Economics

Robert J. Aumann

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2005

Autobiography

Being awarded the Prize in Economic Sciences in Memory of Alfred Nobel is a beautiful, fairy-tale experience, from beginning to end. It is not, however, conducive to research, writing, teaching, or indeed to any ordinary academic work. Instead, one is besieged by hundreds – indeed thousands – of demands to appear and speak at events and congresses, many of which have nothing at all to do with science, to underwrite good (and less good) causes, to sign and send autographs, to be photographed, to be interviewed, etc., etc., etc.

The reader should not misunderstand. This is not a complaint; a Nobelist is glad to do these things, it's worth it. Rather, it's an apology for the biography that follows. This biography was due on February 1, 2006, but I simply did not get to it. After many futile attempts on the part of the Nobel Foundation to wrest the biography from me, I was told that if it were not in by May 15, it would simply not be included. This alarmed me, so finally I started seriously working on it. But being serious did not help me; the requests for interviews, appearances, speeches, photographs, etc., etc., kept pouring in. Somehow, though, I did manage to devote a few minutes a day to the biography. Unfortunately (or perhaps fortunately) the result is rather haphazard and disorganized, jumping from topic to topic; but perhaps it is better – more interesting – that way than a more straight-laced essay would be.

I was born in June of 1930 in Frankfurt-am-Main, Germany, to an orthodox Jewish family. My father was a wholesale textile merchant, financially comfortable, whose family had lived in Germany for centuries; he had fought in World War I for the Germans, and been decorated. My mother grew up in London, and obtained a B.A. at University College, London – a somewhat unusual feat for women in the early 20th century. The Nazi regime in the thirties made life very difficult for Jews in Germany, and my parents saw the handwriting on the wall – realized that far worse was in the offing. In 1938 we obtained American visas with some difficulty, and emigrated from Frankfurt to New York. In that passage my parents lost all their assets, and had to work very hard to make a living; nevertheless, they gave their two children – my brother and me – excellent educations, and we had wonderful childhoods. We attended Jewish parochial schools, and obtained bachelor's degrees at the City College of New York.

In high school I had an extraordinary teacher of mathematics, Abraham Gansler, who taught me to love the subject. What attracted me most were the axioms, theorems, proofs, and constructions of Euclid 's geometry. So in City College, I decided to "major" in (emphasize) mathematics. I was mainly interested in classical mathematics: complex and real functions, Fourier series, differential geometry, and so on. Most of all, analytic and algebraic number theory, which I read voraciously, largely from the books of Edmund Landau. Number theory fascinated me because (i) the problems are very natural; (ii) they are simple to formulate, a schoolchild can understand them; (iii) the solutions are very difficult and deep, they require years of university study even to begin to fathom; and (iv) the whole subject was absolutely useless, had no practical applications, was a purely intellectual endeavor. The vogue of pure mathematics – the "purer," the better – was at its height in the mid-twentieth century, and, young and impressionable, I was drawn into it.

As is customary in American higher education, in addition to mathematics, I studied many other subjects: physics, chemistry, biology, bacteriology, geology, philosophy, English and German literature, history, writing, art, music, public speaking, film, ... There was even a course in economics, which baffled and bored me, and which I "dropped" after a few weeks. But many of the other courses left a deep impression. The best were those with a "hands-on" approach: the art course consisted almost exclusively of showing and analyzing (reproductions of) specific paintings and sculptures; the music course consisted almost exclusively of playing and analyzing specific musical compositions; the literature courses consisted almost exclusively of reading and analyzing specific literary works, mostly English poetry and German drama (an entire semester was devoted to Goethe's *Faust*). An extreme example was the course in geology. For three weeks we did nothing

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but learn the names of various kinds of rock, pieces of which were provided in the classroom. At the end of that period there was an examination: we were provided with twenty-five or thirty pieces of rock – not pictures, but the rocks themselves – which we had to identify. After that, we never saw the inside of the classroom again; the course consisted *exclusively* of field trips, all inside New York City and its immediate surroundings, all accessible by public transportation. I will never forget what I learned there – why rivers meander, what makes rocks different, dikes, glaciation, U- and V-shaped valleys, etc., etc.; when hiking in Israel and all over the world, I teach these things to my children and grandchildren, who do not have the benefit of such a marvellously broad education.

