q_i^2$, which, for a given quantity Q_i offered to each customer of network i, yields a linear demand function:

$$p_i^{ld} \equiv p_i^{ld}(Q_i) = g - bQ_i$$

where γ β are positive and constant and p_i^{ld} is the clearing market price in the long distance sector. We suppose that equilibrium in the long distance sector always exists.[3] Due to the assumptions on the timing of the game (discussed below), this demand function has the same functional form of the aggregate demand function for all the subscribers of a network; for this reason, Q_i will be interpreted hereafter as the global demand of long distance calls for *all* the customers connected to network *i*.

Each long distance operator k, $k = 1,..., n$-2, decides what quantities q_{ki} and q_{kj} of long distance minutes to offer to customers connected to networks i and j; similarly, network operator i decides the quantities $\tilde{q}_{ii}$ and $\tilde{q}_{ij}$ to supply to its customers and to customers of network j. The global quantity of long distance calls supplied to all network i's subscribers is $Q_i \equiv \sum_{k=1}^{n-2} q_{ki} + \tilde{q}_{ii} + \tilde{q}_{ji}$. Similarly for network j.

Given income y, the generic local telephone consumption q and the generic long distance telephone consumption Q, a consumer located at x and joining network i derives utility $y + v_0 - t|x - x_h| + u^{loc}(q) + u^{ld}(Q)$, where v_0 represents a fixed surplus from being connected to either network and is assumed to be large enough so that all customers always choose to be connected to a network, $t|x - x_h|$ denotes the cost of being connected to a network with address x_h ($h = i,j$), and t represents the cost of transportation. Note that we assume that the consumption of long distance calls Q has no influence on the consumption of the local services q of the network operators. Market shares are defined as in Hotelling's model. Thus, the consumer indifferent between the two networks is located at $x = \alpha$ where:

$$\alpha = \alpha(p_i^{loc}, p_j^{loc}, \tilde{a}_i, \tilde{a}_j) = \frac{1}{2} + \sigma\left[\omega_i - \omega_j\right]$$

$\omega_t = v(p_t^{loc}) + v^{ld}(p_t^{ld}(Q_t(\tilde{a}_t)))$, $t = i, j$, is the net surplus of a consumer connected to one network and consuming long distance calls, and $\sigma \equiv 1/2t$ is an index of substitutability between networks. The market shares of the two networks are thus $\alpha_i = \alpha$ and $\alpha_j = 1 - \alpha$. Then, market shares depend both on local prices, as in LRT, and on one-way access charges; in particular, it results:

$$\frac{\partial \alpha}{\partial p_i^{loc}} = -\sigma q(p_i^{loc}) \qquad \frac{\partial \alpha}{\partial p_i^{ld}} = -\sigma Q_i(p_i^{ld}) \qquad \frac{\partial \alpha}{\partial \tilde{a}_i} = \sigma\beta Q_i \frac{\partial Q_i}{\partial \tilde{a}_i}$$

In addition, denote with $\eta_\alpha = \frac{\partial \alpha}{\partial Q_i}\frac{Q_i}{\alpha}$ the elasticity of the market share with respect to the global quantity of long distance minutes; this term shows the impact of a variation in the quantity Q_i on the market share. Let assume that, in equilibrium, $\eta_\alpha > 1$. For any

[3] This implies that the coefficient γ in the demand function is greater than the marginal cost supported by the long distance operators (both the independent and integrated ones) in providing the long distance services to customers.

fixed values of the price variables, this assumption is needed to assure the concavity of the profit function of network operator *i* with respect to $\tilde{a}_i$.

Timing of the Game. The timing of the model is the following:

Step 1. The two-way access charges, as in LRT, is exogenously fixed by a negotiation between the networks.

Step 2. Network operators *i* compete with each other and set both local calls prices, p_i^{loc} and p_j^{loc}, and the one-way access charges, $\tilde{a}_i$ and $\tilde{a}_j$, that maximize non-cooperatively their own profits.

Step 3. Customers decide what networks they want to be connected to; this defines the market share of each network in the upstream (local) sector.

Step 4. Long distance operators, network *i* and network *j* decide simultaneously quantities offered to customers of network *i* and *j*.

3. Competition in the downstream market

Solving backwards, we first determine the equilibrium in the long distance market for customers of network *i*, given $\tilde{a}_i$ and a, both in case of integration and separation of local operators in the downstream market.

3.1. Vertical Integration

The profit of a generic long distance operator *k*, *k* = 1,..., *n*-2, is:

$$p_k^{ld} = a\left[p_i^{ld}(Q_i) - (\tilde{a}_i + c)\right]q_{ki} + (1-a)\left[p_j^{ld}(Q_j) - (\tilde{a}_j + c)\right]q_{kj} \quad \text{with } k = 1,..,n-2$$

where a is the market share of network *i* in the "subscription" market, q_{ki} and q_{kj} the quantity offered by the long distance operator to the customers connected with network *i* and *j*, respectively. The profit of the network *i*'s subsidiary in the downstream sector is (similarly for network *j*):

$$\tilde{p}_i^{ld} = a\left[p_i^{ld}(Q_i) - (c_0 + \tilde{c})\right]\tilde{q}_{ii} + (1-a)\left[p_j^{ld}(Q_j) - (\tilde{a}_j + \tilde{c})\right]\tilde{q}_{ij}$$

The optimal quantities of long distance calls offered to customers connected with network *i* are given by:

$$\tilde{q}_{ii}^* = \frac{g + (n-1)(\tilde{a}_i + c) + \Delta c - n(c_0 + \tilde{c})}{b(n+1)} \qquad (1)$$

$$q_{ji}^* = \frac{g - 2(\tilde{a}_i + c) - n\Delta c + (c_0 + \tilde{c})}{b(n+1)} \qquad (2)$$

$$q_i^* = \frac{g - 2(\tilde{a}_i + c) + \Delta c + (c_0 + \tilde{c})}{b(n+1)} \quad (3)$$

The total quantity supplied downstream to all network *i*'s subscribers is:

$$Q_i^* = \frac{n\,g - (n-1)(\tilde{a}_i + c) - \Delta c - (c_0 + \tilde{c})}{(n+1)b} \quad (4)$$

Similar expression can be derived for network *j*. Notice that the one way access charge that allows network *i* to become the only supplier of long distance calls for its subscribers (so that it results $\tilde{q}_{ii} = Q_i^*$) would be:

$$\tilde{a}_i^{for} = p^{ld^M} - c - \frac{\Delta c}{n-1} \quad (5)$$

where $p^{ld^M} = [\,g + (c_0 + \tilde{c})]/2$ is the monopoly price. It's possible to compare condition (5) with the well-known *Efficient Component Pricing Rule* (ECPR), also known as the Baumol-Willig rule, the imputation rule or the parity-pricing formula.[4] Synthetically, the ECPR implies that the access charge should be equal to the difference between the final price and the marginal cost on the competitive segment. Applying this definition in an unregulated environment, the access charge becomes equal to $\tilde{a}_i^{ECPR} = p^{ldM} - c$. Then, if $n \to \infty$ or $\Delta c \to 0$ the last term in condition (5) tends to zero; therefore, the access charge that forecloses the submarket of long distance calls originated by network *i*'s subscribers corresponds to the one deriving from the application of the ECPR in an unregulated environment. More generally, if both network operators set the access charge according to condition (5) no independent long distance firm will operate in the market and only the network firms, directly or through a subsidiary, will supply long distance calls. Eventually, the market is characterized by a duopoly competition, both in local and long distance sector, between two vertically integrated firms.

3.2. Vertical Separation

Since networks *i* and *j* are not integrated downstream, no firm has any cost advantages as in the previous case. The quantity offered by each of long distance operators to customers of network *i* is the usual optimal solution in the Cournot game with *n* firms. Thus, the optimal quantity supplied by each operators to customers of network *i* results equal to $q_i^{s*} = \frac{g - (\tilde{a}_i^s + c)}{(n+1)b}$, where the superscript *s* stands for "separation case". The corresponding global quantity is $Q_i^{s*} = \frac{g - (\tilde{a}_i^s + c)}{b}\frac{n}{n+1}$.

[4] The rule was originally introduced by Willig (1979) and Baumol (1983). More recently, it has been popularized by Baumol and Sidak (1994a and 1994b) and developed by Armstrong *et al.* (1996). For a critical view, see Kahn e Taylor (1994), Economides and White (1995).

3.3. *The benchmark case*

Suppose that there is no local competition between networks and let us first determine the optimal one way access charge that an integrated local network would set in an unregulated context. We hence turn back to the usual one way bottleneck problem in which there is only one network – say network *i* – which controls the only essential facility necessary to complete a long distance call.

In case of *vertical integration*, total profit of network *i* is given by the sum of retail profit of long distance calls, $\tilde{p}_{ii}^{ld} = b(\tilde{q}_{ii}^{*})^{2}$, interconnection profit, that is the revenues deriving from giving rivals access to its network, $I(\tilde{a}_i) = (\tilde{a}_i - c_0)\left(Q_i^{*} - \tilde{q}_{ii}^{*}\right)$ and retail profit deriving from supplying local calls. However, since this last term does not depend on access charge $\tilde{a}_i$, without loss of generality, we can ignore it. In symbols, we have:

$$\tilde{p}_i^{T}(\tilde{a}_i) = b(\tilde{q}_{ii}^{*})^{2} + (\tilde{a}_i - c_0)\left(Q_i^{*} - \tilde{q}_{ii}^{*}\right) \qquad (6)$$

The optimal one-way access charge derives from the maximization of (6) with respect to $\tilde{a}_i$. When long distance operators face the same marginal costs (Δc = 0), the optimal solution is the following:

$$\tilde{a}_i^{B} = c_0 + \frac{g - (c_0 + c)}{2} = p^{ld^{M}} - c \qquad (7)$$

where the superscript *B* stands for “benchmark”. As shown before, only the subsidiary of network operator remains in the market with this access tariff charging customers with a monopolistic retail tariff. The result is not surprising: in an unregulated environment, an integrated firm which controls an essential facility always has the incentive to foreclose potential rivals and extend its market power also to the downstream sector of the industry. The results change when we consider the case in which independent long distance operators are more efficient (i.e. $\Delta c > 0$). In this case, the optimal access charge becomes:

$$\tilde{a}_i^{B} = \tilde{a}_i^{for} - \frac{n-2}{n+3}\Delta c \qquad (8)$$

Condition (8) states that optimal access charge is lower than the one which forecloses the market; in particular, the higher the difference in costs between independent and integrated long distance operators the lower the one-way access charge.

In case of *vertical separation*, the total profit of network operator *i* is only given by the interconnection profit coming from access services supplied to the long distance operators, that is $\tilde{p}_i^{Ts}(\tilde{a}_i^{s}) = (\tilde{a}_i^{s} - c_0)Q_i^{s*}$. The maximization of profit gives the following condition:

$$\tilde{a}_i^{sB} = c_0 + \frac{g-(c_0+c)}{2} \tag{9}$$

which is equal to condition (7) derived in presence of integration. However, in this case the optimal access tariff does not foreclose the long distance market.[5] Since the network operator is vertically separated, it would have no economic sense to set an access charge that shuts down all the downstream market.

