Global Stability of Financial Networks Against Contagion: Measure, Evaluation and Implications

Bhaskar DasGupta*† and Lakshmi Kaligounder
University of Illinois at Chicago
Chicago, IL 60607, USA
Email: {bdasgup,lkalig2}@uic.edu

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Abstract

Involvements of major financial institutions in the recent financial crisis have generated renewed interests in fragilities of global financial networks among economists and regulatory authorities. In particular, one potential vulnerability of the financial networks is the "financial contagion" process in which insolvencies of individual entities propagate through the "web of dependencies" to affect the entire system. In this paper, we formalize a banking network model originally proposed by researchers from Bank of England and elsewhere that may be applicable to scenarios such as the OTC derivatives market, define a global stability measure for this model, and comprehensively evaluate the stability measure over more than 700,000 combinations of networks types and parameter combinations. Based on such comprehensive evaluations, we discuss some interesting implications of our evaluations of this stability measure, and derive topological properties and parameters combinations that may be used to flag the network as a *possible* fragile network.

Keywords: financial networks, stability, financial contagion, network models.

*Web: http://www.cs.uic.edu/∼dasgupta
†Corresponding author.

1 Introduction

The recent global financial crisis at an unprecedented level have exposed potential weaknesses of the global economic system, renewing interests in the determination of fragilities of various segments of the global economy. Since financial institutions played a crucial role in this crisis, financial system governed by borrowing, lending and participation in risky investments have attracted a major part of the attention of economists; see [1, 2] for a survey. The issue of instability of free market based financial systems is not new and has been under discussion among the economists starting with the early works of Fisher [3] and Keynes [4] during the 1930's great depression era. However, the exact causes of such instabilities have not been unanimously agreed upon yet. For example, economists Ekelund and Thornton contend that a major reason for the recent financial crisis is the enactment of the Gramm-Leach-Bliley Act in 1999 that removed several restrictions on mixing investment and consumer banking [5], whereas many other economists such as Calabria disagree with such an assertion [6]. Some economists such as Minsky in fact have argued that such instabilities are are systemic (*i. e.*, inherent) for many modern market-based economic systems [7].

Our motivation in this paper to investigate the issue of global stabilities of financial networks comes from the point of view of a regulatory agency (as was also done, for example, in [1]). A regulatory agency with sufficient knowledge about a significant part of a global financial network is expected to periodically evaluate the stability of the network, and flag the network *ex ante* for further analysis if it fails some preliminary test or exceeds some minimum threshold of vulnerability. In this motivation, flagging a network as vulnerable does not necessarily imply that such is the case, but that such a network requires further analysis based on other aspects of free market economics that are not or simply cannot be modeled[1]. While too many false positives may drain the finite resources of a regulatory agency for further analysis and investigation, this motivation assumes that vulnerability is too important an issue to be left for an *ex post* analysis.

2 Informal overview of our contribution

Although simpler topological properties such as clustering coefficients have been used by authors to study properties of financial network [8, 9], they are too simplistic to be useful in the type of stability analysis done in this paper. Here we formalize a simplified *ex ante* financial network contagion model similar to what has been used by prior researchers from Bank of England and elsewhere [1, 10–12], and define a global stability measure for this model. We perform a thorough evaluation and analysis of this stability measure *over more than 700,000 combinations of networks types and parameters.* Such a comprehensive analysis allows us to uncover many interesting insights into the relationships of the stability with other relevant parameters of the network, such as:

Effect of unequal asset distribution: More uneven distribution of assets among banks in a banking network tend to make the network more vulnerable. Moreover, a banking network with uneven distribution of assets among banks tends to exhibit the following properties:

- Failures of those banks with disproportionately higher assets contribute more damage to the long-term stability of the network.

[1]For example, some such factors are the rumors and panics caused by the insolvency of a large bank and a possible subsequent credit freeze. While fears, panics and rumors are all real aspects in networked economics, there are hardly any universally agreed upon good way of modelling them.

- These networks have a *minimal instability even if their equity to asset ratio is large and comparable to loss of external assets*. This is not the case for networks where the distribution of assets among banks is uniform.

Thus, in summary, we conclude that banks with disproportionately large external assets affect the stability of the entire network in an *adverse* manner.

Effect of total amount of assets: Stabilities of networks with highly uneven distribution of assets among banks are affected *very little* by the amount of total external assets.

Effect of connectivity: For banking networks where all banks have the same amount of external assets, higher connectivity leads to *lower* stability. In contrast, for banking networks in which few banks have disproportionately higher external assets compared to the remaining banks, higher connectivity leads to *higher* stability.

Correlated versus idiosyncratic insolvencies: *Correlated* initial insolvencies of banks cause significantly more damage to the stability of a banking network as opposed to *idiosyncratic* initial insolvencies.

Phase transition properties of stability: The stability measure exhibits several sharp *phase transitions* for various banking networks and combinations of parameters. We have also provided intuitive theoretical explanation for one such phase transition.

Although the issue of stability of financial systems has been discussed by prior researchers [1, 10–17, 19, 20, 23], *to our knowledge no prior paper has mathematically defined a global stability measure and performed a comprehensive evaluation of such a measure as is done in this paper.*

2.1 Policy implications

Returning to our original motivation of flagging financial networks for potential vulnerabilities, our empirical results suggest that a network model similar to that used in the paper may be flagged for the following cases:

- the equity to asset ratio of banks in the network is extremely low, or
- the network has a highly skewed distribution of external assets and inter-bank exposures among its banks and the network is sufficiently sparse, or
- the network does not have either a highly skewed distribution of external assets or a highly skewed distribution of inter-bank exposures among its banks, but the network is sufficiently dense.

