Uncertain Externalities, Liability Rules, and Resource Allocation

By PETER H. GREENWOOD AND CHARLES A. INGENE*

The rich literature on the economics of externalities has been confined to the analysis of what we term certainty—perfect knowledge of the impact an action taken by one economic unit will have on another unit. The burden of these studies has been that "... if market transactions were costless, all that matters (questions of equity apart) is that the rights of the various parties should be well-defined and the results of of legal actions easy to forecast" (Ronald Coase, p. 19), since "... the affected parties might engage in bargaining and attempt to arrange a solution between themselves" (Otto Davis and Andrew Whinston, p. 113). So long as negotiations (market transactions) are costless, the allocation of resources at the conclusion of the bargaining process is socially optimal because it has the same characteristics as the equilibrium position attained by a merger of the affected parties into a single firm which fully internalizes externalities. This powerful conclusion, independence from the assignment of liability by the legal system (questions of equity apart), has come to be called the "Coase Theorem."

We extend the results of our precursors by analyzing an uncertain distribution of externalities, postulating a situation in which one firm's activities affect another firm in a random fashion. The existence of this uncertainty is sufficient to cause any firm facing it to modify its behavior in a subtle but significant manner: maximization is of the expected utility from profit rather than of profit itself. This response to an inevitably risky situation enables the firm to incorporate into its maximization problem both the distribution of the externality and its own attitude toward risk.

We demonstrate that given uncertainty, the allocation of resources may not be independent of legal liability; thus, the outcomes of bargaining and of merger may not coincide. Outcomes independent of liability and the equivalence of bargaining and merger are dependent both upon the existence of a stock market in which risk may be shared and upon the absence of indivisibilities in wealth. Although the traditional conclusions under certainty are correct and are contained within our model as a special case, in an uncertain world the Coase Theorem is valid only with the more stringent assumptions stated above.

In Section I we establish our mathematical model, a Taylor series expansion of the expected utility from profit, and use it to discuss the result of a merger by the involved parties. The series leads to the Pratt-Arrow measure of absolute risk aversion. We show that the equilibrium position "fully" internalizes the externality; but, the definition of fully may be dependent upon the risk attitude of whoever controls the merged firm. We also discuss the fact that a dominant shareholder or an autonomous manager may prevent the complete spreading of risk through a stock market.

The second section deals with legally permissible, uncompensated externalities. The affected firm will then approach the polluter in a costless "private bribery market" to seek an alteration in the level of the externality creating output.[1] The polluter

*Assistant professor of resource economics, University of New Hampshire; and assistant professor of economics, University of Oklahoma, respectively. We are grateful to G. Borts, R. Dacey, and an anonymous referee for their helpful comments. None of them are sufficiently risk preferring to wish to share responsibility for an article which will have an uncertain reception; thus, common law allocates liability for remaining errors to the authors.

[1]For ease of exposition, we will discuss only a negative externality; our mathematics is perfectly general, however. We do assume that pollution and output are directly related in order to avoid the complication of

will comply so long as the usual, profit-maximizing, marginal conditions are met. At the completion of the bargaining process there will be no gains from trade to further bargaining; the achieved optimum will reflect the risk attitude of the firm which is effectively responsible for damages. We also show that a risk preferrer and a risk averter demand different levels of output reduction in the bribery market. In Section III the polluter is held legally responsible; the results parallel those in Section II, with the polluter's risk attitude incorporated into the equilibrium conditions.

In Section IV we show that the government, by use of lump sum or per unit taxes (subsidies), can alter the allocation of resources; therefore, it can correct any bribery market failure which it perceives. A per unit tax (subsidy) is shown to possess an "income effect" and a "substitution effect" upon output levels whereas a lump sum tax (subsidy) has an income effect only. We state our conclusions in Section V. In the Appendix, Section A, we investigate the sensitivity of the equilibrium output levels to shifts in the probability distribution of damages. In particular, we examine an additive shift of the mean, a multiplicative shift of the variance, and a proportional shift of all moments. Each has an income effect on output levels that is akin to the effect on output of a change in fixed costs. In addition, the last case has a substitution effect. The sign of the change in output levels is shown to be related to whether risk aversion is increasing, invariant, or decreasing with wealth. In the Appendix, Section B, we develop a market which permits the sharing of risk between the polluter and the pollutee, and show that in the absence of a market for perfect risk sharing, the Coase Theorem does not hold.

I. Pareto Optimality

Consider a pair of firms, A and B, which are ineluctably linked via an externality generated by A, randomly impacting upon B. A neoclassical example of a (negative) externality is a factory which pollutes the air; this "side-product" harms a nearby washerwoman; the degree of damage is dependent upon imperfectly predictable climatic conditions.

Firm A's profits are a function of its own output and a set of known parameters, simply written as $\Pi(Q)$. We assume Π to be a continuous, twice-differentiable function with a unique maximum. Firm B's profits are a function of the outputs of both firms, written as $\pi(Q,q)$. Uncertainty is introduced by allowing the impact of Q upon π to vary stochastically. The π is distributed as $(\bar{\pi}, v)$, where $\bar{\pi}$ is the expected value of π and v is the variance of profits about the mean.[2] For expositional ease we will treat the externality as negative; the mathematics imposes no such constraint. Both firms are regarded as price takers in all markets; they need not be perfect competitors.