After completing a B.S. at "City," I entered the Massachusetts Institute of Technology (MIT) for graduate studies. At MIT I became interested in the more modern branches of mathematics, like algebraic topology, to which I was attracted by the excellent lectures of George W. Whitehead. I decided to do a Ph.D. with Whitehead in knot theory, a branch of algebraic topology that deals with the properties of knots (those tied in ropes). Like analytic number theory, knot theory deals with problems that (i) are very natural, have an immediacy that is even greater than that of the distribution of prime numbers or Fermat's last theorem; (ii) are simple to formulate, a schoolchild can understand them; (iii) have solutions that are very difficult and deep; and (iv) the whole subject was absolutely useless.

Whitehead gave me a very difficult problem – one that had been attacked unsuccessfully for a quarter of a century – namely, to show that knots are "aspheric" (we won't explain here what this means). I didn't solve this problem, but did establish asphericity for knots of a special kind: those that are "alternating." That means that when you draw the knot on a piece of paper, and follow along any component of the knot, then the undercrossings alternate with the overcrossings – as, for example, in the famous "Borromean Rings." My thesis was published in the *Annals of Mathematics* in 1956.

After completing the Ph.D., I went to work for an Operations Research consulting outfit – the "Analytical Research Group" (ARG) – affiliated with Princeton University 's mathematics department, and located at the University's Forrestal Center for Applied Research. ARG did highly practical consulting. One of the problems that I was assigned had to do with defending a city from attack by a squadron of aircraft, a few of which are carrying nuclear weapons, but most of which are decoys. At MIT I had met John Nash – who in 1994 shared the Prize in Economic Sciences in Memory of Alfred Nobel with John Harsanyi and Reinhard Selten, and who came to MIT as a young instructor in the early fifties – and had heard a little about game theory from him. At the time it didn't interest me very much, but when I was assigned the problem about the decoys, I remembered the conversations with Nash, and figured that game theory had to be the right tool to attack this problem. So I studied some game theory – just enough for this problem – and then the subject started attracting me in its own right. The rest is history, as the saying goes.

Jews have been yearning for the land of Israel, and for Jerusalem, for close to 2000 years – ever since the destruction of the Temple by the Romans in the year 70, and the ensuing exile of the Jewish people. In our central prayer, which we recite three times a day, we ask the Lord to "return to Jerusalem Your city in mercy, and rebuild it and dwell therein." Jerusalem is mentioned many thousands of times in the scriptures, in our other prayers, in the Talmud, and indeed in all our sources. So when the state of Israel was established in 1948, my brother and I made a determination eventually to make our lives there. My brother fulfilled this ambition shortly thereafter, in 1950, but I decided first to complete my education. In 1953 I met an Israeli girl, Esther Schlesinger, who was visiting the United States; we were married in Brooklyn in April of 1955. In the fall of 1956 I took up a position as instructor of mathematics at the Hebrew University of Jerusalem, and have been there ever since. Esther and I had five beautiful children; the oldest, Shlomo, was killed in action in 1982, while serving in the Israeli Army in "Operation Peace for Galilee." At this writing, I have nineteen grandchildren and two great-grandchildren. Esther died of ovarian cancer in October of 1998, after we had enjoyed forty-four truly magnificent years together. In late November of 2005, about a week before being awarded the Prize in Economic Sciences in Memory of Alfred Nobel, I married Esther's widowed sister, Batya Cohn.