The reader should note that the above policy implications are valid only in the context of the specific network model and parameter ranges considered in the paper.

3 Network model and global stability measure

An investigation of the stability of a financial system may begin by addressing the following items:

network model: How do we precisely model the individual components and their mutual interactions in the financial system under consideration ?

insolvency propagation: How does insolvencies of individual components of the financial system affect the overall network over time ?

global stability evaluation: How do we evaluate the global stability of the financial system ?

We describe these items in the sequel.

3.1 The network model

As was done by prior researchers, for simplicity we formulate our results in terms of balance-sheet insolvency cascades in a network of financial institutions (hereafter called "banks" and "banking networks") with interlinked balance sheets, where losses flow into the asset side of the balance sheets. The same formulation can be reused with appropriate modifications for analyzing cascades of cash-flow insolvency in OTC markets. From now on, we refer to balance-sheet insolvency simply as insolvency.

A banking network is represented by a node-weighted and edge-weighted directed graph $G = (V, F)$ in which the node set V represents the banks and the edge set F represents *direct* inter-bank exposures. Let the notation $\mathsf{in\text{--}degree}(v)$ denote the in-degree of node $v \in V$, and the notation $w(e) = w(u, v) > 0$ denote the weight of an edge $e = (u, v) \in F$. *Throughout, we use E and I to denote the total external asset and the total inter-bank exposure of the banking network, respectively.* Two types of banking networks are possible:

homogeneous network model: E and I are equally distributed between the nodes and the links, respectively, and

heterogeneous network model: E and I may not be equally distributed between the nodes and the links, respectively.

For both types of networks, we define the following quantities:

- σ_v denotes the share of the total external asset E for bank $v \in V$.
- $\gamma \in (0, 1)$ denotes the ratio of equity to asset.
- $\iota_v = \sum_{(v,u) \in F} w(v, u)$ denotes the total interbank asset of bank $v \in V$.
- $b_v = \sum_{(u,v) \in F} w(u, v)$ denotes the total interbank borrowing of bank $v \in V$.
- $e_v = (b_v - \iota_v) + \sigma_v \left(E - \sum_{v \in V} (b_v - \iota_v) \right)$ denotes the effective share of the total external asset E for bank $v \in V$.
- $a_v = e_v + \iota_v$ denotes the total asset of bank $v \in V$.
- $c_v = \gamma \, a_v$ denotes the net worth (equity) of bank $v \in V$.

Both homogeneous and heterogeneous network models are relevant in practice, and have been investigated by prior researchers. Calculations of these parameters for a simple 5-node banking network are shown in Section A in the appendix.

3.2 Initial insolvency via shocks

The initial insolvencies of a banking network at time $t = 0$ are caused by "shocks" received by a subset of banks. Such shocks can occur, for example, due to *operational risks* (*e. g.*, frauds[2]) or *credit risks*, and has the effect of *reducing* the external assets of a selected subset of banks. Mathematically, we are given the

[2] Studies such as by Iyer and Peydro [34] show that fraud is an important cause of bank insolvency.

real number $\Phi \in (0,1] > \gamma$ denoting the *severity* of the initial shock, and the effect of the initial shock is to *simultaneously* decrease the external assets of each shocked bank v from e_v by $s_v = \Phi\, e_v$, thereby reducing the net worth of v from its original value c_v to its new value $c_v - s_v$. *By the phrase "shocking mechanism", we refer to the rule that is used to select the initial subset of nodes to be shocked.*

3.3 Insolvency propagation equation

The insolvencies are assumed to propagate in discrete time units, say $t = 0, 1, 2, \ldots$. We add "(t)" to all relevant variables to indicate their dependences on time. A bank becomes insolvent if its modified net worth becomes negative, and such a bank is *removed* from the network in the next time step. Let $V_{\text{insolvent}}(t) \subseteq V$ be the set of banks that have become insolvent *before* time t. The insolvency of a bank at time t affects the equity of other banks in the network by the following discrete-time "insolvency propagation equation":

$$\forall\, u \in V \setminus V_{\text{insolvent}}(t):\;\; c_u(t+1) = c_u(t) - \sum_{\substack{v:\;\; \left(c_v(t)<0\right) \\ \wedge \left(v \in V \setminus V_{\text{insolvent}}(t)\right) \\ \wedge \left((u,v)\in F\right)}} \frac{\min\left\{ \,|\, c_v(t) \,|\,,\; b_v \right\}}{\textsf{in-degree}(v,\, t)} \qquad (1)$$

In the above equation, the term $\dfrac{|\,c_v(t)|}{\textsf{in}-\textsf{degree}(v,\,t)}$ allows the loss of equity of an insolvent bank to be distributed equitably among its creditors that have not become insolvent yet, whereas the term $\dfrac{b_v}{\textsf{in}-\textsf{degree}(v,\,t)}$ ensures that the total loss propagated cannot be more than the total interbank exposure of the insolvent bank. The Insolvency propagation continues until *no* new bank becomes insolvent. A pseudo-code of the insolvency propagation method starting with the initial shock is shown in Fig. 1.