4. Competition between vertically integrated networks

According to the assumption of the model previously described, the aggregate expression for the profit of network *i* is:

$$p_i = a\left[\left(p_i^{loc} - c^{loc}\right)q(p_i^{loc}) - f + I(\tilde{a}_i) + \tilde{p}_{ii}^{ld}(\tilde{a}_i)\right] + a(1-a)\left[q(p_j^{loc}) - q(p_i^{loc})\right](a - c_0) + \\ +(1-a)\left[p_j^{ld}\left(Q_j(\tilde{a}_j)\right) - \left(\tilde{a}_j + \tilde{c}\right)\right]\tilde{q}_{ij}(\tilde{a}_j) \tag{10}$$

where:

- $p_i^{loc} = \left(p_i^{loc} - c^{loc}\right)q(p_i^{loc}) - f$ is the *retail profit* per customer deriving from local calls;
- $I(\tilde{a}_i) = (\tilde{a}_i - c_0)\left(Q_i - \tilde{q}_{ii}\right)$ is the *interconnection profit* per customer resulting from providing the one-way access to the long distance operators;
- $\tilde{p}_{ii}^{ld}(\tilde{a}_i) = \tilde{q}_{ii}\left[p_i^{ld} - (c_0 + \tilde{c})\right]$ is the *downstream profit* per customer deriving from the provision of long distance calls to own customers;
- $a(1-a)\left[q(p_j^{loc}) - q(p_i^{loc})\right](a - c_0)$ is the *access revenues* (or deficit) which depends on the balancing between off-net and on-net calls;
- $\tilde{p}_{ij}^{ld}(\tilde{a}_j) = \left[p_j^{ld} - \left(\tilde{a}_j + \tilde{c}\right)\right]\tilde{q}_{ij}(\tilde{a}_j)$, which does not depend on $\tilde{a}_i$, is the downstream profits deriving from supplying long distance calls to customers subscribing to rival network *j*.

We assume that, in equilibrium, $\Delta\tilde{p}^{ld}(\tilde{a}_i^*, \tilde{a}_j^*) = \tilde{p}_{ii}^{ld}(\tilde{a}_i^*) - \tilde{p}_{ij}^{ld}(\tilde{a}_j^*) \geq 0$, that is the downstream profit that network *i* obtains from providing long distance calls to its customers is greater than the one it can obtain from providing long distance calls to customers of the rival network. This assumption is needed for the existence and the uniqueness of the equilibrium. However, this assumption seems to be reasonable: in a symmetric equilibrium, if network *i* wants to offer long distance calls to customers of network *j*, it must pay the same marginal cost of terminating and transporting the call as the one paid by network *j*, while the latter does not pay the one-way access charge as the network *i* does. Then, network *i* faces cost disadvantages in supplying its services to the

[5] It is easily shown that global quantity Q_i^{s*} derived in the previous section (3.2) is still positive once the access charge in (9) is applied.

customers subscribed to network *j* and so its profits might be lower in this segment of the downstream market.

4.1. Optimal solutions

Following LRT, we look for a symmetric equilibrium. Network *i* must maximize p_i with respect to p_i^{loc} and $\tilde{a}_i$. At a symmetric equilibrium $p_i^{loc} = p_j^{loc} = p*$, $\tilde{a}_i = \tilde{a}_j = \tilde{a}^*$, a = 1/2. We have the following:

Proposition 1

a) The only candidate symmetric price for local calls is the one which satisfies the following condition:

$$\frac{p^* - \left(c^{loc} + \dfrac{a - c_0}{2}\right)}{p^*} = \frac{1}{h}\left(1 - 2s\left[p^{loc*} + I^* + \Delta\tilde{p}^{ld*}\right]\right) \tag{11}$$

which exists only if *a* and/or s are not too large.

b) The optimal price for local calls increases with *a*: the two-way access charge can be used, as in LRT, as an instrument of tacit collusion.

Proof. See Appendix I.

The main conclusions on the equilibrium price for local calls are qualitatively the same as in LRT and the collusive effect of the reciprocal two-way access charge is confirmed. High interconnection charges sustain collusion by making price cutting very costly: if a firm deviates from collusion and lowers its price, it will induce customers to originate many calls. This results in a net outflow of calls with corresponding expensive interconnection payments to the rival network. Then, since lowering the access charge generates an access deficit at the margin, the incentive to lower the local retail price is reduced and competition is weakened.

Condition (11) suggests that if there is no competition in the local area (i.e. $s = 0$), the networks set $a = c_0$, in order to avoid problem of double marginalization, and the upstream price is equal to the monopoly case, i.e. $p^* = p^M$, where $p^M = (h/h-1)c^{loc}$. Conversely, if there is competition for customers (i.e. $s > 0$) the optimal price is lower than the one in LRT because of the tougher competition for market share related to the profitability of the downstream sector in terms of interconnection profit and retail profit. Thus, the presence of downstream competition influences the way local competition is carried on by network operators: the unit profit per customer is higher in this case respect to the "pure" two-way situation and this makes local competition more intense.

The next proposition shows how network operators can implement monopoly prices on network services using the reciprocal access charge *a*.

Proposition 2

The monopoly network access charge, denoted with a^M, which implements the monopoly price on local calls p^M exceeds the monopoly access charge of a "pure" two-way bottleneck situation considered in LRT.

<u>*Proof*</u>. Using the monopoly condition $\frac{p^M - c^{loc}}{p^M} = \frac{1}{h}$, from equation (11) we obtain:

$$\frac{a^M - c_0}{2} = \frac{2\, s p^M}{h}\left[p^{loc^M} + I^* + \Delta \tilde{p}^{ld*}\right]$$

where $p^{loc^M} = (p^M - c^{loc})q(p^M) - f$ is the monopoly profit. Thus, prices grow slower respect to the "pure" two-way bottleneck situation. ■

The higher the interconnection and retail profits on the downstream market, the greater the mark up on the reciprocal access charge and so the lower the final prices for local calls. For a given a larger than c_0, local prices are thus always lower than in LRT.

4.2. Determination of the one way access charge

Let us focus on the one-way access charges. We have the following:

Proposition 3

a) The symmetric one-way access charge ($\tilde{a}_i = \tilde{a}_j = \tilde{a}^*$) that maximizes the total profit of network i is the following:

$$\frac{\tilde{a}_i^* - c_0}{p_i^{ld}} = \frac{1}{2h_{ld}}\left\{\frac{n+1}{n-1} + \frac{n-3}{n-1}\frac{\tilde{q}_{ii}^*}{Q_i^*} - 2\, s\left[p^{loc*} + I^* + \Delta \tilde{p}^{ld*}\right]\right\} \qquad (12)$$

where $h_{ld} = -\frac{\partial Q_i}{\partial p_i^{ld}}\frac{p_i^{ld}}{Q_i}$ is the demand elasticity of long distance calls. After manipulations, condition (12) can be rewritten as:

$$\tilde{a}_i^* = \tilde{a}^* = \tilde{a}^{for} - \frac{n+1}{n+3}\left(\frac{n-2}{n-1}\Delta c + s\, b Q_i^*\left[p^{loc*} + I^* + \Delta \tilde{p}^{ld*}\right]\right) \qquad (13)$$

where $\tilde{a}^{for}$ is the one-way access charge that allows network i to foreclose the segment of downstream market originated by its own subscribers (see condition 8).

b) The optimal one way access charge is inversely related to s.

c) An increase in the reciprocal access charge a lowers the downstream one way access charge $\tilde{a}_i$.

Proof. See Appendix I.

From the analysis of Proposition 3, we can draw some relevant insights. Firstly, if the long distance operators and the networks operator are equally efficient ($\Delta c = 0$) and if competition in the local area does not exist (s = 0), the optimal access charge that each network operator sets is the one corresponding to the benchmark case, that is the one that forecloses the downstream market. In this case, the market is characterized by a duopoly competition, both in local and long distance sector, between two vertically integrated firms. Otherwise, if the independent long distance operators are more efficient than the networks operators ($\Delta c > 0$) and if the competition in the subscriber market is strong enough (s > 0), then the one-way access charge is lower than the one that could foreclose the downstream sector, even if the industry is still unregulated. Then, local competition pushes access prices down without any regulatory intervention.

The above mentioned effects are reinforced when the number of firms in the downstream sector is large. In fact, for $n \to \infty$, equation (13) becomes:

$$\tilde{a}^* = \tilde{a}^{for} - \left(\Delta c + s\, bQ_i^* \left[p^{loc*} + I * + \Delta \tilde{p}^{ld*}\right]\right)$$

In this case the entire difference in efficiency is directly translated into lower access charge. In addition, the effect of competition in the subscribers' market is also stronger.

The most important results deriving from Proposition 3 are the relationships between s and $\tilde{a}_i$ and between a and $\tilde{a}_i$. First of all, the more intense is the local competition between network operators, the lower is the one-way access charge set by each network operator. Intuitively, customers will subscribe to the network that can provide both local and long distance calls (directly or through interconnection agreement with other firms) at the lowest price. If the local competition is tough, it means that customers can easily switch from one supplier to the other according to the retail prices offered to them. Then, since lowering the one way access charge reduces the retail price of long distance calls, each network operator is induced to set a low one way access charge in order to attract more customers to its own network. For these reasons, when local competition is strong (s > 0), a reduction in the one way access charge has a positive effect on the subscribers' market share and it enhances competition also in the long distance sector. The result formally confirms what suggested in a qualitatively way by Armstrong (1997), who wrote that "a possible benefit from local competition concerns its potentially desiderable effect on other vertically related markets, as long distance telecommunication". Once a firm controls the point of access of a customer to the network, it has also the incentive to control, directly or indirectly, other services consumed by the same customer who generally prefers to receive all services from and paying a bill to a single supplier, i.e. using the so called one-stop shop. Then, local competition enhances competition also in the other vertically related markets, such as the long distance sector, with substantial benefits for consumers in welfare terms.

Secondly, the reciprocal access charge, that can be used by the networks as an instrument of tacit collusion, influences the level of the downstream (one-way) access charge. Intuitively, if we fix $\tilde{a}_i = \tilde{a}^*$, an increase in the two-way access charge *a* raises the retail price of local calls (p^*), as shown in Proposition 1. Thus, as long as $p^* \leq p^M$, the retail profits in the local area increases. As retail profits per customer are now higher, competition for market share gets tougher. The higher profitability of the local market leads network operators to lower their downstream access charge in order to attract more customers. Obviously, this decreases interconnection and retail profit from the downstream activities, but this reduction will always be smaller than the increase in the retail profits of the local market due to the larger market share. Eventually, total profits could reach their maximum value when retail profits in the local market are at their monopoly level, that is the reciprocal access charge is set at the value shown in Proposition 2. Thus, downstream access charges could be at their lowest level as the profitability of the local market increases.