A generally accepted economic doctrine is that full internalization of the externality is necessary if Pareto optimality is to obtain. The (conceptually) simplest method of accomplishing this is a merger of A and B. However, even this merged firm cannot know a priori the exact effect its output decisions will have on profits; it must consider myriad possibilities, weighting them on the basis of their likelihood and desirability. Thus, the merged firm will maximize its expected utility of profits.

We utilize a Taylor series to describe the utility function.[3,4] While the series may be

inferior factors which Charles Plott has noted. Our model can be extended to incorporate such factors.

[2]We assume that there exists a known relationship between Q and a physical measure of "pollution at the smokestack" which is independent of the state of nature. Uncertainty occurs as "pollution enters the air." Both $\bar{\pi}$ and v are functions of Q and q.

[3]Otto Loistl has shown that this assumption is not as innocent as it appears at first blush. However, Kenneth Arrow has argued that for a utility function to satisfy the von Neumann-Morgenstern axioms it must be bounded from above. A Taylor series expansion of such a utility function will converge to the function; therefore, our use of a Taylor series is legitimate. Note that truncating the series at, say, the fourth-order term would complicate our mathematics without materially affecting the results.

[4]To avoid the possibility of cyclical majorities, decisions are made by an individual At this time the

expanded about any arbitrary values, wise choices ought to lead to an economically enlightening conclusion. We believe such numbers to be Π^* and $\bar{\pi}^*$. They are the optimal level of profits from part *A* and the optimal level of expected profits from part *B* of the merged firm. We start by expanding the utility of profits:

$$(1)\quad u(\Pi + \pi) = u(\cdot) + u'(\cdot)\{(\Pi - \Pi^*) + (\pi - \bar{\pi}^*)\} + (1/2)u''(\cdot)\{(\Pi - \Pi^*)^2 + 2(\Pi - \Pi^*)(\pi - \bar{\pi}^*) + (\pi - \pi^*)^2\} + 0^{(3)}$$

Primes denote derivatives, $(\cdot) \equiv (\Pi^* + \bar{\pi}^*)$, and $0^{(3)}$ denotes terms of order 3 and above. We truncate the series by treating $0^{(3)}$ as negligible. Retention of the remainder would complicate the analysis without affecting its essential components. Take the expected value of (1), noting that the variance is defined as $E[(\pi - \bar{\pi})^2] = E[\pi^2] - \bar{\pi}^2$ and that the expansion of $u(\Pi + \bar{\pi})$ about Π^* and $\bar{\pi}^*$ (rather than $u(\Pi + \pi)$ about Π^* and $\bar{\pi}^*$, as in equation (1)) is very similar to (1). It follows that the expected value of (1) may be expressed as

$$(2)\quad E[u(\Pi + \pi)] = u(\Pi + \bar{\pi}) + (1/2)u''(\cdot)v$$

In (2), $(1/2)u''(\cdot)$ is a specific number while v and $u(\Pi + \bar{\pi})$ are functions of both Q and q.

The merged firm will maximize (2) by setting the partial derivatives to zero (throughout this paper we assume that second-order conditions are satisfied):

$$(3a)\quad u'(\Pi + \bar{\pi})(\Pi_Q + \bar{\pi}_Q) + (1/2)u''(\cdot)v_Q = 0$$

$$(3b)\quad u'(\Pi + \bar{\pi})(\bar{\pi}_q) + (1/2)u''(\cdot)v_q = 0$$

a subscript denotes the partial derivative with respect to the argument (i.e., $\partial\Pi/\partial Q \equiv \Pi_Q$). When equations (3) hold, $\Pi = \Pi^*$ and $\bar{\pi} = \bar{\pi}^*$, so we may rewrite (3) as:

$$(4a)\quad \Pi_Q + \bar{\pi}_Q = -\frac{1}{2}\frac{u''(\cdot)}{u'(\cdot)}v_Q = \frac{1}{2}r(\cdot)v_Q$$

decision maker is the manager of part *B*, the expansion is of his utility function.

$$(4b)\quad \bar{\pi}_q = -\frac{1}{2}\frac{u''(\cdot)}{u'(\cdot)}v_q = \frac{1}{2}r(\cdot)v_q$$

where $r(\cdot)$ is the Pratt-Arrow measure[5] of absolute risk aversion evaluated at profit level $(\cdot)$. Since profit is a "good," $u'(\cdot) > 0$. The sign of $u''(\cdot)$ defines the attitude towards risk. If it is positive the firm is a risk preferrer; if zero, the firm is risk neutral. Risk aversion is defined as $u''(\cdot) < 0$. Obviously, $r(\cdot)$ is of the opposite sign from $u''(\cdot)$.