3.4 Rationale for the network and insolvency propagation model

As several prior researchers such as [10, 23, 24] have commented:

> "conceptual frameworks from the theory of weighted graphs with additional parameters may provide a powerful tool for analysis of banking network models".

Several parametric graph-theoretic models have been used by prior researchers in finance, economics and banking industry to study various properties and research questions involving banking systems [1, 10, 19, 20, 25–30], differing in the way edges are interpreted and additional parameters are used to characterize the contagion. As noted by researchers such as [10, 29]:

> "the modelling challenge in studying banking networks lies not so much in analyzing a model that is flexible enough to represent all types of insolvency cascades, but in studying a model that can mimic the empirical properties of these different types of networks".

The insolvency propagation model formalized and evaluated in this paper using a mathematically precise abstraction is similar in spirit to the models used in [1, 10–12, 25], and is represented by *cascades of cash-flow insolvencies*. As observed in [29], OTC *derivatives markets* and similar other markets are prone to such type of cascades. In such markets parties deal directly with one another rather than passing through an exchange, and thus each party is subject to the risk that the other party does not fulfill its payment obligations. The following example from [29] illustrates chains of such interactions:

> [29] "Consider two parties A and B, such that A has a receivable from party B upon the realization of some event. If B does not dispose of enough liquid reserves, it will default on the payment. Now consider that B has entered an off-setting contract with another party C, hedging its exposure to the random event. If C is cash-flow solvent, then the payment will flow through the intermediary B and reach A. However, if C is cash-flow insolvent and defaults, then the intermediary B might become cash-flow insolvent if it depends on receivables from C to meet its payment obligations to A".

The length of such chains of interactions in some OTC markets, like the *credit default swap market*, is significant [21, 22], thereby increasing the probability of a cascade of cash-flow insolvencies [23]. As observed in [10] and elsewhere, a insolvency propagation model such as studied in this paper

> "conceptualises the main characteristics of a financial system using network theory by relating the cascading behavior of financial networks both to the local properties of the nodes and to the underlying topology of the network, allowing us to vary continuously the key parameters of the network".

Although the cascading effect studied is a simplified one, as noted by Haldane and May from Bank of England in their paper [1]:

> "This is a deliberate oversimplification, aimed at a clearer understanding of how an initial failure can propagate shocks throughout the system".

3.5 The global stability measure

Consider a banking network with all parameters as described before, and let $0 < \mathcal{K} \leq 1$ be a real number denoting the fraction of nodes in V that received the initial shock under a shocking mechanism Υ. The *vulnerability index* of the network is then defined as

$$\xi\left(\mathcal{K}, G, \gamma, \Phi, \Upsilon\right) = \frac{1}{|V|} \times \max_{V' \subseteq V \,:\, |V'| = \mathcal{K}} \left\{ \lim_{t \to \infty} \left| V_{\text{insolvent}}(t) \right| \right\}$$

For example, $\xi\left(0.1, G, 0.3, 0.5, \Upsilon\right) = 0.9$ means it is possible to make 90% nodes of the network G become insolvent with $\gamma = 0.3$ and $\Phi = 0.5$ if we provide an initial shock to a suitably selected subset of 10% of nodes of G under the shocking mechanism Υ. *Note that lower values of ξ imply higher global stability of a network.* For simplicity, we will omit the arguments of ξ when they are clear from the context.

3.6 Rationale for the global stability measure

It is possible to think of other alternate measures of global stability than the one quantified above. However, the measure introduced above is in tune with the ideas that references [1, 10, 25] directly (and, some other references such as [12, 26, 27] implicitly) used to empirically study their networks. Thus, we decided to follow the cue provided by other researchers in the banking industry in studying various insolvency propagation models in defining our stability measure. Similar kinds of measures have also been used by prior researchers in social networks in the context of influence maximization issues [31, 32].

3.7 Network topology generation

We consider two topology models previously used by economists to generate random banking networks:

- the *directed scale-free* (SF) network [33] that has been used by prior banking network researchers such as [29, 30, 35, 36], and
- the *directed Erdös-Rényi* (ER) network [41] that has been used by prior banking network researchers such as [11, 37–40].

The directed scale-free networks in this paper are generated using the algorithm outlined by Bollobas *et al.* [41]. The graph grows by adding single edges at discrete time steps. Let α, β, η, δ_{in} (in-degree) and δ_{out} (out-degree) be non-negative real numbers with $\alpha + \beta + \eta = 1$. The initial graph $G(0)$ at time $t = 0$ has just one node with no edges. At any time t the graph $G(t)$ has exactly t edges, and a random number $n(t)$ of nodes. For $t > 0$, the graph $G(t+1)$ is obtained from the graph $G(t)$ by using the following rules:

- With probability α, add a new node v together with an edge from v to an existing node w, where w is chosen randomly such that $\Pr[w = u] = \frac{(d_{in}(u)+\delta_{in})}{(t+\delta_{in}n(t))}$ for every existing node u, where $d_{in}(u)$ is the in-degree of node u in $G(t)$.
- With probability β, add an edge from an existing node v to an existing node w, where v and w are chosen independently, such that

$$\begin{aligned}\Pr[v = u] &= \tfrac{(d_{out}(u)+\delta_{out})}{(t+\delta_{out}n(t))} \text{ for every existing node } u \\ \Pr[w = u] &= \tfrac{(d_{in}(u)+\delta_{in})}{(t+\delta_{in}n(t))} \text{ for every existing node } u\end{aligned}$$

  with $d_{out}(u)$ being the out-degree of node u in $G(t)$.
- With probability η, add a new node w and an edge from an existing node v to w, where v is chosen such that $\Pr[v = u] = \frac{(d_{out}(u)+\delta_{out})}{(t+\delta_{out}n(t))}$ for every existing node u.