Finally, it is also possible to compare condition (13) with the *Efficient Component Pricing Rule* (ECPR). It results that $\tilde{a}_i^{ECPR} \geq \tilde{a}_i^*$ if $\Delta c > 0$ and/or $s > 0$. Then, the global quantity in the downstream market derived here is larger than under the ECPR.

4.3. The separation case

Suppose now that each network is not integrated downstream. Network operator *i* maximizes its profit given by:

$$p_i = a\left[\left(p_i^{loc} - c^{loc}\right)q(p_i^{loc}) - f + I(\tilde{a}_i^s)\right] + a\,(1-a)\left[q(p_j^{loc}) - q(p_i^{loc})\right](a - c_0)$$

where as before $I(\tilde{a}_i^s)$ is the interconnection profit. We have the following

Proposition 4

a) In the case of separation, the optimal symmetric local price, is:

$$\frac{p^s * - \left(c^{loc} + \dfrac{a - c_0}{2}\right)}{p^s *} = \frac{1}{h}\left(1 - 2\,s\left[p^{s^{loc*}} + I^s *\right]\right) \qquad (14)$$

b) The optimal symmetric one-way access charge is given by:

$$\frac{\tilde{a}_i^{s*} - c_0}{p_i^{ld}} = \frac{1}{h_{ld}}\left\{\frac{n+1}{n} - 2s\left[p^{sloc*} + I^s *\right]\right\} \qquad (15)$$

and after some manipulations:

$$\tilde{a}_i^{s*} = \tilde{a}^{s*} = \frac{g+(c_0+c)}{2} - c - s\,bQ_i^{s*}\left[p^{sloc*} + I^{s}*\right] \tag{16}$$

where { g+ (c_0+c)}/2 is the monopoly price in the downstream sector.

Proof. See the Proof for proposition 1 and 3.

The intuition behind Proposition 4 is very similar to the analysis done in the previous section. Unfortunately, it is difficult to make a comparison between the above optimal conditions with the optimal solutions derived in the case of integration. Since both the local tariff and the one-way access charge are endogenous variables in the maximization problem, the level of one variable influences the optimal value of the other. Fortunately, a more precise comparison can be carried out in the following sections where we assume that local competition is in two-part tariffs.

5. Competition in two-part tariffs

Suppose that network operators can offer to customers its services using non linear tariffs. Since the customers' demand function is known, networks cannot do better than offering two-part tariffs:

$$T_t(q) = F_t + p_t^{loc}q, \quad t = i,j$$

where the fixed fee F_t can be interpreted, as in LRT, as a subscriber line charge and p_t^{loc} as the marginal price for a local calls or usage fee.

Using the above notation, the net surplus offered to network' t consumers is now:

$$w_t = v(p_t^{loc}) + v^{ld}(p_t^{ld}(Q_t(\tilde{a}_t)) - F_t\ , \quad t = i,j$$

Thus, network i's market share is determined by the net surplus:

$$a_i = a(p_i^{loc}, p_j^{loc}, F_i, F_j, \tilde{a}_i, \tilde{a}_j) = \frac{1}{2} + s\left[w_i - w_j\right]$$

In case of integration, network i's profit becomes:

$$p_i = a\left[\left(p_i^{loc} - c^{loc}\right)q(p_i^{loc}) - f + F_i + I(\tilde{a}_i) + \tilde{p}_{ii}^{ld}(\tilde{a}_i)\right] + a(1-a)\left[q(p_j^{loc}) - q(p_i^{loc})\right](a - c_0) +$$
$$+(1-a)\left[p_j^{ld}\left(Q_j(\tilde{a}_j)\right) - \left(\tilde{a}_j + \tilde{c}\right)\right]\tilde{q}_{ij}(\tilde{a}_j) \tag{17}$$

In addition, suppose that in the downstream market all the competitive firms are equally efficient (i.e. $Dc = 0$); we have the following:

Proposition 5

a) When the degree of substitutability (s) or the access markup ($a - c_0$) is not too high, a symmetric equilibrium exists in non linear tariffs.

b) A network's optimal usage fee is its perceived marginal cost; for network i we have $p_i^{loc} = c^{loc} + \alpha_j(a - c_0)$. So at a symmetric equilibrium:

$$p^* = p_i^{loc} = p_j^{loc} = c^{loc} + \frac{a - c_0}{2} \tag{18}$$

c) The symmetric equilibrium subscriber fee F^* is equal to:

$$F^* = \frac{1}{2s} - q(p^*)\left(\frac{a - c_0}{2}\right) + f - I(\tilde{a}^*) - \Delta\tilde{p}^{ld} \tag{19}$$

that is, the "Hotelling markup" ($1/2s$) plus the net marginal cost of adding a subscriber to the local network ($f - q(p^*)\frac{a - c_0}{2}$) minus the per unit profit deriving from downstream activities (retail and interconnection).

d) The symmetric one-way access charge is given by:

$$\tilde{a}_i = \tilde{a}_j = \tilde{a}^* = c_0 + \frac{3\left[g - (c_0 + c)\right]}{n + 7} \tag{20}$$

e) As in LRT, the symmetric equilibrium profit is independent of the two-way access charge a and it is equal to:

$$p^* = \frac{1}{4s} + \frac{1}{2}\tilde{p}_{ij}^{ld}(\tilde{a}^*) = \frac{1}{4s} + \frac{(g - c_0 - c)^2}{b(n + 7)^2} \tag{21}$$

Proof. See Appendix II.

As in LRT, the equilibrium profit is independent of the two-way access charge. An increase in the two-way access charge leads to an increase of the retail usage fee, and so to a decrease in the customers' level of consumption, but does not affect the operators' profit. Then, the two-way access charge can no longer be used as an instrument of tacit collusion as in the linear case. Networks can build market share using the fixed fee and not the marginal retail tariff. Competition is thus more intense than in the case of linear tariff. In particular, the symmetric fixed fee is lower than the one suggested in LRT according to the profitability of the downstream activities (both in the retail and in the interconnection segment). Intuitively, since networks can offer both local and long distance calls, they have an incentive to decrease their fixed fee in order to attract more customers; as shown before, if a network operator control the access to a customer, it knows that it could extend its control also to all the other complementary services consumed by its customers. Thus, local competition is even stronger here than in the “pure” network competition model. Finally, note that the equilibrium profit is higher than the one obtained in LRT and, in particular, it is equal to the profit that would be obtained under unit demands plus a term related to the profit deriving from providing long distance calls to customers connected with the rival network.

The optimal one-way access charge, given by condition (20), does no longer depend on the level of substitutability s as in the linear case. However, tougher competition in the local area has a relevant impact on the way the downstream access charge is set. In order to illustrate this statement, let us compare condition (20) with condition (7) which defines the optimal unregulated one way access charge that the local network operator would set in absence of local competition. We obtain:

$$\tilde{a}^* - \tilde{a}_i^B = -\frac{1}{2}\frac{(n+1)(g-(c_0+c))}{n+7} < 0 \qquad (22)$$

From condition (22) it results that the optimal one way access charge derived in a contest of competition between networks is always lower than the one way access charge that an unregulated network operator sets in absence of local competition. Thus, in presence of competition in the local area, the access charge is lower than the foreclosure level, even if the industry is not regulated.

5.1. The separation case

In case of separation, network *i*’s profit becomes:

$$p_i = a\left[\left(p_i^{loc} - c^{loc}\right)q(p_i^{loc}) - f - F_i^s + I(\tilde{a}_i^s)\right] + a(1-a)\left[q(p_j^{loc}) - q(p_i^{loc})\right](a - c_0)$$

The maximization of the above condition leads to the following:

Proposition 6

The equilibrium, which always exists when the degree of substitutability (s) or the access markup ($a - c_0$) are not too high, is given by the following set of equations:

- for the optimal usage fee:

$$p^{S*} = p_i^{loc} = p_j^{loc} = c^{loc} + \frac{a - c_0}{2} \qquad (23)$$

- for the optimal subscriber fee $F*$:

$$F^{s*} = \frac{1}{2s} - q(p*)\left(\frac{a-c_0}{2}\right) + f - I(\tilde{a}^{s*}) \qquad (24)$$

- for the optimal one-way access charge:

$$\tilde{a}_i^s = \tilde{a}_j^s = \tilde{a}^{s*} = c_0 + \frac{g-(c_0+c)}{n+2} \qquad (25)$$

Thus, the symmetric equilibrium profit is equal to $p^{s*} = \frac{1}{4s}$.

Proof. See the Proof for Proposition 5.

The main intuitions are identical to the one drawn in the integration case and for these reasons we don't go further into details. However, notice that the equilibrium profit is lower than the one that can be obtained by a vertically integrated local operator. Besides, comparing the optimal one way access charge in (25) with the access charge in (9) obtained in absence of local competition between network operators, we have:

$$\tilde{a}^{s*} - \tilde{a}_i^{sB} = -\frac{n}{2(n+2)}(g-c_0-c) < 0 \qquad (26)$$

as in case of integration, the optimal access charge in presence of competition in local area is lower that the interconnection tariff in absence of local competition.

5.2. Integration versus separation: a welfare comparison

It is now possible to make a welfare comparison among the two different configurations of the market in order to investigate what is the best market structure from a social point of view. Firstly, let's compare the level of the optimal one-way access charges in (20) and (25) obtained in case of integration and separation, respectively. In symbols, it results:

$$\tilde{a}^* - \tilde{a}^{s*} = \frac{(2n-1)(g-c_0-c)}{(n+7)(n+2)} > 0 \qquad (27)$$

Then, the one way access charge in case of integration is always higher than the one derived in case of separation. The result is related to the competitive advantage of an integrated operator that faces competition of non integrated firms. In an unregulated environment, an integrated network operators always tend to raise rivals' costs and put them at a disadvantage in order to extend their market power to the downstream competitive sector. For these reasons, fear of possible abuses of the vertically integrated local network asks for regulatory interventions in the telecommunications market even if the market is more competitive.

Concerning the level of the fixed fees, after the necessary manipulations, we obtain:

$$F^{*} - F^{s*} = -\frac{(g - c_0 - c)^2}{b}\left[\frac{2n^3 + 10n^2 + 11n + 48}{(n+2)^2(n+7)^2}\right] < 0 \qquad (28)$$

The optimal fixed fee in case of integration is always lower than in case of separation. This implies that local competition is stronger if network operators are vertically integrated since they can directly provide customers with not only local but also long distance calls or some other complementary services. The reduction in the fixed fee is completely passed on to customers who benefit most from the growing competition in the local segment. However, the difference in fixed fee depends on the number of competitors in the downstream sector: when the number of long distance operators is small, and so network operators have a strong incentive to extend their market power downstream, the difference is remarkable and local competition is more intense; on the contrary, when $n \to \infty$, that is when the downstream market is perfectly competitive, the fixed fees turn out to be identical since the retail downstream profit per customers tends to zero and the downstream profits are only given, both in case of separation and integration, by the interconnection profits. Finally, note that the usage fees are always equal to the perceived marginal cost and so they remain the same in both the different market structures; then, there is no distortion on consumers' consumption of local calls.