The firm chooses the output combination $\{Q^*, q^*\}$ which satisfies equations (4). Thus, it sets its marginal profit from each type of output (net of damages) equal to one-half $r(\cdot)$ times the rate of change of the variance of π with respect to output.[6] Call $(1/2)r(\cdot)v_i$, $i = Q, q$, the firm's adjusted risk attitude. If in equilibrium $v_i = 0$, the firm's inclination towards risk is irrelevant; it acts as if it were an expected profit maximizer. This can occur if v has a maximal value, if the randomness of π is unrelated to output (known as system uncertainty), or if the variance itself is zero. Thus, the traditional analysis of certainty is contained within our model as a special case.

Because equations (4) reflect the utility function of the decision maker, we ask if output levels are dependent upon whoever controls the corporation. In particular, if the manager of part *A* were elevated to control, his measure of absolute risk aversion $R(\cdot)$ would replace $r(\cdot)$ in equations (4). Does $R(\cdot) = r(\cdot)$? A further question concerns nationalization, or at least governmental intervention: what is $S(\cdot)$, society's attitude towards risk? If $S(\cdot) \neq R(\cdot), r(\cdot)$, society may wish to intervene in the allocative process.

Arrow and Robert Lind argue that if the profits of the (merged) firm are statistically independent of other components of national income, if there is no corporate in-

[5]See Arrow or John Pratt.

[6]If Q or q is zero, $v = 0$. Thus both v_Q and v_q must be positive over some output range; this is Hayne Leland's "principle of increasing uncertainty." However, saturation levels of pollution may be reached, thus v_Q and v_q may become nonpositive at sufficiently high output levels

come tax, if there are a large number of shareholders (each holding a small portion of his wealth in this firm), and "... if managers were acting in the interest of the firm's shareholders, they would essentially ignore risks ..." (p. 376). Thus, $r(\cdot) = 0 = R(\cdot)$. They also argue that the government should act in the same manner ($S(\cdot) = 0$). Clearly, if the Arrow-Lind assumptions hold we have certainty equivalence and the Coase Theorem always holds.

However, indivisibilities may negate these results. An autonomous manager who receives a significant segment of his income from the firm will obey his own, not the market's, risk measure. Similarly, if "... in order to control the firm, some shareholder [holds] a large block of stock which is a significant component of his wealth" (Arrow-Lind, p. 376), the firm should use his, not the market's, risk measure. With indivisibilities, corporate control matters. The externality will always be fully internalized, although the sense of fully will not always be the same. Society may then wish to intervene to obtain a Pareto optimal resource allocation.

Prior to discussing potential governmental intervention, we investigate liability rules under which the independent firms A and B may bargain privately to lessen the impact of the externality. Before bargaining can occur, "[I]t is necessary to know whether the damaging business is liable or not for damage caused since without the establishment of this initial delimitation of rights there can be no market transactions to transfer and recombine them" (Coase, p. 8).

II. Legally Permissible Pollution

Consider the case of a legal system freely granting the right of unlimited generation of an externality to firm A. Firm B is injured by the externality and, therefore, has cause to seek an improvement in its own situation.[7] While protective measures such as physical movement of its plant or alteration of its productive process to a less affectable technology are possibilities, they will not concern us here. We are interested in the extent to which the two firms may bargain to their mutual advantage—firm B by obtaining a profitable reduction in Q and firm A by profitably reducing its own output.

For every unit reduction in Q, firm B's profits rise; it follows that B must be willing to pay some sum of money, not exceeding its marginal profit gain, to obtain a unit reduction in Q. In fact, B must have a demand curve for a reduction in Q, a curve which measures the marginal benefit to B of such a reduction. If the loss to B rises at an increasing rate with added units of A's output, the demand curve will have the usual negative slope. In contrast, firm A will agree to reduce its own output if its marginal profit loss is compensated by a payment from B. Firm A possesses a supply curve for the reduction of its output which is exactly its own marginal foregone profit curve. If A's profits increase at a decreasing rate, its supply curve will have a positive slope.

The intersection of supply and demand curves[8] defines the optimal level of reduction in output from $\hat{Q}$, firm A's output in the absence of communications between the firms.[9] The intersection does not necessarily define the optimum from society's view; society may prefer a different intersection, for the demand curve incorporates B's adjusted risk attitude, an attitude which may not coincide with society's.

We utilize a neoclassical analysis to determine the supply curve, the demand curve and their intersection, the latter a point which possesses the characteristic that all possible gains from trade have been exhausted. In short, we treat both firms as price takers in the bribery market.

[7]Were A's production to create a positive externality, B would be interested in obtaining an expansion of A's output, the formal analysis would be the same.

[8]Ken-Ichi Inada and Koyoshi Kuga have shown the conditions under which there is no intersection, or no unique intersection

[9]$\hat{Q}$ is defined by $\Pi_Q = 0$, $\Pi_{QQ} < 0$, since A is a riskless profit maximizer who ignores the impact Q has on B.

Following the tradition established by Coase, we assume that transactions between the firms are costless. Barring obtuseness by the managers, bargaining will continue so long as there are gains from trade to be realized. We determine the reduction in Q as a result of bargaining: an allocational issue. The actual division of the gains is a distributional question which does not concern us, although it may be of interest in its own right.