To study the affect of connectivity on network stability, we generated random SF and ER networks with average degrees of 3 and 6. In addition, to study the effect of sparse hierarchical topology of networks on its stability, we used the Barábasi-Albert *preferential-attachment* SF model [33] to generate random *in-arborescence* networks. In-arborescences are directed rooted trees with all edges oriented towards the root, and have the following well-known topological properties:

- They belong to the class of *sparsest connected directed acyclic* graphs.
- They are *hierarchical* networks, *i. e.*, the nodes can be partitioned into levels $L_1, L_2, \ldots, L_p$ such that L_1 has exactly one node (the "root") and nodes in any level L_i have directed edges only nodes in level L_{i-1} (see Fig. 2). In particular, the root models a "central bank" that only lends money to other banks but does not borrow money from any bank.

A random in-arborescence network of n nodes is generated using Barábasi-Albert preferential-attachment model [33] in the following manner:

- Start with one node as the root of the tree.

- At every successive time step, add a new node to the tree, and add an undirected edge from the new node to a randomly selected node u that is already in the tree. Node u is selected such that $\Pr[u = v] = \frac{\text{degree}(v)}{\sum_w \text{degree}(w)}$, where $\text{degree}(y)$ denotes the degree of node y. Increment the degree of node u and the new node. Repeat this step till the tree has n nodes.

- Orient all the edges towards the root.

The minimum difference between two non-identical values of the average vulnerability index over 10 runs for two n-node networks is $1/(10\,n)$. Thus, to allow for minor statistical biases introduced by any random graph generation method, we consider two vulnerability indices to be same (within the margin of statistical error) if their absolute difference is no more than $1/(3\,n)$, which is above $1/(10\,n)$ but no more than 0.7% of the total range of the vulnerability indices. For example, if ξ_1 and ξ_2 are the average vulnerability indices (over 10 runs) of two 50-node networks then we consider ξ_1 to be at least ξ_2 if $\xi_1 > \xi_2 - 0.0066$, and consider ξ_1 to be the same as ξ_2 if $\left|\,\xi_1 - \xi_2\right| \leq 0.0066$.

3.8 Shocking mechanisms Υ

We used the following two mechanisms to select the nodes to receive the initial shock:

Idiosyncratic (random) shocks: We select a subset of nodes uniformly at random. This corresponds to the case of random idiosyncratic initial insolvencies of banks, and is a choice that has been used by prior researchers such as [10, 25].

Coordinated shocks: In this type of non-idiosyncratic correlated shocking mechanism, we seek to play an *adversarial* role in selecting a subset of nodes for the initial shock that may contribute more damage to the stability of the network[3].

For homogeneous networks, all banks have the same share of the total external asset E. However, note the total interbank exposure b_v of a bank v is directly proportional to the in-degree of the corresponding node v, and, as Equation (1) also suggests, nodes with higher inter-bank exposures are the banks that are more likely to transmit the shock to a larger number of other banks. Thus, we play an adversarial role by selecting a set of $\kappa|V|$ nodes in *non-increasing* order of their in-degrees starting from a node with the highest in-degree.

For heterogeneous networks, nodes with higher "weighted" in-degrees (*i. e.*, higher values of the sum of the weights of incoming edges) represent banks that are larger than other banks in terms of their external assets, and have more inter-bank exposures. Thus, for heterogeneous networks, we play an adversarial role by selecting $\kappa|V|$ nodes in *non-increasing order of their weighted in-degrees* starting from a node with the highest weighted in-degree.

The coordinated shocking mechanism falls under the general category of non-idiosyncratic *correlated shocks* where the nodes with high in-degrees or high weighted in-degrees are correlated.

3.9 Parameter spaces

3.9.1 Homogeneous networks

For homogeneous networks, *all* combinations of the following range of parameters were explored exhaustively:

[3] See, for example [42], about the role of adversarial strategies in measuring the worst-case bounds for network properties.

- $|V| = 50, 100, 300$;
- $E/I = 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, 2, 2.25, 2.5, 2, 75, 3, 3.25, 3.5$;
- $\Phi = 0.5, 0.6, 0.7, 0.8, 0.9$;
- $\gamma = 0.05, 0.1, 0.15, \ldots, \Phi - 0.05$; and
- $\mathcal{K} = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$.

Thus, we explored a total of 24,570 combinations of parameters for each of the 2 types of shocks (idiosyncratic or coordinated) for each of the 5 network models (in-arborescence, ER of average degree **3**, ER of average degree **6**, SF of average degree **3** or SF of average degree **6**), *resulting in 245,700 different homogeneous banking networks*. To correct statistical biases, for each such combinations of parameters, shock types and network types, we generated 10 corresponding random networks and computed the average value of the vulnerability index over these 10 runs. For generation of ER and SF random networks, we selected the values of network generation parameters such that the expected number of edges of the network is $3|V|$ or $6|V|$ based on whether we require the average degree of the network to be 3 or 6, respectively.

3.9.2 Heterogeneous networks

Recall that in a heterogeneous network it is possible to have a few banks whose external assets or interbank exposures are *significantly larger* than the rest of the banks in the networks, *i. e.*, it is possible to have a few banks that are "too big", and thus heterogeneous networks permit investigation of the effect of such big banks on the global stability of the entire network.