The difference in the one-way access charge shown in (27) implies that the retail prices in the long distance market also change accordingly to the different market structure. In particular, since the downstream access charge is higher in case of integration, we have:

$$p^{ld}(\tilde{a}^{*}) - p^{ld^{s*}}(\tilde{a}^{s*}) = \frac{2(g - c_0 - c)(n-3)}{(n+2)(n+7)} > 0 \qquad (29)$$

when $n \geq 3$ the retail price in case of integration is equal or higher than the one in case of separation.[6] Thus, since the demand function of long distance calls is supposed to be linear, the loss in consumers' surplus in long distance market is given by:

$$\Delta CS^{D} = CS^{ld} - CS^{ld^{s*}} = \frac{2(g - c_0 - c)^2(n-3)^2}{b(n+2)^2(n+7)^2} \qquad (30)$$

Adding condition (28) and (30), it results that the global change in consumers' surplus is:

[6] Notice that $n = 2$ means that only the networks' subsidiaries operate in the downstream sector. Then, long distance tariffs in case of integration are lower since the double marginalization problem is avoided.

$$\Delta CS = \frac{(g - c_0 - c)^2}{b(n+2)^2(n+7)^2}\left[2n^3 + 8n^2 + 23n + 30\right] > 0$$

The reduction in fixed fee related to the more intense competition in the local segment is higher than the increase in the retail long distance tariff and so consumers' surplus is greater in case of integration than in case of separation.

In addition, since $p^* > p^{s*}$, global welfare is higher in case of integration. Local competition between vertically integrated networks leads to both higher benefits for customers and higher profits for network operators and so it results socially preferable respect to the separation case even if competition in the downstream market might be partially softened.

5.3 The asymmetric cases

By now we have taken into consideration situations which presented two full coverage network operators both vertically integrated or separated. The results show that, in case of competition in two-part tariffs, in a mature phase of the industry there is no need to regulate the two-way access charge since it can no longer be used as a collusive device by network operators. However, these conclusions could change if the competition between networks is no more symmetric because not all local operators are vertically integrated or, in the entry phase, one network has a smaller coverage than the incumbent network. Let us focus on these asymmetric cases and analyse how the level of the two-way and one-way access charges affects the "level playing field" between networks.

5.3.1. Competition between integrated and non-integrated networks

Assume that only one network, say network i, is integrated whereas the other one (say network j) isn't and let us analyse the equilibrium of the game between the two networks in case of competition in two-part tariffs. In addition, suppose that there is no difference in efficiency between long distance operators ($\Delta c = 0$). Competition in quantity between long distance operators gives the following optimal conditions:

$$\tilde{q}_{ii}^* = \frac{g + (n-1)\tilde{a}_i - nc_0 - c}{b(n+1)}, \quad q_i^* = \frac{g - 2\tilde{a}_i - c + c_0}{b(n+1)}, \quad q_j^* = \frac{g - (\tilde{a}_i + c)}{(n+1)b}$$

The global quantities supplied to customers of network i and j are respectively:

$$Q_i^* = \frac{n\,g - (n-1)\tilde{a}_i - (c_0 + nc)}{(n+1)b} \quad \text{and} \quad Q_j^* = \frac{g - (\tilde{a}_j + c)}{b}\,\frac{n}{n+1}$$

According to the above assumptions, the total profits of network operators are:

$$p_i = a_i\left[\left(p_i^{loc} - c^{loc}\right)q(p_i^{loc}) - f + F_i + I(\tilde{a}_i) + \tilde{p}_{ii}^{ld}(\tilde{a}_i)\right] + a_i a_j\left(q(p_j^{loc}) - q(p_i^{loc})\right)(a - c_0)$$
$$+ a_j \tilde{p}_{ij}^{ld}(\tilde{a}_j) \quad (31)$$

$$p_j = a_j\left[\left(p_j^{loc} - c^{loc}\right)q(p_j^{loc}) - f + F_j + I(\tilde{a}_j)\right] + a_i a_j\left(q(p_i^{loc}) - q(p_j^{loc})\right)(a - c_0) \quad (32)$$

where $a_i = 1/2 + s\left[v(p_i^{loc}) - F_i + v^{ld}(\tilde{a}_i) - v(p_j^{loc}) + F_i - v^{ld}(\tilde{a}_j)\right]$ and $a_j = 1 - a_i$.

Denoting with $A_i = (a - c_0)(q(p_j^{loc}) - q(p_i^{loc}))$ and $A_j = (a - c_0)(q(p_i^{loc}) - q(p_j^{loc})) = -A_i$, the asymmetric equilibrium game is given by the following conditions. For the vertically integrated network *i* we have:

$$p_i^{loc}* = c^{loc} + (1 - a_i)(a - c_0) \quad (33)$$
$$F_i^* = \frac{a_i}{s} - (p_i^{loc} * - c^{loc})q(p_i^{loc}*) - (1 - 2a_i)A_i + f - I(\tilde{a}_i^*) - \Delta\tilde{p}^{ld}(\tilde{a}_i^*, \tilde{a}_i^*) \quad (34)$$
$$\tilde{a}_i^* = c_0 + \frac{3(g - c_0 - c)}{n + 7} \quad (35)$$

leading to the following profit:

$$p_i^* = \frac{(a_i)^2}{s} + (a_i)^2 A_i + \tilde{p}_{ij}^{ld}(\tilde{a}_j^*)$$

For the network *j* it results:

$$p_j^{loc}* = c^{loc} + a_i(a - c_0) \quad (36)$$
$$F_j^* = \frac{a_j}{s} - (p_j^{loc} * - c^{loc})q(p_j^{loc}*) - (1 - 2a_j)A_j + f - I(\tilde{a}_j^*) \quad (37)$$
$$\tilde{a}_j^* = c_0 + \frac{(g - c_0 - c)}{n + 2} \quad (38)$$

while the profit is equal to:

$$p_j^* = \frac{(a_j)^2}{s} + (a_j)^2 A_j = \frac{(a_j)^2}{s} - (a_j)^2 A_i$$

The above conditions show that the equilibrium of the game is not symmetric because of the different vertical organization of both operators. The two local operators set different local and long distance tariffs and different one-way access charges. In particular, the usage fees are set both equal to the perceived marginal cost as before but now market shares are not the same. Finally, the one-way access charges are equal to the ones derived in the previous sections; thus, as shown in condition (27), the integrated local operators set an higher one way access charge with respect to the separated operator. This leads to a higher retail price for long distance calls for customers of network i (i.e $p_i^{ld} > p_j^{ld}$) and less minutes consumed.

The most interesting implications derive from the analysis of the equilibrium market shares which emerges in this context. The following proposition shows how the equilibrium is affected by the level of the two-way access charge.

Proposition 7

Denoting gross utility minus cost of local calls provided by network i and network j with s_i and s_j, respectively, (i.e. $s_i = v(p_i^{loc}*)+(p_i^{loc}*-c^{loc})q(p_i^{loc}*)$ and $s_j = v(p_j^{loc}*)+(p_j^{loc}*-c^{loc})q(p_j^{loc}*)$), the asymmetric equilibrium market share is given by:

$$a_i = \frac{1}{2} + \frac{s}{3}\left[s_i - s_j\right] + \frac{s}{3}\left[\frac{(g - c_0 - c)^2(4n^2 + 41n + 66)}{b(n+2)^2(n+7)^2}\right] \quad (39)$$

Thus,

1) if $a = c_0$, and so $s_i = s_j$, the subscription market share of the integrated network is higher than the one of the separated operator. The difference depends on the substitutability between networks, s, and on the number n of firms in the downstream sector.

2) If $a > c_0$, $s_i > s_j$, then the two-way access charge influence the equilibrium market share and hence the equilibrium profit of each network. In particular, the higher is the mark-up on the two-way access charge the higher results the market share of the integrated operator.

Proof. See Appendix II.

Proposition 7 states that if network operators don't have the same vertical organization in the market, competition can be influenced both by the level of the two-way access charge and the profitability of the downstream activities.

Firstly, consider the case that the two-way access charge is fixed at its lowest level, that is at the marginal cost ($a = c_0$). Even in this case, the equilibrium is no longer symmetric. Condition (39) shows that the difference in market shares depends on two main factors: the substitutability between networks, i.e. s, and the number n of firms in the downstream sector. When customers can easily switch from one network to the other, that is s is high, the difference in market share is higher. Intuitively, subscribers may place a large premium on receiving all services from and paying a bill to a single supplier which might naturally be considered to be the chosen local network. Thus, when competition in the local area is tough, the integrated network could have a competitive advantage with respect to the non-integrated firm. For this reason, the former one sets a lower fixed fee in order to attract many customers who remain "captive" once the subscription decision has been taken. Besides, the last term in (39) tends to zero when n tends to infinity. Then, when competition in the downstream sector is approximately perfect there is no difference between integrated and separated network behaviour, for $a = c_0$. Thus, equilibrium becomes symmetric once again, for $a = c_0$, even if the two networks have a different vertical structure. On the contrary, the less intense competition in the long distance sector is, the higher results the value of the third term in (39) and therefore the difference in subscription market share.

If $a > c_0$, the two-way access charge essentially changes the level playing field between the two networks. An increase in the mark up on the reciprocal access charge raises the market share and so the profit of the integrated network. Then, it is in the interest of the integrated network to insist on a high access charge.[7] Even if ex post it leads to an increase in the usage fee, and so in a reduction in the consumption level of local calls, ex ante the possibility to raise the two-way access charge may also deter the entry of potential rival especially if the connection of an additional customers to the network requires substantial fixed investment (Dessein, 1999a). Thus, contrary to the symmetric case, the level of the two-way access charge matters when the competitors are no more symmetric and it can hardly alter the playing field of network operators.

The possible abuses of the integrated network towards the rival separated network or other potential entrants through the use of acces tariffs were recently analysed by Mini (1999). Mini compares (two way) access arrangements with the RBOCs (which are vertically separated) and GTE (a vertically integrated rival company) of the AT&T attempting to enter local markets in 22 states in which both GTE and a RBOC are present. Mini shows that access arrangments are more likely to be reached and to be reached more quickly under vertical separation. As of March 1999, AT&T had failed to obtain interconnection arrangments with the Bells in only 2 of the 22 sample states, but failed with GTE in 10 of these states. In addition, the arrangments were reached first with Bell 11 times and only once with GTE. The average delay in reaching agreement is 70% longer with GTE and the pricing request of GTE on interconnection are consistently “tougher” than the one of the RBOCs.

These results confirms the difficulty of obtaining access and reaching a fair interconnection agreement in presence of a vertical integrated network. The regulatory Authority has to take into consideration the possible anticompetitive behaviour of the integrated network trying to soften competition both in local and log distance segments and so regulation turns out to be still necessary to protect new entrants.