Firm A's supply function of Q reduction is unaffected by the uncertain impact its output has on B. From A's view, all relevant parameters are known in advance of the production decision, including the bribery payments it receives. Thus, A maximizes its augmented profit function $\Pi(Q) + P(\hat{Q} - Q)$ where P is the per unit bribery payment and $(\hat{Q} - Q)$ is the level of output reduction.

Firm B continues to confront uncertainty. What it has done in the bribery market is buy a certain reduction in Q and an expected reduction in damages. Firm B maximizes its expected utility from post-bribery profits, found by a Taylor series expansion about $\bar{\pi}^*$, $\{P(\hat{Q} - Q)\}^*$ (the latter term is the optimal bribe).

$$(5)\quad E[u(\pi)] = u[\bar{\pi} - P(\hat{Q} - Q)] + (1/2)u''[\bar{\pi}^* - \{P(\hat{Q} - Q)\}^*]v$$

Firm A's supply function in the bribery market is

$$(6a)\qquad \Pi_Q - P = 0$$

Marginal profits from increased output are balanced against marginal gain from reduced output. For B:

$$(6b)\qquad P + \bar{\pi}_Q = (1/2)r[\cdot]v_Q$$

$$(6c)\qquad \bar{\pi}_q = (1/2)r[\cdot]v_q$$

The latter equation is the standard equilibrium condition: set marginal profits from q equal to the adjusted risk attitude. The former equation says that B's adjusted risk attitude should equal the net marginal profit from a reduction in Q (composed of two parts: the cost of buying the reduction and the savings as a result of the purchase). Firm B's demand curve is (6b) given that (6c) holds.[10]

Combining equations (6) gives the output levels in a world where pollution is permissible and firms may bargain costlessly:

$$(7a)\qquad \Pi_Q + \bar{\pi}_Q = (1/2)r[\cdot]v_Q$$

$$(7b)\qquad \bar{\pi}_q = (1/2)r[\cdot]v_q$$

The effect of imposing legal liability on B is to impose B's adjusted risk attitude on the equilibrium. While these results are similar to those in a merger controlled by B, $r[\cdot]$ is evaluated at a different wealth level. Does this effect the output levels?

THEOREM: *The Coase Theorem is not valid in an uncertain world if the legally liable firm is controlled by a dominant shareholder or an autonomous manager.*

PROOF:

Assume (4) and (7) are identical and define $\{Q^*, q^*\}$. All objective values $(\bar{\pi}_q, \bar{\pi}_Q, \Pi_Q, \bar{\pi}^*, v_Q, v_q)$ are identical in both sets of equations. Thus, $r[\bar{\pi}^* - \{P(Q - Q)\}^*] = r(\Pi^* + \bar{\pi}^*)$; but, $\Pi^* \geq 0$ while $-\{P\cdot(Q - Q)\}^* < 0$. Since the risk function is monotonic in wealth, $r[\cdot] \neq r(\cdot)$ and (7) cannot define $\{Q^*, q^*\}$.[11] Of course, if the Arrow-Lind assumptions stated in Section I hold, the Coase Theorem is valid under uncertainty.[12]

[10]Demand is a function of $r[\cdot]$ and, therefore, of B's wealth level. A fortiori, demand is dependent on the bargaining process chosen; the total gains from trade are not independent of their distribution.

[11]Two brief comments are in order: 1) It is not clear that either (4) or (7) define a first best Pareto optimal allocation since we have precluded the possibility of risk sharing. 2) In the event of a merger one would expect that B would compensate A. When compensation is paid (4) and (7) may define the same allocation by coincidence.

[12]An alternative approach to the stock market, proposed by Franco Modigliani and Merton Miller, assigns firms to risk classes, Jan Mossin has shown that a firm's risk class is $-C\sum_k \sigma_{jk}$, where C is "... the same for all companies and can be given an interpretation as market risk aversion" (p. 753). The term $\sum_k \sigma_{jk}$ is the sum of the covariances of profit of company j with all other companies, including itself (i.e., its variance). If the firm's manager takes his risk attitude from the market ($r = -C\sum_k \sigma_{jk}$), the optima defined by equa-

Within the bribery market, the position of B's demand curve is affected by its adjusted risk attitude. If this is zero (certainty equivalence) the supply-demand intersection occurs at a particular price-reduction combination. In contrast, a risk-averse firm will obtain more reduction at a higher price if v_Q is positive, but less reduction at a lower price if v_Q is negative. The reason for this latter case is that extra Q lessens the variance of profits, an event which the risk-averse firm finds attractive. Correspondingly, if $v_Q > 0$, a risk preferrer will demand a smaller reduction and offer a lower per unit bribe than a risk-neutral firm.[13] The risk preferrer is a "tougher bargainer" because it perceives itself as having less to gain from trade—in fact, if B is a sufficiently strong preferrer of risk, there will be no gains from trade available to anyone.[14] Notice that if there are potential gains from trade available, they will generally be divided between the firms. Both firms have market power—the power to block an agreement is the power to obtain part of the gains. There is absolutely no validity to the naive view expressed by James Marchand and Keith Russell that because B is (effectively) liable, A obtains all the gains from trade.