To this end, we define a (α, β)-*heterogeneous network* $G = (V, F)$ as one in which we select a random subset V' of $\alpha|V|$ nodes and distribute βE part of the total external asset E equally among these nodes in V'. Let F' be the edges involving these $\alpha|V|$ nodes (*i. e.*, edges in F' have at least one end-point from V'). Then we distribute βI part of the total interbank exposure I equally among a random subset of $\alpha|F'|$ of edges from the edges in F'. The remaining parts of E and I, namely $(1-\beta)E$ and $(1-\beta)I$ respectively, are distributed equally among the remaining $(1-\alpha)|V|$ nodes and remaining $|F| - \beta|F'|$ edges, respectively.

We performed our simulations for (α, β)-heterogeneous networks for two pairs of α and β, namely $(\alpha, \beta) = (0.1, 0.95)$ and $(\alpha, \beta) = (0.2, 0.6)$, *thus we obtained a total of 491,400 different heterogeneous banking networks*. The combination $(\alpha, \beta) = (0.1, 0.95)$ corresponds to the extreme situation in which 95% of the assets and exposures involve 10% of banks, thus creating a minority of banks that are significantly larger than the remaining banks. The other combination $(\alpha, \beta) = (0.2, 0.6)$ corresponds to a less extreme situation in which there are a larger number of moderately large banks.

Relevant data of all networks used in this paper will be made publicly available in the final version of the paper.

4 Results and discussion

4.1 Effect of unequal distribution of assets

4.1.1 On global stability

We comprehensively compared the values of the vulnerability index ξ for homogeneous and heterogeneous networks of same types and same values of common global parameters. For this purpose, for the same value of the common parameters $|V|$, E, κ, Φ and γ, for the same for network type (ER, SF or in-arborescence) of same average degree (6, 3 or 1) and for the same shocking mechanism Υ (coordinated or idiosyncratic), we compared the value of ξ for the homogeneous network with the corresponding values of ξ for both $(0.1, 0.95)$-heterogeneous and $(0.2, 0.6)$-heterogeneous networks. The comparison results are tabulated in Table 1, where most of the entries are significantly higher than 50% (and in fact, are at least 90%). Thus, we conclude:

> (A) *networks where all banks have roughly the same external assets display higher global stability over similar networks in which fewer banks have a disproportionately higher external assets compared to the remaining banks.*

4.1.2 On residual instability

For homogeneous networks, if the equity to asset ratio γ is close enough to the severity of the shock Φ then the network almost always tends to be perfectly stable, as one would intuitively expect. However, the above property is *not* true in general for highly heterogeneous networks in the sense that, even when γ is close to Φ, these networks (irrespective of their topologies and densities) have a *minimum* amount of global instability[4]. In Tables 4– 13 we tabulate residual instabilities for different types of homogeneous and heterogeneous networks under coordinated and idiosyncratic shocks. The numbers in these tables show, for each combination of network types, $|V|$, shocking mechanism and values of Φ and γ such that $|\Phi - \gamma| = 0.05$, the percentage of networks with this combination for which the vulnerability index ξ was less than 0.05, 0.1 or 0.2. As the reader will observe, all the numbers for heterogeneous networks are significantly lower than their homogeneous counter-parts. Thus, we conclude:

> (B) *a heterogeneous network of any type and any density, in contrast to its corresponding homogeneous version, has a non-trivial minimum global instability even if its equity to asset ratio is very large and close to the severity of the shock.*

4.2 Effect of total external assets

The Glass-Steagall Act of 1933 was enacted in USA to control speculation by banks by separating consumer banking and investment banking. Later, the Gramm-Leach-Bliley Act of 1999 effectively removed this separation between investment and consumer banking, and subsequently in 2007 a collapse of major financial institutions was on the horizon. A controversial belief correlates these events by asserting that a repeal of Glass-Steagall act allowed a ripple effect of insolvencies of individual banks to many other banks in the network. In our setting, the quantity E/I controls the total (normalized) amount of external investments of all banks in the network. Thus, varying the ratio E/I allows us to investigate the role of the magnitude of total external investments on the stability of our banking network[5]. Our results are tabulated in Table 2. All the entries in Table 2 are vanishingly close to 0, showing that heterogeneous networks exhibited *very small* average changes in the vulnerability index ξ when E/I was varied. Thus, we conclude:

[4]For graphical illustrations to this phenomena, the reader is referred to *supplemental* Fig. S1 and Fig. S2– S3. For example, in *supplemental* Fig. S1, when γ is 45% and Φ is only 5% more than γ, the vulnerability index ξ is approximately 0 for *all* the 9 combinations of parameters, but in *supplemental* Fig. S2– S3 *all* the 18 networks have a value of $\xi \geq 0.1$ even when the equity to asset ratio γ is 45% and the severity of the shock is only 5% more than γ.

[5]Visual illustrations of Conclusion (C) are shown in *supplemental* Fig. S4 for homogeneous networks and in *supplemental* Fig. S5 for heterogeneous networks.

(C) *for all heterogeneous banking networks under consideration, global stabilities are affected very little by the amount of total external assets in the system*[6].