When the Prize was announced, the work of mine most prominently mentioned by the Committee was my 1959 paper "Acceptable Points in General Cooperative *n*-Person Games," which is perhaps the first rigorous treatment of repeated games that has some generality. Briefly put, the finding of that paper is that the strong equilibrium payoffs of a repeated game coincide with the core (more precisely, β-core) payoffs in the one-shot game. Frankly, I don't recall well the genesis of that paper. As mentioned above, I became interested in game theory while at Princeton in the years 1954–56. The 1957 Luce–Raiffa book *Games and Decisions,* which made a big impression on me, has an interesting – though inconclusive – discussion of repeated games, and this may have piqued my interest. I vividly recall working on "Acceptable Points" while at the Bureau of Standards in Washington in the summers of 1957 and 58; the yellow pads are still before my eyes. In the course of that work I became aware of what later became known as the "Folk Theorem" (see my 1981 "Survey of Repeated Games"), but it seemed to me at the time that it did not have sufficient mathematical depth to merit publication. That was a big mistake. Both "Acceptable Points" and the Folk Theorem are expressions of the relation between equilibrium behavior in the repeated game and cooperative behavior in the one-shot game; but while "Acceptable Points" is undoubtedly interesting, and much the deeper and more subtle, the Folk Theorem is by far the more fundamental and important.

At some point during the academic year 1959–60 I gave a colloquium lecture at the mathematics department of the Hebrew University; the colloquium is a weekly gathering of the entire department, at which a faculty member or guest talks about his own research or a related topic. I chose to discuss the von Neumann–Morgenstern "solutions" of cooperative *n*-person games, sometimes called *stable sets.* Historically, this is the first "solution concept" for cooperative games, and to this day it remains one of the most subtle and beautiful. Michael Maschler, an expert in the theory of functions of a complex variable, was in the audience; after the talk he asked a question. This question led to a lifelong scientific partnership with Maschler.

The specific question that Maschler asked led eventually to our 1964 joint paper on the *bargaining set* for coalitional (a.k.a. cooperative) games. In turn, this led to a very considerable literature, encompassing related concepts such as the *kernel* (Davis and Maschler 1965) and *nucleolus* (Schmeidler 1969). But I myself made only two additional contributions to this subject after 1964. One was a computation and tabulation of the kernels of several classes of games with up to five players, done jointly with Bezalel Peleg from the Hebrew University and Pinny Rabinowitz from the Weizmann Institute of Science; this work led to several conjectures on the structure of the kernel, which turned out to be very fruitful and led to important theoretical advances and a much better understanding of this structure. The other is the 1985 paper "Game-Theoretic Analysis of a Bankruptcy Problem from the Talmud," also joint with Maschler; it is an explanation of a difficult passage in the Babylonian Talmud (Ketubot 93a), the key to which was Schmeidler's nucleolus. This is undoubtedly the work of mine that is best known; not so much to the scientific public – though there, too, it is widely cited – but to the general public. I have lectured on it dozens – perhaps hundreds – of times, to scientific audiences as well as to high school students and teachers, synagogue groups, Talmudic academies, and so on. It has been quoted and explained and reworked by many different people, innumerable times; I cannot possibly keep track of it.

Another important joint work with Maschler is that on repeated games of incomplete information. This began as part of a project initiated by the United States Arms Control and Disarmament Agency (ACDA) in the mid1960s, to help shape United States policy in the arms control negotiations that were taking place with the Soviet Union at that time. Involved in this project, in addition to Maschler and me, were John Harsanyi, Reinhard Selten, and Gerard Debreu – all three of whom eventually became Nobelists; Dick Stearns – eventually awarded the Turing prize in theoretical computer science; Herb Scarf and Harold Kuhn – eventually awarded the John von Neumann Prize in operations research theory; Jim Mayberry, whom I had gotten to know at ARG; and others. Several times a year the members of this stellar group would converge on Washington to discuss arms control and disarmament with each other and with the personnel of the agency; between meetings, we would work alone or in small groups in our home environments. I doubt that this work did indeed have much practical influence on policy, though one can never know; a side discussion with an agency staff member at lunch or during a coffee break can sometimes leave a deep impression – conscious or subconscious – that may eventually profoundly affect policy.

Be that as it may, the work of Maschler and me on repeated games of incomplete information, in which Dick Stearns also played an important role, spawned a large scientific literature. For many years the original work was difficult to obtain; it was available only in the form of the original ACDA reports, and one had to scrounge around for it. Eventually, in 1995, the original reports were published in book form by the MIT Press, together with extensive notes describing much of the subsequent work (up to 1995, of course). This book was awarded the Lanchester Prize of the Operations Research (OR) Society of America for the best OR book in 1995.