5.3.2. Entry process

Suppose that one network operator, say network j, initially has a limited coverage, that is, in an entry phase of the industry, it cannot provide its services to all the subscribers market. However, it has to face competition in the subscriber market of the full coverage incumbent operator. In addition, both the network operators are vertically integrated downstream. Denote with $m \in [0,1]$ the coverage chosen by network j when it decides to enter the local market. Market shares are thus given by $a_i = 1 - m(1-a)$ and $a_j = m(1-a)$, where $a = 1/2 + s\left[w_i - w_j\right]$ is the market share that network i commands

[7] See also Dessein (1999a) for further considerations.

among those consumers that can be served by both networks and w_t, $t = i,j$, is the variable net surplus offered by network t to its customers.

Network i's profit becomes:

$$p_i = a_i\left[\left(p_i^{loc} - c^{loc}\right)q(p_i^{loc}) - f - F_i + I(\tilde{a}_i) + \tilde{p}_{ii}^{ld}(\tilde{a}_i)\right] + a_i a_j\left[q(p_j^{loc}) - q(p_i^{loc})\right](a - c_0) + \\ + a_j \tilde{p}_{ij}(\tilde{a}_j)$$

and similarly for network j.

The maximization of the above function leads to the following optimal conditions:

$$p_i^{loc}{*} = c^{loc} + (1\text{-}a_i)(a - c_0)\ ,\ p_j^{loc}{*} = c^{loc} + a_i(a - c_0)$$
$$F_i^* = \frac{a_i}{ms} - (p_i^{loc}{*} - c^{loc})q(p_i^{loc}{*}) + (1 - 2m(1 - a))A_i + f - I(\tilde{a}^*) - \Delta\tilde{p}^{ld}(\tilde{a}^*)$$
$$F_j^* = \frac{a_j}{ms} - (p_j^{loc}{*} - c^{loc})q(p_j^{loc}{*}) - (1 - 2m(1 - a))A_j + f - I(\tilde{a}^*) - \Delta\tilde{p}^{ld}(\tilde{a}^*)$$
$$\tilde{a}_i^* = \tilde{a}_j^* = \tilde{a}^* = c_0 + \frac{3\left(g - c_0 - c\right)}{n + 7}$$

Notice that the coverage level in the subscriber market does not influence the optimal level of the downstream (one-way) access charge that remains the same as in the symmetric case. The global profits of each network operator become:

$$p_i^* = \frac{(a_i)^2}{ms} + (a_i)^2 A_i \quad , \quad p_j^* = \frac{(a_j)^2}{ms} + (a_j)^2 A_j = \frac{(a_j)^2}{ms} - (a_j)^2 A_i$$

plus a fixed term regarding the retail downstream profit, which can be ignored, without any loss of generality. The asymmetric equilibrium market share is given by:

$$a_i = \frac{1}{2} + \frac{1 - m}{3} + \frac{ms}{3}\left[s_i - s_j\right] \qquad (40)$$

where s_i and s_j are the gross utility minus cost of local calls provided by network i and network j, respectively. The above result is equivalent to the one derived in Dessein (1999a), who firstly showed the effect of the two-way access charge in case of a "pure" horizontal competition in non-linear tariffs between two network firms with different coverage. If $m < 1$, even if a is set at marginal cost (i.e. $s_i = s_j$), market share of the full coverage network is higher than the one of the new entrant; in addition, the difference in market share raises as the mark up of two-way access charge a on marginal cost of access increases. Then, the two-way access charge increase profits of the full coverage network and lowers profits of the other. If the bargaining power is unequally distributed among network operators, the two-way access charge can be used by the full coverage network to obtain a competitive advantage. Thus, in an entry phase of the industry, the regulatory Authority has to intervene in order to level the playing field of networks and yield an efficient bargaining outcome among local operators.[8]

6. Concluding remarks

In this paper we have developed a model of network competition between two vertically integrated telecommunications operators. These operators control a local network and operate also in a long distance sector facing competition from other independent non-integrated firms, which need the (one way) access to local network as an essential facility. The analysis has been carried out in a game structure consistent with the assumption that customers' choice of long distance supplier is less flexible than their subscription decision. Once having decided which local network they want to subscribe to, they become "captive" and they cannot change their subscription decision after having observed the prices per minutes of long distance calls. We believe that this assumption might be realistic in a contest of network competition between telecommunications operators especially when customers face high switching costs if they decide to change their local providers. The main results can be summarized as follows. In a mature phase of the industry, the game between two vertically integrated networks leads to a symmetric equilibrium, where network operators share each other the subscriber market and set the same retail (local and long distance) and interconnection tariffs. In particular, the optimal one way access charge set by the integrated networks is lower than the one that could enable them to impede the entry of potential rivals in the downstream sector even if the market is assumed to be unregulated. Then, local competition pushes access prices down without any regulatory intervention and it enhances competition also in the long distance market lowering retail prices. However, the optimal one way access charge in case of integration is higher than the optimal one in case of separation. Thus, the regulatory intervention on access prices could still be required in order to prevent the vertically integrated networks from softening competition in the long distance. As in LRT, the two-way access charges can still be used as a collusive device only if competition is in linear tariffs but not if competition is in two-part tariffs. However, also in this case, if network operators are not symmetric, two-way access charge still play a crucial role in the market. In fact, the level playing field among networks can be altered in favor of an integrated firm or of a network with higher coverage which would have an incentive to raise the access charge in order to increase the rival's cost and put the rival at a disadvantage. Thus, in the entry phase of the industry, regulation of the (two-way and one-way) access charges is still necessary in order to avoid abuses and to create a more competitive environment.

[8] For further considerations and a formal proof, see Dessein (1999a).

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APPENDIX I

Proof of Proposition 1 and Proposition 3

Explanation of condition (8)

Using the same Proposition 1 from LRT it can be shown that no equilibrium exists for s large, just replace f by $f' = f - \left[I^* + \Delta \tilde{p}^{ld*}\right]$. Differentiating global profit with respect to p_i^{loc}, we have:

$$\frac{\partial p_i}{\partial p_i^{loc}} = a\left[q(p_i^{loc}) + \left(p_i^{loc} - c^{loc}\right)\frac{\partial q}{\partial p_i^{loc}}\right] - sq(p_i^{loc})\left[p^{loc} + I + \Delta\tilde{p}^{ld}\right]$$

$$-a(1-a)(a-c_0)\frac{\partial q}{\partial p_i^{loc}} + (1-2a)\frac{\partial a}{\partial p_i^{loc}}\left[q(p_j^{loc}) - q(p_i^{loc})\right](a-c_0) = 0 \qquad \text{(I.1)}$$

In a symmetric equilibrium, $p_i^{loc} = p_j^{loc} = p^*$, $\tilde{a}_i = \tilde{a}_j = \tilde{a}^*$, $a = 1/2$ and $d[a(1-a)]/da = 0$; thus, the above condition can be rewritten:

$$\left.\frac{\partial p_i}{\partial p_i^{loc}}\right|_{\substack{p_i^{loc} = p_j^{loc} = p^* \\ \tilde{a}_i = \tilde{a}_j = \tilde{a}^*}} = \frac{1}{2}\left[q(p_i^{loc}) + \left(p_i^{loc} - c^{loc}\right)\frac{\partial q}{\partial p_i^{loc}}\right] - sq(p_i^{loc})\left[p^{loc^*} + I^* + \Delta\tilde{p}^{ld*}\right]$$

$$-\frac{1}{4}(a-c_0)\frac{\partial q}{\partial p_i^{loc}} = 0$$

$$\text{(I.2)}$$

or, denoting with $h = -\frac{\partial q}{\partial p_i^{loc}}\frac{p_i^{loc}}{q}$ the demand elasticity of local calls:

$$\left.\frac{\partial p_i}{\partial p_i^{loc}}\right|_{\substack{p_i^{loc} = p_j^{loc} = p^* \\ \tilde{a}_i = \tilde{a}_j = \tilde{a}^*}} = \frac{1}{2}\frac{q(p^*)}{p^*}\left\{(h-1)\left(\frac{h}{h-1}c^{loc} - p^*\right) + h\frac{a-c_0}{2} - 2sp^*\left[p^{loc*} + I^* + \Delta\tilde{p}^{ld*}\right]\right\} = 0$$

$$\text{(I.3)}$$

After manipulations, we obtain condition (8). ■

Relation between p and a*

Since in a symmetric equilibrium $\Delta\tilde{p}^{ld*} \geq 0$, it is straightforward to show that $p(.)$ increases with a. Indeed, applying the implicit function theorem to (I.3), we have:

$$\frac{\partial p^*}{\partial a} = \frac{h/2}{h - 1 + 2s\left(p^{loc*} + I^* + \Delta\tilde{p}^{ld*} + p^* p'(p^*)\right)}$$

$\partial p^*/\partial a$ is positive since $p(p) \geq 0$ (non-negativity condition), $p'^{loc}(p^*) \geq 0$ (i.e. $p^* \leq p^M$), where $p^M = \frac{h}{h-1} c^{loc}$ is the monopoly price in the local segment, and $\Delta\tilde{p}^{ld*} \geq 0$. So for a large enough, $\partial p^*/\partial a$ will be positive and bounded away from zero, implying that p^* goes to infinity with a. But then for a large and thus p^* high enough, each network could corner the market and get the full monopoly profit cutting its price; so no equilibrium exist for a large. ■

Explanation of condition (10)

The F.O.C with respect to $\tilde{a}_i$ is the following:

$$\frac{\partial p_i}{\partial \tilde{a}_i} = a\left\{Q_i^* - \tilde{q}_i^* + (\tilde{a}_i - c_0)\frac{\partial\left[Q_i^* - \tilde{q}_i^*\right]}{\partial \tilde{a}_i} + \frac{\partial p_{ii}^{ld}}{\partial \tilde{a}_i}\right\} + \frac{\partial a}{\partial \tilde{a}_i}\left[p^{loc} + I + \Delta\tilde{p}^{ld}\right]$$

$$+ (1 - 2a)\frac{\partial a}{\partial \tilde{a}_i}\left[q(p_j^{loc}) - q(p_i^{loc})\right] = 0 \qquad \text{(I.4)}$$