tions (4) and (7) are identical given the present formulation of the problem. That society's risk attitude should be the same as the private risk attitude for this risk class has been shown by Agnar Sandmo. However, a slight reformulation of our problem will cause the optima of (4) and (7) to diverge. Write A's profits as $\Pi(Q) + \epsilon$; $\epsilon \sim (0, \sigma^2)$, so that $\epsilon_q = 0 = \epsilon_Q$. (This construction of A's profit function will leave its first-order maximization conditions unaffected.) Let the covariance of A's and B's profits be zero with respect to all other firms in the market. Then the merged firm has as its risk attitude $-C(\sigma^2 + v + COV(\pi, \Pi))$. When pollution is legally permissible, the market assigns it to the risk class $-C(v + COV(\pi, \Pi))$. Firm B does not take cognizance of the system uncertainty which confronts A; thus, B has a different risk class than does the merged firm and equations (4) and (7) are not identical

[13] Ira Horowitz, p. 367, reaches a similar conclusion for a related problem.

[14] If B is an extreme risk preferrer, the supply and demand curves intersect to the left of the vertical axis. Firm B demands an increase in Q.

III. Pollution not Permissible without Compensation

Suppose the legal system permits firm A to befoul the environment to some limit at no penalty but requires that a firm harmed by excessive pollution be fully compensated for its lost profits. If the legal limit is effective—if A exceeds the limit in the presence of penalties—and if B's profits rise with its own output *ceteris paribus*, then A's legally mandated damage payments to B are positively related to q. Firm A has dual incentives: to lower its own output and to persuade B to lessen its production; both events will improve A's profits.

While reduction of Q is an internal matter, reduction of q requires the cooperation of B. Firm B is always willing to curtail its production if it is amply rewarded; its minimal supply price is its relinquished marginal profits. If B's profits increase at a decreasing rate with additional q, B's supply of reduced output will have a positive slope. Firm A will demand an output reduction of q so long as it can buy the reduction for no more than its maximal demand price. This is the marginal decrease in its legally mandated "excessive pollution" payment; therefore, A's demand curve is a marginal benefit curve. It will have a negative slope if, as q is curtailed, there is a greater profit reduction for firm B at lower (more acceptable) pollution levels than at higher ones.

Once again, the intersection of the supply and demand curves in the bribery market will define the optimum from the viewpoint of the involved parties. We utilize a neoclassical pricing approach to define the intersection and to distribute the gains from trade, treating the firms as price takers in the bribery market. Reduction occurs from the level $\hat{q}$, the amount of output produced by B in the absence of interfirm negotiation but in the presence of the legally mandated payments.

Firm A's profit is given by

(8)
$$\Pi(Q) - [\bar{\pi}(M,q) - \pi(Q,q)] - p(\hat{q} - q)$$

where $p(\hat{q} - q)$ is the bribe paid; its level is determined by costless negotiation prior to production. Firm A's profit from its own output $(\Pi(Q))$ is also known a priori. The legally mandated damage payment to B—the bracketed term—is known only after the fact. Thus, A confronts uncertainty. Firm B is guaranteed the difference between (a) the expected level of its own profits when $Q = M (M \geq 0)$, the output creating the mandated level of expected externalities, and (b) the actual level of its profits. Notice that when the state of nature retards pollution, $\pi(Q,q)$ rises and damage payments fall. Firm A, not B, benefits from a favorable state of nature; A, of course, will maximize its expected utility from net profits.

Firm B's guaranteed profits are $\pi(Q,q) + [\bar{\pi}(M,q) - \pi(Q,q)] + p(\hat{q} - q) = \bar{\pi}(M,q) + p(\hat{q} - q)$; B no longer confronts uncertainty.[15] In effect, the legal system causes A to insure B against risk. Firm B's supply curve is

$$\text{(9a)} \qquad \bar{\pi}_{q|M} - p = 0$$

where $\bar{\pi}_{q|M} \equiv \partial\bar{\pi}(M,q)/\partial q$. A's first-order conditions, found after utilizing a Taylor series to expand the expected utility of (8) about Π^*, $\bar{\pi}^*$, $\bar{\pi}^*(M,q)$, $\{p(\hat{q} - q)\}^*$, are

$$\text{(9b)} \qquad \Pi_Q + \bar{\pi}_Q = (1/2)R[\cdot]v_Q$$

$$\text{(9c)} \qquad p = (1/2)R[\cdot]v_q + (\bar{\pi}_{q|M} - \bar{\pi}_q)$$

Firm B's supply of reduced output (9a) is defined by the condition that marginal bribery gain equal marginal profit loss. Equations (9b) and (9c) define A's demand curve. The former says that marginal profits from Q, net of external damage to B, should equal A's adjusted risk attitude. The latter sets the marginal bribery cost p equal to the adjusted risk attitude plus the marginal legal required expected damage payment. Combining equations (9) yields a set which characterizes the equilibrium position:

$$\text{(10a)} \qquad \Pi_Q + \bar{\pi}_Q = (1/2)R[\cdot]v_Q$$

$$\text{(10b)} \qquad \bar{\pi}_q = (1/2)R[\cdot]v_q$$

These are identical to the Pareto optimal conditions (4) only under the same restrictions given for equations (7) in Section II.