4.3 Effect of network connectivity

Although it is clear that connectivity and other similar topological properties of a banking network should have a crucial effect on its stability, prior researchers have drawn mixed conclusions on this. For example, Allen and Gale [23] concluded that networks with less connectivity are *more* prone to contagion than networks with higher connectivity due to improved resilience of banking network topologies with higher connectivity via transfer of proportion of the losses in one bank's portfolio to more banks through interbank agreements. On the other hand, Babus [17] observed that, when the network connectivity is higher, liquidity can be redistributed in the system to make the risk of contagion lower, and Gai and Kapadia [11] observed that higher connectivity among banks leads to more contagion effect during a crisis. The mixed conclusions arise because links between banks have conceptually two conflicting effects on contagion, namely,

- more interbank links may increase the opportunity for spreading insolvencies to other banks, but
- more interbank links may also provide banks with a type of co-insurance against fluctuating liquidity flows.

As our findings show below, these two conflicting effects have different strengths in homogeneous versus highly heterogeneous networks, thus justifying the mixed conclusions of past researchers.

Recall that in a homogeneous network all banks have the same external asset. Table 14 shows sparser homogeneous networks with lower average degrees to be more stable for same values of other parameters. Thus, we conclude:

(D) *for homogeneous networks, higher connectivity leads to lower stability.*

In a heterogeneous network, there *are* banks that are "too big" in the sense that these banks have much larger external assets and inter-bank exposures compared to the remaining banks. Table 3 shows that for heterogeneous network models denser networks with higher average degree are more stable than the corresponding sparser networks of lower average degree for same values of other parameters, specially when the heterogeneity of the network is larger (*i. e.*, when $\alpha = 0.1$ and $\beta = 0.95$). Thus, we conclude:

(E) *for highly heterogeneous networks, higher connectivity leads to higher stability.*

4.4 Correlated versus idiosyncratic shocks

For most parameter combinations, our results showed that coordinated shocks, which is a type of correlated shock, resulted in a insolvencies of higher number of nodes as opposed to idiosyncratic shocks for the same network with the same parameters, *often by a factor of two or more.* Statistics of relevant comparisons for various network type are shown in Table 15. For example, Table 15 shows that for homogeneous

[6] It may be tempting to provide an intuitive interpretation of this conclusion by trying to relate this lack of change of ξ with the term $\frac{b_v}{\text{in–degree}(v,t)}$ of Equation (1) that bounds the total shock propagated between successive time steps. This explanation may not be fully correct for at least two reasons. Firstly, the above conclusion does not hold for homogeneous networks which also use Equation (1) for insolvency cascading. Secondly, in our simulations we have found this term to limit insolvency cascading only in a minority of cases.

in-arborescence networks the vulnerability index under coordinated shocks is at least as much as the vulnerability index under idiosyncratic shocks 84.62% of the time, and for $(0.1, 0.95)$-heterogeneous ER networks of average degree 6 the vulnerability index under coordinated shocks is at least as much as the vulnerability index under idiosyncratic shocks 98.99% of the time[7]. Thus, we conclude:

> (F) *correlated shocking mechanisms are more appropriate to measure the worst-case stability compared to idiosyncratic shocking mechanisms.*

4.5 Phase transition properties of stability

Phase transitions are quite common when one studies various topological properties of graphs [43]. The vulnerability index ξ shows several sharp phase transitions[8]. Two such interesting cases are discussed below.

4.5.1 Dense homogeneous networks

Based on the behavior of ξ with respect to $\Phi - \gamma$, we observe that, for smaller value of κ and for denser ER and SF networks under coordinated or idiosyncratic shocks, there is often a *sharp* decrease of stability when γ was decreased beyond a particular threshold value. For example, with $\Phi = 0.5$ and $\mathcal{K} = 0.1$, both 100 node dense (average degree 6) SF and ER homogeneous networks exhibited more than ninefold increase in ξ around $\gamma = 0.15$ and $\gamma = 0.1$, respectively[9].

To investigate the extent of such a sharp decrease around a threshold value of γ in the range $\big[0.05, 0.2\big]$, we computed, for each of the five homogeneous network types under coordinated shocks and for each values of the parameters $|V|$, Φ, ${}^{E}\!/{}_{I}$ and κ, , the ratio

$$\Lambda\left(n, \Phi, {}^{E}\!/{}_{I}, \kappa\right) = \frac{\displaystyle\max_{0.05 \leq \gamma \leq 0.2} \big\{\xi\big\} - \min_{0.05 \leq \gamma \leq 0.2} \big\{\xi\big\}}{\displaystyle\max_{\text{entire range of } \gamma} \big\{\xi\big\} - \min_{\text{entire range of } \gamma} \big\{\xi\big\}}$$

that provides the *maximum* percentage of the total change of the vulnerability index that occurred within this narrow range of γ. The values of $\Lambda\left(n, \Phi, {}^{E}\!/{}_{I}, \kappa\right)$ for the dense ER and SF homogeneous networks under coordinated shocks are shown in *supplemental* Table 16 for $\Phi = 0.5, 0.8$ and $\kappa = 0.1, 0.2, 0.3, 0.4, 0.5$ (the behaviour of ξ is similar for other intermediate values of Φ). If the growth of ξ with respect to γ was uniform (linear) or near uniform over the entire range of γ, Λ would be approximately $\lambda = \frac{0.2-0.05}{0.45-0.05} = 0.375$; thus, any value of Λ significantly higher than λ indicates a sharp transition within the above-mentioned range of values of γ. As Table 16 shows, a *significant majority* of the entries for $\Phi \leq 0.8$ and $\kappa \leq 0.2$ are $2\,\lambda$ or more.