In a symmetric equilibrium, $p_i^{loc} = p_j^{loc} = p^*$, $\tilde{a}_i = \tilde{a}_j = \tilde{a}^*$, $a = 1/2$ and $d\left[a(1-a)\right]/da = 0$; thus, condition (I.4) can be rewritten:

$$\frac{\partial p_i}{\partial \tilde{a}_i}\Big|_{\substack{p_i^{loc} = p_j^{loc} = p^* \\ \tilde{a}_i = \tilde{a}_j = \tilde{a}^*}} = Q_i^* - \tilde{q}_i^* + (\tilde{a}_i - c_0)\frac{\partial\left[Q_i^* - \tilde{q}_i^*\right]}{\partial \tilde{a}_i} + \frac{\partial p_{ii}^{ld}}{\partial \tilde{a}_i} + 2sbQ_i^*\frac{\partial Q_i^*}{\partial \tilde{a}_i}[\Pi] = 0 \qquad \text{(I.5)}$$

where $\Pi = p^{loc*} + I^* + \Delta\tilde{p}^{ld*}$ is evaluated at the equilibrium. Since:

$$\frac{\partial\left[Q_i^* - \tilde{q}_i^*\right]}{\partial \tilde{a}_i} = \frac{\partial\left[(n-2)q_i^* + \tilde{q}_{ji}^*\right]}{\partial \tilde{a}_i} = -\frac{2}{b(n+1)}(n - 2 + 1) = -\frac{2(n-1)}{b(n+1)}$$

$$\frac{\partial p_{ii}^{ld}}{\partial \tilde{a}_i} = 2b\tilde{q}_{ii}^*\frac{\partial \tilde{q}_{ii}^*}{\partial \tilde{a}_i} = 2b\tilde{q}_{ii}^*\frac{n-1}{b(n+1)} = 2\tilde{q}_{ii}^*\frac{n-1}{n+1}$$

we obtain after manipulations:

$$\frac{\tilde{a}_i^* - c_0}{p_i^{ld}} = \frac{1}{2h_{ld}}\left\{\frac{n+1}{n-1} + \frac{n-3}{n-1}\frac{\tilde{q}_{ii}^*}{Q_i^*} - 2s[\Pi]\right\} \qquad \text{(I.6)}$$

that is condition (9) in Proposition 3, where $h_{ld} = -\frac{\partial Q_i}{\partial p_i^{ld}} \frac{p_i^{ld}}{Q_i}$ is the demand elasticity of long distance calls. Using the elasticity formula, we have:

$$\tilde{a}_i^* - c_0 = \frac{b}{2}\frac{n+1}{n-1}Q_i^* + \frac{b}{2}\frac{n-3}{n-1}\tilde{q}_{ii}^* - sbQ_i^*[\Pi] \qquad \text{(I.7)}$$

Substituting (3) and (6) into (I.7) we have:

$$\tilde{a}_i^* - c_0 = \frac{b}{2}\frac{n+1}{n-1}\left[\frac{n\,g-(n-1)(\tilde{a}_i + c) - \Delta c - (c_0 + \tilde{c})}{(n+1)b}\right]$$
$$+ \frac{b}{2}\frac{n-3}{n-1}\left[\frac{g+(n-1)(\tilde{a}_i + c) + \Delta c - n(c_0 + \tilde{c})}{b(n+1)}\right] - sbQ_i^*[\Pi]$$

We obtain:

$$2(n+3)\tilde{a}_i^* = c_0(n+3) + (n+3)\,g - (n-1)\tilde{c} - 4c - \frac{4\Delta c}{n-1} - 2(n+1)\,sbQ_i^*[\Pi]$$

Adding and substracting $3\tilde{c}$ and $(n+3)\tilde{c}$ it results:

$$\tilde{a}_i^* = \frac{g+(c_0+\tilde{c})}{2} - \frac{2(n+3)\tilde{c} - 4\tilde{c} + 4c}{2(n+3)} - \frac{4\Delta c}{2(n-1)(n+3)} - \frac{n+1}{n+3}\,s\,bQ_i^*[\Pi]$$

Once again, adding and substracting $2(n+3)c$ to the numerator of the second element in the RHS of the above formula, we have:

$$\tilde{a}_i^* = \frac{g+(c_0+\tilde{c})}{2} - c - \frac{n+1}{n+3}\Delta c - \frac{4\Delta c}{2(n-1)(n+3)} - \frac{n+1}{n+3}\,sbQ_i^*[\Pi]$$

that is, after manipulations, condition (10) defined in Proposition 3. ■

Second order sufficient condition

To analyse if our candidate equilibrium, represented by the set of equations in (8) and (10), can be a local maximum, we study the Hessian matrix of our optimization problem, evaluated at the equilibrium.

The Hessian matrix is the following:

$$H\Big|_{\substack{p_i^{loc}=p_j^{loc}=p^* \\ \tilde{a}_i=\tilde{a}_j=\tilde{a}^*}} = \begin{bmatrix} \partial^2 p_i \big/ \partial (p_i^{loc})^2 & \partial^2 p_i \big/ \partial p_i^{loc}\partial \tilde{a}_i \\ \partial^2 p_i \big/ \partial p_i^{loc}\partial \tilde{a}_i & \partial^2 p_i \big/ \partial \tilde{a}_i^{\,2} \end{bmatrix}$$

Following the proof of Proposition 1 in LRT, we have:

$$\frac{\partial^2 \pi_i}{\partial (p_i^{loc})^2}\Big|_{\substack{p_i^{loc}=p_j^{loc}=p^* \\ \tilde{a}_i=\tilde{a}_j=\tilde{a}^*}} = \frac{p''^{loc}(p^*)}{2} - q''(p^*)\frac{(a-c_0)}{4} - sq'(p^*)\left[p^{loc*} + I^* + \Delta\tilde{p}^{ld*}\right]$$
$$-2\,sq(p^*)p'^{loc}(p^*) \qquad \text{(I.8)}$$

that is:

$$\frac{\partial^2 \pi_i}{\partial (p_i^{loc})^2}\Big|_{\substack{p_i^{loc}=p_j^{loc}=p^* \\ \tilde{a}_i=\tilde{a}_j=\tilde{a}^*}} = -\frac{1}{2}\frac{q(p^*)}{p^*}\Bigg\{(h-1)\left((h+1)\frac{p^M - p^*}{p^*} + 1\right) + h\frac{a-c_0}{2}\frac{h+1}{p^*}$$
$$-2s\,h\left[p^{loc*} + I^* + \Delta\tilde{p}^{ld*}\right] + 4\,sp^*\,p'^{loc}(p^*)\Bigg\} \qquad \text{(I.9)}$$

From (I.3) it results $h\frac{a-c_0}{2} = 2\,sp^*\left[p^{loc*} + I^* + \Delta\tilde{p}^{ld*}\right] - (h-1)\left(p^M - p^*\right)$; then, condition (I.9) can be rewritten as:

$$\frac{\partial^2 \pi_i}{\partial (p_i^{loc})^2}\Big|_{\substack{p_i^{loc}=p_j^{loc}=p^* \\ \tilde{a}_i=\tilde{a}_j=\tilde{a}^*}} = -\frac{1}{2}\frac{q(p^*)}{p^*}\left\{h - 1 + 2\,s\left[p^{loc*} + I^* + \Delta\tilde{p}^{ld*}\right] + 4\,sp^*\,p'^{loc}(p^*)\right\} \qquad \text{(I.9bis)}$$

which is strictly negative since $h > 1$, $p^{loc*} \geq 0$ and $\Delta\tilde{p}^{ld*} \geq 0$ (non-negativity condition) and $p'^{loc}(p^*) \geq 0$ (i.e. $p^* \leq p^M$).

For simplicity, denote with $\Theta^{ld}(\tilde{a}_i) = I(\tilde{a}_i) + \tilde{p}_{ii}^{ld}(\tilde{a}_i)$ the global profit of network *i* deriving from its downstream activities (interconnection and retail). The F.O.C. in (I.5), evaluated at the candidate equilibrium, can be rewritten as:

$$\frac{\partial \pi_i}{\partial \tilde{a}_i}\Big|_{\substack{p_i^{loc}=p_j^{loc}=p^* \\ \tilde{a}_i=\tilde{a}_j=\tilde{a}^*}} = \frac{1}{2}\left\{\frac{\partial \Theta^{ld}}{\partial \tilde{a}_i}\right\} + \frac{\partial \alpha}{\partial \tilde{a}_i}[\Pi] = 0 \;\Rightarrow\; \frac{\partial \Theta^{ld}}{\partial \tilde{a}_i} = -2\frac{\partial \alpha}{\partial \tilde{a}_i}[\Pi] \qquad \text{(I.5bis)}$$

Thus, the second order derivative with respect to $\tilde{a}_i$, evaluated at the optimal solution, is the following:

$$\frac{\partial^2 \pi_i}{\partial \tilde{a}_i^2}\Big|_{\substack{p_i^{loc}=p_j^{loc}=p^* \\ \tilde{a}_i=\tilde{a}_j=\tilde{a}^*}} = 2\frac{\partial \alpha}{\partial \tilde{a}_i}\left[\frac{\partial \Theta^{ld}}{\partial \tilde{a}_i}\right] + \frac{1}{2}\left[\frac{\partial^2 \Theta^{ld}}{\partial \tilde{a}_i^2}\right] + \frac{\partial^2 \alpha}{\partial \tilde{a}_i^2}[\Pi] \qquad \text{(I.10)}$$

Substituting (I.5bis) in (I.10), we obtain:

$$\frac{\partial^2 p_i}{\partial \tilde{a}_i^2}\Big|_{\substack{p_i^{loc}=p_j^{loc}=p^* \\ \tilde{a}_i=\tilde{a}_j=\tilde{a}^*}} = -4\left(\frac{\partial a}{\partial \tilde{a}_i}\right)^2 [\Pi] + \frac{1}{2}\left[\frac{\partial^2 \Theta^{ld}}{\partial \tilde{a}_i^2}\right] + \frac{\partial^2 a}{\partial \tilde{a}_i^2}[\Pi] \qquad \text{(I.11)}$$

It results:

$$\frac{\partial^2 \Theta^{ld}}{\partial \tilde{a}_i^2} = -\frac{2}{b}\frac{(n-1)(n+3)}{(n+1)^2} < 0 \text{ since } n \geq 2 \ , \ \frac{\partial a}{\partial \tilde{a}_i} = -sQ_i^*\frac{n-1}{n+1} \ , \ \frac{\partial^2 a}{\partial \tilde{a}_i^2} = \frac{s}{b}\left(\frac{n-1}{n+1}\right)^2$$

Then, condition (I.11) becomes:

$$\frac{\partial^2 p_i}{\partial \tilde{a}_i^2}\Big|_{\substack{p_i^{loc}=p_j^{loc}=p^* \\ \tilde{a}_i=\tilde{a}_j=\tilde{a}^*}} = \frac{1}{2}\left[\frac{\partial^2 \Theta^{ld}}{\partial \tilde{a}_i^2}\right] + [\Pi]\left\{\frac{s}{b}\left(\frac{n-1}{n+1}\right)^2 - 4\left(-sQ_i^*\frac{n-1}{n+1}\right)^2\right\}$$

After manipulations we have:

$$\frac{\partial^2 p_i}{\partial \tilde{a}_i^2}\Big|_{\substack{p_i^{loc}=p_j^{loc}=p^* \\ \tilde{a}_i=\tilde{a}_j=\tilde{a}^*}} = \frac{1}{2}\left[\frac{\partial^2 \Theta^{ld}}{\partial \tilde{a}_i^2}\right] + s\,[\Pi]\left(\frac{n-1}{n+1}\right)^2\left\{\frac{1}{b} - 4\,sQ_i^{*2}\right\} \qquad \text{(I.12)}$$

Since $\frac{\partial a}{\partial Q_i^*} = sbQ_i^*$ the last term in the RHS of (I.12) can be rewritten as:

$$\frac{1}{b} - 4sQ_i^{*2} = \frac{1}{b} - \frac{4}{b}Q_i^*\frac{\partial a}{\partial Q_i^*}$$

Hence

$$\frac{1}{b} - \frac{4}{b}Q_i^*\frac{\partial a}{\partial Q_i^*} < 0 \ \Rightarrow \ Q_i^*\frac{\partial a}{\partial Q_i^*} > \frac{1}{4} \ \Rightarrow \ h_a = \frac{Q_i^*}{a}\frac{\partial a}{\partial Q_i^*} > \frac{1}{4a}$$

and so, since $a = 1/2$ in equilibrium, $h_a > 1/2$; thus, according to our assumptions, the RHS of (I.12) is negative.