Maschler is by no means my only collaborator. I have collaborated – and am collaborating – with more than thirty scientists; a good part of my prize is attributable to them. Prominent among them is Lloyd Shapley, with whom I coauthored the 1974 book *Values of Non-Atomic Games;* it concerns games with many players, who impinge significantly on the outcome only when they form large coalitions, but not as individuals. Examples are national elections or large economies or markets. Another collaboration with Shapley concerns perfect equilibria in repeated games; it is a sharpening of the Folk Theorem, and it, too, was cited by the Nobel Committee. At around the same time that Lloyd and I were looking into this at the Rand Corporation in Santa Monica, California, in 1976, Ariel Rubinstein was doing a master's thesis – with Bezalel Peleg in Jerusalem – on precisely the same question, and he reached essentially the same conclusions that we did. Although it became widely known soon thereafter, this work was not published until 1994 – most appropriately, in a festschrift honoring Maschler.

Others with whom I've collaborated extensively – and intensively – throughout the years include Jacques Drèze, Mordecai Kurz, Sergiu Hart, Bezalel Peleg, Adam Brandenburger, Frank Anscombe, Abraham Neyman, Benjy Weiss, Micha Perles, Joe Kruskal, Roger Myerson, and others. At this writing I'm collaborating with Roberto Serrano on a project on which we've already been working for several years; with Sergiu Hart on another project; and with Hart and Motty Perry on yet another project.

Influence is not limited to joint papers. Innumerable individuals impinge on one, both in person and in what one reads and hears. Prominent among them are one's students, but there are many, many others. Science is one huge cooperative venture. The Nobel Prizes focus attention on important scientific work by focusing on a small number of individuals; but really, any scientific work – including work that merits a Nobel Prize – is the product of many minds.

For the finale, we flash half a century back. Though writing it up took a little longer, my thesis was essentially complete in October of 1954; I remember standing in the shower and being hit with the mathematical idea that enabled its completion. Fifty years later – almost to the day – at 10:00 pm, the phone in my flat rings. My grandson Yakov Rosen, who is in the second year of medical school, is on the line. "Grandpa," he says, "can I pick your brain? We are studying knots. I don't understand the material, and think that our lecturer does not understand it either. For example, what, exactly, are 'linking numbers'?" "Why are you studying knots?" I ask. "What do knots have to do with medicine?" "Well," says Yakov, "sometimes the DNA in a cell gets knotted up. Depending on the characteristics of the knot, this may lead to cancer. So, we have to understand knots."

I was completely bowled over. Fifty years later, the "absolutely useless" – the "purest of the pure" – is taught in the second year of medical school, and my grandson is studying it.

Science is exploration – exploration for the sake of exploration, and for nothing else. We must go where our curiosity leads us, we must go where we want to go. And eventually, it is sure to lead us to the beautiful, the important, and the useful.

For me, life has been – and still is – one tremendous joyride, one magnificent tapestry. There have been bad – very bad – times, like when my son Shlomo was killed and when my wife Esther died. But even these somehow integrate into the magnificent tapestry. In one of his beautiful letters, Shlomo wrote that there can be no good without bad. Both Shlomo and Esther led beautiful, meaningful lives, affected many people, each in his own way.

And there have been a *lot* of very good times. The excitement of research, of groping in the dark and then hitting the light. The satisfaction of teaching, of meeting someone at a party who tells you that the course in complex variables that he heard from you twenty-five years ago was the most beautiful that he *ever* heard. The exhilaration of climbing – seeking and finding foot- and handholds – on an almost vertical rock face. The beauty of a walk in the woods with a four-year-old grandchild, who spots and correctly identifies a tiny wild orchid about which you told him last week. Dancing with your wife at your child's wedding. Unraveling an intricate passage in the Talmud with your eighteen-year-old granddaughter, or with a study partner with whom you have studied for thirty years. Slipping on a "black" (expert) ski slope, tumbling two hundred meters down, and then going back and doing the same slope again – this time without a slip. Cooking a meal and hearing from a guest that the soup was the best she *ever* tasted. Raising a beautiful family. Seeing the flag of Israel fluttering in the wind, right next to that of Sweden, from the roof of the Grand Hotel in Stockholm.

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