4.5.2 Homogeneous in-arborescence networks

Homogeneous in-arborescence networks under coordinated shocks (and to a lesser extent under idiosyncratic shocks) exhibited a *sharp* increase in stability as the ratio ${}^{E}\!/{}_{I}$ of the total external asset to the total interbank exposure the system is increased beyond a particular threshold *provided the equity to asset ratio* γ *was approximately* 50% *of the shock parameter* Φ. For example[10], for a 50-node homogeneous in-arborescence network under coordinated shock, ξ exhibited a sharp decrease from 0.76 to 0.18 for ${}^{E}\!/{}_{I} \in \big[0.75, 1\big]$, $\mathcal{K} = 0.1$, $\Phi = 0.5$ and $\gamma = 0.25 = {}^{\Phi}\!/{}_{2}$.

[7]Visual illustrations of Conclusion (F) are shown in *supplemental* Fig. S6– S11.
[8]For visual illustratation, see *supplemental* Fig. S1– S4.
[9]See *supplemental* Fig. S1 for a visual illustration.
[10]See *supplemental* Fig. S4 for a visual illustration.

To investigate the extent of such a sharp decrease of ξ around a threshold value of ${}^{E}\!/_{I}$ in the range $\big[0.5, 1\big]$ with $\gamma \approx {}^{\Phi}\!/_{2}$, we computed, for each type of shocking mechanism, and for each values of the parameters n, Φ, $\gamma \approx {}^{\Phi}\!/_{2}$, and κ of the homogeneous in-arborescence network, the ratio

$$\Delta(n, \Phi, \gamma, \kappa) = \frac{\max\limits_{0.5 \leq {}^{E}\!/_{I} \leq 1} \big\{\xi\big\} - \min\limits_{0.5 \leq {}^{E}\!/_{I} \leq 1} \big\{\xi\big\}}{\max\limits_{\text{entire range of } E} \big\{\xi\big\} - \min\limits_{\text{entire range of } E} \big\{\xi\big\}}$$

that provides the *maximum* percentage of the total change of the vulnerability index that occurred within this range of ${}^{E}\!/_{I}$. If the growth of ξ with respect to ${}^{E}\!/_{I}$ was uniform (linear) or near uniform over the entire range of ${}^{E}\!/_{I}$, Δ would be approximately $\delta = \frac{1-0.5}{3.5-0.25} \approx 0.16$; thus, any value of Δ significantly higher than δ indicates a sharp transition within the above-mentioned range of ${}^{E}\!/_{I}$. As Table 17 shows, when $\gamma = {}^{\Phi}\!/_{2}$ a *significant* majority of the entries are coordinated shocks and many entries under idiosyncratic shocks are at least $2\,\delta$.

An informal intuition behind the type of sharp decrease of ξ is as follows. Suppose that at time $t = 0$ a node u is shocked and becomes insolvent. For an in-arborescence network, the amount of shock transmitted from u to each of its in-neighbors is $\mu \approx (\Phi - \gamma) \times \left(1 + \frac{E}{n\, \mathsf{in\text{--}degree}(u)}\right)$. Suppose that an in-neighbor v of u has $\mathsf{in\text{--}degree}(v) = 0$. Then, $c_v(0) = \gamma \times (E/n)$ and

$$\nu = \mu - c_v(0) = (\Phi - \gamma) \times \left(1 + \frac{E}{n\, \mathsf{in\text{--}degree}(u)}\right) - \gamma \times \frac{E}{n}$$

Assuming $\gamma \approx {}^{\Phi}\!/_{2}$, we have

$$\nu \approx \gamma \times \left(1 + \frac{E}{n\, \mathsf{in\text{--}degree}(u)} - \frac{E}{n}\right)$$

Thus, assuming $\mathsf{in\text{--}degree}(u) > 1$, beyond the threshold value of $E = \left(1 - \frac{1}{\mathsf{in\text{--}degree}(u)}\right) \times n$, ν would be strictly negative, the node v will not become insolvent at any time $t \geq 1$, and the shock will not propagate any further through v. If u has many such in-neighbors of zero in-degrees, then the stability of the network will improve sharply around such a threshold value of E. If the in-degrees of the in-neighbors of u are not zero but bounded by a small constant, the same effect will be produced around a different threshold value of E.

5 Concluding remarks

In this paper, we have initiated a methodology for systematic investigation of the global stabilities of financial networks that arise in OTC derivatives market and elsewhere. Our results can be viweed as a much needed beginning of a systematic investigation of these issues, with future research works concentrating on further improving the network model, the stability measure and parameter choices.