We have shown that the assignment of liability for externalities may be of consequence. The final allocation of resources is dependent upon liability rules when there is uncertainty as to the state of nature and when indivisibilities preclude the complete sharing of risk.

IV. Social Intervention and Pareto Optimality

Suppose costless negotiation does not lead to the optimum defined by equations (4). In addition to the reasons stated above, "... if one accepts the proposition that the state is more than a collection of individuals and has an existence and interests apart from those of its individual members, then it follows that government policy need not reflect individual preferences" (Arrow-Lind, p. 365). Thus, the government may wish to intervene even if equations (7) and/or (10) define $\{Q^*,q^*\}$. Second best questions aside, can the government improve (in its view) the allocation of resources? The answer is yes; per unit and/or lump sum taxes (or subsidies) may be utilized.

To economize on space, we investigate only the case of permissible pollution. We start by stating a set of simplifying assumptions. They are not crucial to the analysis. Let $S(\cdot) = 0$, $r(\cdot) > 0$, and the Principle of Increasing Uncertainty hold (i.e., $v_t > 0$). The optimum is defined as $\Pi_Q + \bar{\pi}_Q = 0 = \bar{\pi}_q$. Let there be a tax T per unit of output Q levied on firm A and another tax t per unit of output q levied on firm B. Bargaining still occurs. Profits for firms A and B, respectively, are

[15] B could refuse to bargain with A, produce the (then optimal) output level $\hat{q}$, and sue A for damages done: damages which are related to Q, $\hat{q}$, and the state of nature known to have prevailed when production occurred. Firm B rejects this avenue because its total profits $[\bar{\pi}(M, \hat{q})]$ from the lawsuit would be less than those it can obtain by bargaining. Notice that if $Q \leq M$ in the absence of negotiations, or if $\bar{\pi}_Q > 0$, we belong in Section II.

(11a) $\Pi(Q) + P(\hat{Q} - Q) - TQ$

(11b) $\pi(Q,q) - P(\hat{Q} - Q) - tq$

Firm B continues to confront uncertainty; thus, it continues to maximize its expected utility of profits. First-order conditions are now:

(12a) $\Pi_Q - P - T = 0$

(12b) $\bar{\pi}_Q + P - (1/2)r\{\cdot\}v_Q = 0$

(12c) $\bar{\pi}_q - t - (1/2)r\{\cdot\}v_q = 0$

where $\{\cdot\} \equiv \{\bar{\pi}^* - [P(\hat{Q} - Q)]^* - (tq)^*\}$. Note that T is not independent of P. While we treat P as parametric for ease of presentation, it is in practice determined by negotiation. Thus, when establishing tax/subsidy levels, the government must consider its own impact upon the bargaining process. Equations (12) combine to form

(13a) $\Pi_Q + \bar{\pi}_Q = (1/2)r\{\cdot\}v_Q + T$

(13b) $\bar{\pi}_q = (1/2)r\{\cdot\}v_q + t$

The optimal tax levels are $T = -(1/2)r\{\cdot\}v_Q < 0$ and $t = -(1/2)r\{\cdot\}v_q < 0$; the optimal taxes are subsidies because firm B is more risk averse than society.[16]

There are three points of interest here. First, apart from questions of income distribution, $-TQ$ could be replace by $+T(\hat{Q} - Q)$. The effect on the shadow price of the marginal unit of Q is what matters and it would be unaltered. Second, $-TQ$ (or $+T(\hat{Q} - Q)$) could appear in (11b) instead of (11a); equations (13) would be of the same form although the value of $r\{\cdot\}$ would in general change. Third, neither T nor t can be set equal to the right-hand side of equations (7) because nonzero taxes have an effect upon the wealth level at which $r\{\cdot\}$ is evaluated. This final point can be seen by taking the total derivative of (12b) and (12c) and rearranging terms. We obtain

(14a) $$\frac{\partial Q}{\partial t} = \frac{qr'(v_q L_{Qq} - v_Q L_{qq})}{2D} - \frac{L_{Qq}}{D}$$

(14b) $$\frac{\partial q}{\partial t} = \frac{qr'(v_Q L_{Qq} - v_q L_{QQ})}{2D} + \frac{L_{QQ}}{D}$$

where r' is the rate of change of the Pratt-Arrow measure of absolute risk aversion due to a change in wealth; $L_{qq} < 0$, $L_{QQ} < 0$, $D \equiv L_{QQ}L_{qq} - L_{Qq}^2 > 0$, all from second-order conditions; and P is parametric.

Equations (14) may be expressed more simply as:

(15a) $$\frac{\partial Q}{\partial t} = q\frac{\partial Q}{\partial f} - \frac{L_{Qq}}{D}$$

(15b) $$\frac{\partial q}{\partial t} = q\frac{\partial q}{\partial f} + \frac{L_{QQ}}{D}$$

where $(\partial Q/\partial f)$ and $(\partial q/\partial f)$ are the effect on output levels of a change in fixed costs.[17] Their signs are dependent upon whether the firm's risk attitude is increasing, invariant, or decreasing in wealth ($r' \gtreqless 0$) and upon the value of the parenthetical term in (14). If $L_{Qq} \geq 0$, our earlier assumptions with the second-order conditions guarantee that $(\partial Q/\partial f)$ and $(\partial q/\partial f)$ are of the same sign[18] as r'. However, if $L_{Qq} < 0$, we cannot in general determine the effect a change in wealth has upon output. Now, from equations (15), we see that a change in the per unit subsidy (from, say, zero) has a dual impact upon output levels. The first is the income effect: q times a pure wealth effect; the second is a substitution effect which is negative for $(\partial q/\partial t)$ and of uncertain sign for $(\partial Q/\partial t)$. Note that a lump sum tax has a pure wealth effect on output levels since it is equivalent to a change in fixed costs.