Besides, from condition (I.5) we have:

$$\frac{\partial^2 p_i}{\partial p_i^{loc}\partial \tilde{a}_i}\Big|_{\substack{p_i^{loc}=p_j^{loc}=p^* \\ \tilde{a}_i=\tilde{a}_j=\tilde{a}^*}} = -sq(p^*)\left[\frac{\partial \Theta^{ld}}{\partial \tilde{a}_i}\right] + \frac{\partial a}{\partial \tilde{a}_i}\,p'(p^*) \qquad \text{(I.13)}$$

The determinant of the Hessian matrix H is the following:

$$|H| = \frac{\partial^2 p_i}{\partial (p_i^{loc})^2}\frac{\partial^2 p_i}{\partial \tilde{a}_i^2}$$
$$- s^2 q(p^*)^2 \left(\frac{\partial \Theta^{ld}}{\partial \tilde{a}_i}\right)^2 - \left(\frac{\partial a}{\partial \tilde{a}_i}\right)^2 p'^{loc}(p^*)^2 + 2sq(p^*)p'^{loc}(p^*)\frac{\partial \Theta^{ld}}{\partial \tilde{a}_i}\frac{\partial a}{\partial \tilde{a}_i}$$

As shown before, the equilibrium exists only if a and/or s are not too large (i.e. $s \to 0$ and/or $a \to c_0$). Then, simplifying the above condition we obtain:

$$|H| \cong \frac{\partial^2 p_i}{\partial (p_i^{loc})^2}\frac{\partial^2 p_i}{\partial \tilde{a}_i^2} > 0$$

Thus, the Hessian matrix is negative definite so that the candidate equilibrium is a local maximum. ■

Symmetry

Fix $\tilde{a}_i = \tilde{a}_j = \tilde{a}^*$. Then, Proposition 1 from LRT shows that a unique symmetric equilibrium exists for a and/or s small, just replace f by $f' = f - \left[I^* + \Delta\tilde{p}^{ld*}\right]$. In the same way, fix $p_i^{loc} = p_j^{loc} = p^*$, profit in function of $\tilde{a}_i$ and $\tilde{a}_j$ have exactly the same functional form as profits in LRT in function of p_i^{loc} and p_j^{loc} for $a = c_0$, just replace f by $f'' = f - [(p^* - c)q(p^*)]$. Indeed, for $p_i^{loc} = p_j^{loc} = p^*$, profits are strictly concave in $\tilde{a}_i$ (see condition I.12) and there is only one equilibrium in which $\tilde{a}_i = \tilde{a}_j = \tilde{a}^*$. ■■

Relation between $\tilde{a}_i$ and s

Applying the implicit function theorem to the F.O.C., denoted by X, we have:

$$\frac{\partial \tilde{a}_i^*}{\partial s} = -\frac{\partial X / \partial s}{\partial X / \partial \tilde{a}_i^*} \qquad \text{(I.14)}$$

The sign of $\partial X / \partial \tilde{a}_i^*$ is negative from the second order condition (I.12). Hence, the sign of (I.14) only depends on $\partial X / \partial s$:

$$\frac{\partial X}{\partial s} = 2bQ_i^* \frac{\partial Q_i^*}{\partial \tilde{a}_i}[\Pi] = -2Q_i \frac{n-1}{n+1}[\Pi]$$

which is negative. Then, the higher the subsitutability in the local sector, the lower the one way access charge. ■

Relation between $\tilde{a}_i$ and a

Applying the implicit function theorem to condition (I.5bis), denoted once again by X, we have:

$$\frac{\partial \tilde{a}_i^*}{\partial a} = -\frac{\partial X / \partial a}{\partial X / \partial \tilde{a}_i^*} \qquad \text{(I.15)}$$

The sign of $\partial X / \partial \tilde{a}_i^*$ is negative from the second order condition (I.12). Hence, the sign of (I.15) only depends on $\partial X / \partial a$:

$$\frac{\partial X}{\partial a} = 2\,sQ_i^* \frac{\partial Q_i^*}{\partial \tilde{a}_i} p'^{\,loc}(p^*) \frac{\partial p^*}{\partial a} = -2\,sQ_i^* \frac{n-1}{n+1} p'^{\,loc}(p^*) \frac{\partial p^*}{\partial a} < 0$$

since, as shown before, $p'^{\,loc}(p^*) \geq 0$ (i.e. $p^* \leq p^M$) and $\partial p^* / \partial a > 0$. Then, the higher the two-way access charge in the local sector, the lower the one way access charge in the complementary downstream sector. ■

APPENDIX II

Proof of Proposition 5

First Order Conditions

Using the same Proposition 7 from LRT it can be shown that no equilibrium exists for s large and/or (a – c_0) large, just replace f by $f' = f - \left[I + \Delta\tilde{p}^{ld}\right]$. Differentiating global profit with respect to p_i^{loc}, F_i and $\tilde{a}_j$, we have:

$$\frac{\partial p_i}{\partial p_i^{loc}} = a\left[q(p_i^{loc}) + \left(p_i^{loc} - c^{loc}\right)\frac{\partial q}{\partial p_i^{loc}}\right] - sq(p_i^{loc})\left[p^{loc} + F_i + I + \Delta\tilde{p}^{ld}\right]$$
$$-a(1-a)(a-c_0)\frac{\partial q}{\partial p_i^{loc}} + (1-2a)\frac{\partial a}{\partial p_i^{loc}}\left[q(p_j^{loc}) - q(p_i^{loc})\right](a-c_0) \qquad \text{(II.1)}$$

$$\frac{\partial p_i}{\partial F_i} = a - s\left[p^{loc} + F_i + I + \Delta\tilde{p}^{ld}\right] - s(1-2a)\left[q(p_j^{loc}) - q(p_i^{loc})\right](a-c_0) \qquad \text{(II.2)}$$

$$\frac{\partial \pi_i}{\partial \tilde{a}_i} = \alpha \left\{ Q_i^* - \tilde{q}_i^* + (\tilde{a}_i - c_0) \frac{\partial \left[Q_i^* - \tilde{q}_i^* \right]}{\partial \tilde{a}_i} + \frac{\partial \pi_{ii}^{ld}}{\partial \tilde{a}_i} \right\} + \frac{\partial \alpha}{\partial \tilde{a}_i} \left[p^{loc} + F_i + I + \Delta \tilde{p}^{ld} \right]$$
$$+ (1 - 2\alpha) \frac{\partial \alpha}{\partial \tilde{a}_i} \left[q(p_j^{loc}) - q(p_i^{loc}) \right] \qquad \text{(II.3)}$$

In a symmetric equilibrium, $p_i^{loc} = p_j^{loc} = p*$, $F_i = F_j = F*$, $\tilde{a}_i = \tilde{a}_j = \tilde{a}^*$, $\alpha = 1/2$ and $d[\alpha(1-\alpha)]/d\alpha = 0$; thus, the above condition can be rewritten:

$$\frac{\partial \pi_i}{\partial p_i^{loc}}_{|opt} = \frac{1}{2} \left[q(p_i^{loc}) + \left(p_i^{loc} - c^{loc} \right) \frac{\partial q}{\partial p_i^{loc}} \right] - \sigma q(p_i^{loc}) \left[p^{loc^*} + F^* + I^* + \Delta \tilde{p}^{ld*} \right]$$
$$- \frac{1}{4} (a - c_0) \frac{\partial q}{\partial p_i^{loc}} = 0 \qquad \text{(II.4)}$$

$$\frac{\partial \pi_i}{\partial F_i}_{|opt} = \frac{1}{2} - \sigma \left[p^{loc*} + F_i + I^* + \Delta \tilde{p}^{ld*} \right] = 0 \qquad \text{(II.5)}$$

$$\frac{\partial \pi_i}{\partial \tilde{a}_i}_{|opt} = \frac{1}{2} \left\{ Q_i^* - \tilde{q}_i^* + (\tilde{a}_i - c_0) \frac{\partial \left[Q_i^* - \tilde{q}_i^* \right]}{\partial \tilde{a}_i} + \frac{\partial \pi_{ii}^{ld}}{\partial \tilde{a}_i} \right\} + \frac{\partial \alpha}{\partial \tilde{a}_i} \left[p^{loc*} + F^* + I^* + \Delta \tilde{p}^{ld*} \right] = 0 \qquad \text{(II.6)}$$

After manipulations (for condition II.6 see also Appendix I), we obtain the optimal conditions in Proposition 5. ■

Second Order Conditions

The second order derivatives, evaluated at the equilibrium, are the following:

- with respect to the local price p_i^{loc}:

$$\frac{\partial^2 \pi_i}{\partial (p_i^{loc})^2}_{|opt} = \frac{\pi''^{loc}(p^*)}{2} - q''(p^*) \frac{(a - c_0)}{4} - \sigma q'(p^*) \left[p^{loc*} + F^* + I^* + \Delta \tilde{p}^{ld*} \right]$$
$$- 2 \sigma q(p^*) \pi'^{loc}(p^*) \qquad \text{(II.7)}$$

which becomes, using condition (II.5):

$$\frac{\partial^2 \pi_i}{\partial (p_i^{loc})^2}_{|opt} = \frac{1}{2} q'(p^*) - 2 \sigma q(p^*) \pi'^{loc}(p^*) < 0 \qquad \text{(II.7bis)}$$

since $p*$ is lower than the monopoly price.

b) with respect to the fixed fee F_i:

$$\frac{\partial^2 p_i}{\partial (F_i)^2}_{|opt} = -2s < 0 \tag{II.8}$$

c) with respect to $\tilde{a}_j$:

$$\frac{\partial^2 p_i}{\partial \tilde{a}_i^2}_{|opt} = 2\frac{\partial a}{\partial \tilde{a}_i}\left[\frac{\partial \Theta^{ld}}{\partial \tilde{a}_i}\right] + \frac{1}{2}\left[\frac{\partial^2 \Theta^{ld}}{\partial \tilde{a}_i^2}\right] + \frac{\partial^2 a}{\partial \tilde{a}_i^2}\left[\frac{1}{2s}\right] \tag{II.9}$$

where, as in Appendix I, $\Theta^{ld}(\tilde{a}_i) = I(\tilde{a}_i) + \tilde{p}_{ii}^{ld}(\tilde{a}_i)$ denotes the global profit of network i deriving from its downstream activities (interconnection and retail). After manipulations, we have:

$$\frac{\partial^2 p_i}{\partial \tilde{a}_i^2}_{|opt} = 2\frac{\partial a}{\partial \tilde{a}_i}\left[\frac{\partial \Theta^{ld}}{\partial \tilde{a}_i}\right] - \frac{1}{b}\left(\frac{n-1}{(n+1)^2}\right)\left(\frac{n+7}{2}\right) < 0 \tag{II.9bis}$$

since $n \geq 2$, $\partial a / \partial \tilde{a}_i < 0$ and $\partial \Theta^{ld} / \partial \tilde{a}_{i|opt} = \dfrac{(n-1)(n+3)(g-c-c_0)}{b(n+1)(n+7)} > 0$.