Consider now an impediment to bargaining which prevents any interfirm negotiations. Then $P = 0$ in equations (11) and (12a), while (12b) does not "exist" because

[16]There would be a positive tax if society were the more risk averse. Note that if $v_Q > 0$ and $v_q < 0$, society would be in the peculiar position of subsidizing the polluting output and taxing the output of q.

[17]They are obtained by explicitly considering fixed costs f, by writing profits as $\pi(Q, q) - f$, and totally differentiating the first-order conditions. The effect of f on the first-order conditions occurs only in the wealth level at which $r[\cdot]$ is evaluated.

[18]$L_{Qq} = 0$ is not guaranteed by an additively separable profit function.

Q is not a choice variable for firm B. The government can still create an optimal allocation of resources by setting $T = -\bar{\pi}_Q$ and $t = -(1/2)r\{\cdot\}v_q$, $\{\cdot\} \equiv \{\bar{\pi}^* - (tq)^*\}$. The first point mentioned in association with equations (13) remains valid, as does the third. The second no longer holds due to the lack of communication. This communicative absence is corrected by the tax T. The subsidy t is used to rectify misallocations caused by the deviation of B's adjusted risk attitude from society's attitude.

In the case of (bribery) market failure the government can, in principle, intervene to create a Pareto optimal allocation of resources. However, the wealth effect (absent with certainty equivalence) compels the government to know firm B's risk function, not just its value at a (nonoptimal) set of output levels defined by equation (7).

V. Conclusion

When one firm's productive process imposes an externality, positive or negative, upon another firm, those enterprises have cause to attempt to interact in a private bribery market in order to improve both of their profit levels. The nature of the bribery market is determined by the legal system. If A is liable for damages, it will demand of B a reduction in B's output (and thereby achieve a cutback in its damage payments). Firm B will supply an output curtailment so long as its marginal profits foregone are covered by A. The intersection of supply and demand in the bribery market defines the equilibrium position from which no gains from trade remain; thus, at the close of bargaining no Pareto-relevant externalities exist, although there are (in general) externalities present. When A is not liable for damages, the same type of analysis and the same conclusions apply, the difference is that it is B which demands an output reduction from A. These are the standard externality results from which many authors have concluded that the socially optimal allocation of resources occurs as an outcome of the costless bargaining process. The allocation is, they say, independent of the assignment of liability.

When the externality is randomly distributed, uncertainty confronts the firm which is effectively liable. This firm must then incorporate its own attitude toward risk along with the distribution of the externality into its decision rule. We have utilized a Taylor series expansion to embody these facts of economic life into the enterprise's optimization process, a maximization which occurs across the expected utility of profits.

In an uncertain world, liability rules may determine resource allocation. Costless bargaining is not sufficient to guarantee that the Coase Theorem holds; it is also necessary that risk may be shared (say through a stock market) and that there be no indivisibilities. Without the possibility of sharing risk the involved firms will have the same attitude towards risk only by accident. Indivisibilities—an autonomous manager or a dominant shareholder—also preclude a complete sharing of risk even in the presence of a stock market.

When these stronger assumptions do not obtain, the risk attitude of the manager of the firm made responsible for damages is embodied in the equilibrium conditions which derive from bargaining. Resource allocation is affected by legal liability as well as by the bargaining skills of the involved firms. However, the government, by use of a tax/subsidy scheme, can intervene to create a Pareto optimal level of outputs. The attractive results of the Coase Theorem (as usually stated) are a special case in an uncertain world.

Appendix

A

Here we examine the output effect upon both firms of a change in the distribution of π, concentrating upon the legal rule of permissible pollution. We investigate three types of change: a linear shift of expected profits, holding all moments about the mean constant; a spreading of the distribution about a constant mean, summarized as a multiplicative shift of the variance; and a proportional shift of all moments.

The three cases may be stated as

$$A: \pi \sim (\alpha + \bar{\pi}, v)$$
$$B: \pi \sim (\bar{\pi}, \beta v)$$
$$C: \pi \sim (\lambda\bar{\pi}, \lambda^2 v)$$

At this time we explicitly introduce fixed costs by writing profits as $\pi(Q,q) - f$, ($f \geq 0$), in order to isolate an income effect. Of course, f is functionally equivalent to a lump sum tax.

Case A: The procedure is to substitute $\alpha + \bar{\pi}$ for $\bar{\pi}$ in equations (6). Consider a marginal change in α from its initial level such that the expected utility of profits remains maximized. This requires that we take the total derivative and evaluate it at $\alpha = 0$. Manipulation shows

$$\text{(A1)} \qquad \partial X/\partial\alpha = -\partial X/\partial f$$

where X designates Q or q. A marginal increase in expected profits has the same effect as a marginal decrease in fixed costs. There is a pure income effect upon both output levels.

To obtain (A1) we assumed that the firm incurred no cost to create a change in α. Since the firm might invest in pollution abatement equipment, it seems worth pointing out that such a situation would add another term to the right-hand side of (A1): + $(\partial X/\partial f)(\partial f/\partial\alpha)$. Because the firm would voluntarily invest in abatement equipment only if $(\partial\alpha/\partial f) > 1$, we can state that the sign of $(\partial X/\partial\alpha)$ is the negative of the sign of $(\partial X/\partial f)$ whether a change in α is endogenous or exogenous.

Case B: The procedure is the same as in Case A, with $(\partial f/\partial\beta) = 0$ and the total derivative evaluated at $\beta = 1$, the original variance level. Thus,

$$\text{(A2)} \qquad \frac{\partial X}{\partial\beta} = -\left[\frac{\partial X}{\partial f}\right]\left[\frac{r}{r'}\right]$$

We now obtain a weighted income effect. The weight is the ratio of firm B's measure of absolute risk aversion to its rate of change. An increasingly risk-averse firm ($r, r' > 0$) will respond to spreading distribution in the same manner as to an increase in expected profit. Conversely, a decreasingly risk-averse firm ($r' < 0 < r$) will react oppositely to an increased variance than to an increased mean.

Case C: The procedure is as above, with evaluation occurring at $\lambda = 1$. The output effect of a proportional change in all moments is

$$\text{(A3)} \qquad \frac{\partial Q}{\partial\lambda} = \left[\frac{\partial Q}{\partial\beta} + \bar{\pi}\left(\frac{\partial Q}{\partial\alpha}\right)\right] + \frac{PL_{qq}}{D}$$

$$\text{(A4)} \qquad \frac{\partial q}{\partial\lambda} = \left[\frac{\partial q}{\partial\beta} + \bar{\pi}\left(\frac{\partial q}{\partial\alpha}\right)\right] - \frac{PL_{Qq}}{D}$$

There is an income effect and a substitution effect. The bracketed term is similar to the preceding case, although determination of the sign is less obvious if r' and r are not of the same sign. The substitution effect is of determinable sign only for (A3). Since $D > 0$ and $L_{qq} < 0$ by the second-order conditions, and since the marginal bribe (P) is positive, the substitution effect for $(\partial Q/\partial\lambda)$ is negative. The substitution effect for $(\partial q/\partial\lambda)$ is of the opposite sign from the (unknown) sign of L_{Qq}.

B

In the body of this paper we considered what is essentially a polar case; there is no mechanism for the sharing of risk. In this polar case the Coase Theorem does not hold strictly. It was argued that in the opposite polar case, in which there is perfect sharing of risk, the Coase theorem holds. This part of the appendix considers briefly an intermediate case in which the parties may share risk. Assume that B is liable for damages and as a consequence has uncertain profits. The essential trade is for B to transfer to A a portion of the deviation of his profit from its expected value and for B to compensate A for the service A renders. Since A absorbs some risk his expected utility depends in part on q and there is the possibility of a market existing for a change in the level of q. Allowing for these possibilities we write B's augmented profits as

$$\text{(A5)}\quad \bar{\pi}(Q,q) + \delta[\pi(Q,q) - \bar{\pi}(Q,q)] - P(\hat{Q} - Q) + p(\hat{q} - q) - b(1 - \delta)$$

where δ is the share of B's profit deviation retained by B, and b is the unit price B pays A for taking a share. Firm B's expected utility is

$$\text{(A6)}\quad u[\bar{\pi}(Q,q) - P(\hat{Q} - Q) + p(\hat{q} - q) - b(1 - \delta)] + (1/2)u''(\cdot)\delta^2 v$$

This is maximized where:

$$\text{(A7)}\quad \begin{aligned} \bar{\pi}_Q + P &= r(\delta)^2 v_Q \\ \bar{\pi}_q - p &= r(\delta)^2 v_q \\ b &= 2r\delta v \end{aligned}$$

By means of a similar exercise A's expected utility is

$$\text{(A8)}\quad U[\Pi(Q) + P(\hat{Q} - Q) - p(\hat{q} - q) + b(1 - \delta)] + (1/2)U''(\cdot)(1 - \delta)^2 v$$

which is maximized when

$$\text{(A9)}\quad \begin{aligned} \Pi_Q - P &= R(1 - \delta)^2 v_Q \\ p &= R(1 - \delta)^2 v_q \\ b &= 2R(1 - \delta)v \end{aligned}$$

Letting each market clear simultaneously leaves

$$\text{(A10)}\quad \begin{aligned} \Pi_Q + \bar{\pi}_Q &= (Rr/(R + r))v_Q \\ \bar{\pi}_q &= (Rr/(R + r))v_q \end{aligned}$$

When A is liable a similar set of conditions is found, but they will not in general imply the same allocation since the risk attitudes will not be evaluated at the same income levels.

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