Besides, we have:

$$\frac{\partial^2 p_i}{\partial p_i^{loc}\,\partial F_i}_{|opt} = -sq(p^*) - s p'^{loc}(p^*) < 0 \tag{II.10}$$

$$\frac{\partial^2 p_i}{\partial p_i^{loc}\,\partial \tilde{a}_i}_{|opt} = -sq(p^*)\left[\frac{\partial \Theta^{ld}}{\partial \tilde{a}_i}\right] + \frac{\partial a}{\partial \tilde{a}_i} p'^{loc}(p^*) < 0 \tag{II.11}$$

$$\frac{\partial^2 p_i}{\partial F_i\,\partial \tilde{a}_i}_{|opt} = -s\left[\frac{\partial \Theta^{ld}}{\partial \tilde{a}_i}\right] + \frac{\partial a}{\partial \tilde{a}_i} < 0 \tag{II.12}$$

The second and the third leading submatrix of the Hessian, evaluated at the equilibrium, are given respectively by:

$$D_{|opt} = \begin{bmatrix} \partial^2 p_i / \partial (p_i^{loc})^2 & \partial^2 p_i / \partial p_i^{loc}\,\partial F_i \\ \partial^2 p_i / \partial p_i^{loc}\,\partial F_i & \partial^2 p_i / \partial F_i^2 \end{bmatrix}$$

$$G|_{opt} = \begin{bmatrix} \partial^2 p_i / \partial (p_i^{loc})^2 & \partial^2 p_i / \partial p_i^{loc} \partial F_i & \partial^2 p_i / \partial p_i^{loc} \partial \tilde{a}_i \\ \partial^2 p_i / \partial p_i^{loc} \partial F_i & \partial^2 p_i / \partial F_i^2 & \partial^2 p_i / \partial F_i \partial \tilde{a}_i \\ \partial^2 p_i / \partial p_i^{loc} \partial \tilde{a}_i & \partial^2 p_i / \partial F_i \partial \tilde{a}_i & \partial^2 p_i / \partial (\tilde{a}_i)^2 \end{bmatrix}$$

After manipulations, it results:

$$|D|_{opt} = -s\,q'(p^*)\left[1 + sq'(p^*)\frac{(a-c_0)^2}{4}\right] \qquad \text{(II.13)}$$

$$|G|_{opt} = skq'(p^*)\left[1 + sq'(p^*)\frac{(a-c_0)^2}{4}\right] \qquad \text{(II.14)}$$

where $k = \frac{1}{b}\left[\frac{n-1}{(n+1)^2}\right]\left(\frac{n+7}{2}\right) > 0$.

The sufficient second order conditions for the equilibrium to be a local maximum are that (II.13) should be positive and (II.14) should be negative. It is straightforward to see that, as suggested in LRT, if $s \to 0$ and/or $a \to c_0$, the sign of the above determinats are satisfied. But if s or a are not so small, the following condition ensures the global second order conditions:

$$1 + sq'(p^*)\frac{(a-c_0)^2}{4} > 0 \qquad \text{(II.15)}$$

From condition (18), it results $p^* = c^{loc} + \frac{a-c_0}{2}$. Then, condition (II.15) can be rewritten as $1 - sh\frac{x^2}{(x+c^{loc})^{h+1}} > 0$, where $x = \frac{a-c_0}{2}$ and h is the demand elasticity of local calls. Let's define

$$j\,(x) = 1 - sh\frac{x^2}{(x+c^{loc})^{h+1}} \qquad \text{(II.15bis)}$$

Condition (II.15) will be automatically satisfied if the function $f(x)$ evaluated at its minimum level of $x = x^*$ is positive. Deriving (II.15bis) we find that the function $f(x)$ reach a minimun in $x^* = 2c^{loc}/h - 1$. Then, after manipulations, we have that $f(x^*) \geq 0$, and so $f(x) > 0\ \forall x$, if:

$$c^{loc} \geq \frac{h-1}{h+1}\sqrt[h-1]{\frac{4sh}{(h+1)^2}} \qquad \text{(II.16)}$$

Thus, if s is not large enough (i.e. $s \to 0$) and/or the mark up one the two-way access charge is not large enough (i.e. $a \to c_0$) and/or condition (II.16) is satisfied, then, the candidate equilibrium given by the set of equations (II.4), (II.5) and (II.6) is a local maximum. For the symmetry of the equilibrium, see LRT and Appendix 1. ■

Proof of Proposition 7

Maximizing the aggregate profits shown in conditions (31) and (32) for the integrated (i) and the separated (j) networks, respectively, and using the same methodology shown at the begining of this Appendix, it is straighforward to determine the optimal conditions (33), (34) and (35) for the integrated network and (36), (37) and (38) for the separated one.

Market shares are given by:

$$a_i = \frac{1}{2} + s\left[v(p_i^{loc}) - F_i + v^{ld}(\tilde{a}_i) - v(p_j^{loc}) + F_i - v^{ld}(\tilde{a}_j)\right] \qquad \text{(II.17)}$$

The difference in net surplus deriving from consuming long distance calls is given by:

$$v^{ld}(\tilde{a}_i^*) - v^{ld}(\tilde{a}_j^*) = \frac{b}{2}\left(Q_i^* - Q_j^*\right)$$

Since, in equilibrium, it results $Q_i^* = \frac{n+3}{n+7}\frac{(g-c-c_0)}{b}$ and $Q_j^* = \frac{n}{n+2}\frac{(g-c-c_0)}{b}$, we have:

$$v^{ld}(\tilde{a}_i^*) - v^{ld}(\tilde{a}_j^*) = -2\left[\frac{(g-c-c_0)^2}{b}\right]\frac{(n+3)(n^2+6n+3)}{(n+7)^2(n+2)^2} \qquad \text{(II.18)}$$

Using conditions (34) and (39), the difference in the fixed fees in the asymmetric equilibrium is given by:

$$-F_i^* + F_j^* = -\frac{a_i}{s} + (p_i^{loc}* - c)q(p_i^{loc}*) + (1-2a_i)A_i - f + I(\tilde{a}_i^*) + \Delta\tilde{p}^{ld}(\tilde{a}_i^*, \tilde{a}_i^*)$$
$$+\frac{(1-a_i)}{s} - (p_j^{loc}* - c)q(p_j^{loc}*) - (1-2(1-a_i))A_j + f - I(\tilde{a}_j^*) \qquad \text{(II.19)}$$

Since $A_j = -A_i$ and evalutiang the interconnection and retail profits at the optimal one-way access charges, condition (II.19) can be rewritten as:

$$-F_i^* + F_j^* = \frac{1-2a_i}{s} + (p_i^{loc}* - c)q(p_i^{loc}*) - (p_j^{loc}* - c)q(p_j^{loc}*)$$
$$+\frac{(g-c-c_0)^2}{b}\left[\frac{2n^3+10n^2+11n+48}{(n+2)^2(n+7)^2}\right] \qquad \text{(II.20)}$$

Substituting conditions (II.18) and (II.20) into (II.17) and denoting with $s_i = v(p_i^{loc}*)+(p_i^{loc}*-c^{loc})q(p_i^{loc}*)$ and $s_j = v(p_j^{loc}*)+(p_j^{loc}*-c^{loc})q(p_j^{loc}*)$) the gross utility minus cost of local calls provided by network i and network j, the asymmetric equilibrium market share is given by condition (39), i.e.:

$$a_i^* = \frac{1}{2} + \frac{s}{3}\left[s_i - s_j\right] + \frac{s}{3}\left[\frac{(g-c_0-c)^2(4n^2+41n+66)}{b(n+2)^2(n+7)^2}\right]$$

Let's analyse the effect of the two-way access charge a on market share. Following Dessein (1999a), we have:

$$\frac{da_i^*}{da} = \frac{s}{3}\left[\frac{ds_i}{da} - \frac{ds_j}{da}\right]$$

For $a = c_0$, it results $p_i^{loc}* = p_j^{loc}* = c^{loc}$; thus, $s_i = s_j$ and $da_i^*/da_{|a=c_0} = 0$. Then, it results:

$$a_i^* = \frac{1}{2} + \frac{s}{3}\left[\frac{(g-c_0-c)^2(4n^2+41n+66)}{b(n+2)^2(n+7)^2}\right] > a_j^* \qquad \text{(II.21)}$$

and so the market share of the integrated network is greater than the market share of the separated operator.

If $a > c_0$, the level of the two-way access charge influences the equilibrium market share. In particular, if

$$\frac{d^2 s_i}{da^2}_{|a=c_0} > \frac{d^2 s_j}{da^2}_{|a=c_0} \qquad \text{(II.22)}$$

then, an increase in the reciprocal access charge would permit to the integrated network to get a greater market share in the subscriber market.

From the assumptions of the model, it results:

$$s_i = u(q(p_i^{loc}*)) - c^{loc}q(p_i^{loc}*) = \frac{(p_i^{loc*})^{1-h}}{1-(1/h)} - c^{loc}(p_i^{loc*})^{-h}$$

Then, we have:

$$\frac{ds_i}{dp_i^{loc*}} = h^2(p_i^{loc*})^{-h-1} - h(h+1)c^{loc}(p_i^{loc*})^{-h-2} \quad \Rightarrow \quad \frac{ds_i}{dp_i^{loc*}}\Big|_{a=c_0} = -\frac{h}{(c^{loc})^{h+1}} < 0$$

and equivalently for network *j*. Condition (II.22) can be rewritten as:

$$\frac{d^2 s_i}{d(p_i^{loc*})^2}\left(\frac{dp_i^{loc*}}{da}\right)^2_{|a=c_0} > \frac{d^2 s_j}{d(p_j^{loc*})^2}\left(\frac{dp_j^{loc*}}{da}\right)^2_{|a=c_0} \qquad \text{(II.23)}$$

Since $\left(dp_i^{loc*}/da\right)_{|a=c_0} = a_j^*$ and $\left(dp_j^{loc*}/da\right)_{|a=c_0} = a_i^*$, in $a = c_0$, we have:

$$-\frac{h}{(c^{loc})^{h+1}}\left(a_j^*\right)^2 > -\frac{h}{(c^{loc})^{h+1}}\left(a_i^*\right)^2 \quad \Rightarrow \quad a_i^* > a_j^*$$

which is always verified from (II.21). Thus, market share of the integrated network is an increasing function in the reciprocal access charge *a*. ■