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	In-arborescence		ER average degree 3		ER average degree 6		SF average degree 3		SF average degree 6	
	$\alpha = 0.1$ $\beta = 0.95$	$\alpha = 0.2$ $\beta = 0.6$	$\alpha = 0.1$ $\beta = 0.95$	$\alpha = 0.2$ $\beta = 0.6$	$\alpha = 0.1$ $\beta = 0.95$	$\alpha = 0.2$ $\beta = 0.6$	$\alpha = 0.1$ $\beta = 0.95$	$\alpha = 0.2$ $\beta = 0.6$	$\alpha = 0.1$ $\beta = 0.95$	$\alpha = 0.2$ $\beta = 0.6$
coordinated shock	**66.91%**	**60.22%**	**99.26%**	**98.91%**	**98.46%**	**98%**	**98.22%**	**91.68%**	**99.13%**	**97.4%**
idiosyncratic shock	**92.75%**	**81.79%**	**97.76%**	**96.81%**	**98.16%**	**97.61%**	**98.86%**	**94.84%**	**98.83%**	**97.22%**

Table 1: Comparison of stabilities of (α, β)-heterogeneous networks with their homogeneous counter-parts over all parameter ranges. The numbers are the percentages of data points for which $\xi_{(\alpha,\beta)-\text{heterogeneous}}$ was at least $\xi_{\text{homogeneous}}$.

average values of	$\left\| \max_{0.25 \le E/I \le 3.5} \{\xi\} - \min_{0.25 \le E/I \le 3.5} \{\xi\} \right\|$ under coordinated shock	under idiosyncratic shock
$(0.1, 0.95)$-heterogeneous in-arborescence	**0.017**	**0.045**
$(0.2, 0.6)$-heterogeneous in-arborescence	**0.007**	**0.017**
$(0.1, 0.95)$-heterogeneous ER, average degree 3	**0.066**	**0.073**
$(0.2, 0.6)$-heterogeneous ER, average degree 3	**0.040**	**0.041**
$(0.1, 0.95)$-heterogeneous ER, average degree 6	**0.111**	**0.116**
$(0.2, 0.6)$-heterogeneous ER, average degree 6	**0.084**	**0.078**
$(0.1, 0.95)$-heterogeneous SF, average degree 3	**0.119**	**0.094**
$(0.2, 0.6)$-heterogeneous SF, average degree 3	**0.034**	**0.032**
$(0.1, 0.95)$-heterogeneous SF, average degree 6	**0.200**	**0.179**
$(0.2, 0.6)$-heterogeneous SF, average degree 6	**0.054**	**0.054**

Table 2: Absolute values of the largest change of the vulnerability index ξ in the range $0.25 \le E/I \le 3.5$.

$t = 0$; $V_{\text{insolvent}}(0) = \emptyset$; continue$=$TRUE
for every node v, let in–degree$(v, 0) =$ in–degree(v)
let V' be the set of nodes selected for initial insolvency

for every node $u \in V'$ **do**
  $c_u(0) = c_u - \Phi\, e_u$
endfor

while $\Big(\big(\text{continue}=\text{TRUE}\big) \bigwedge \big(V_{\text{insolvent}}(t) \neq V \big) \Big)$ **do**
  for every node $u \in V \setminus V_{\text{insolvent}}(t)$ **do**
    $c_u(t+1) = c_u(t)$

    for every node $v \in V \setminus V_{\text{insolvent}}(t)$ **do**
      if $\Big(\big(c_v(t) < 0 \big) \bigwedge \big((u, v) \in F \big) \Big)$ **then**
        decrease $c_u(t+1)$ by $\dfrac{\min\left\{ \left| c_v(t) \right| ,\ b_v \right\}}{\text{in–degree}(v,\ t)}$
      endif
    endfor

    $V_{\text{insolvent}}(t+1) = V_{\text{insolvent}}(t)$

    if $c_u(t) < 0$ **then**
      add the node u to $V_{\text{insolvent}}(t+1)$
    endif
  endfor

  if $\big(V_{\text{insolvent}}(t+1) = V_{\text{insolvent}}(t) \big)$ **then**
    continue$=$FALSE
  endif

  increase t by 1

  for every node $u \in V \setminus V_{\text{insolvent}}(t)$ **do**
    in–degree$(u, t) = \Big| \Big\{ v \colon \big((v, u) \in F \big) \wedge \big(v \in V \setminus V_{\text{insolvent}}(t) \big) \Big\} \Big|$
  endfor

endwhile

Figure 1: Pseudo-code of insolvency propagation method.

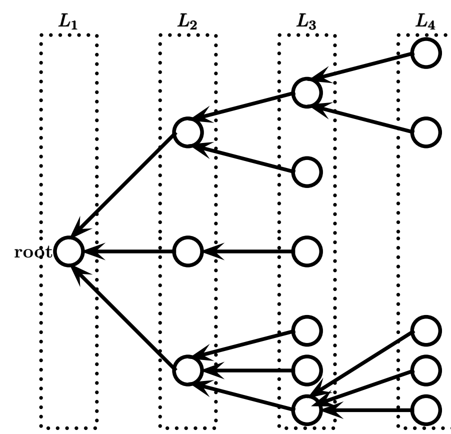


Figure 2: An in-arborescence graph.

(0,1,0.95) ER average degree 6 versus average degree 3		(0.2,0.6) ER average degree 6 versus average degree 3		(0,1,0.95) SF average degree 6 versus average degree 3		(0.2,0.6) SF average degree 6 versus average degree 3	
coordinated shock	idiosyncratic shock	coordinated shock	idiosyncratic shock	coordinated shock	idiosyncratic shock	coordinated shock	idiosyncratic shock
89.3%	**82.39%**	**68.12%**	**61.46%**	**85.51%**	**73.81%**	**69.29%**	**73.07%**

(A)

(0,1,0.95) SF average degree 3 and average degree 6 versus (0.1,0.95)-heterogeneous in-arborescence (SF average degree 1)		(0.2,0.6) SF average degree 3 and average degree 6 versus (0.2,0.6)-heterogeneous in-arborescence (SF average degree 1)	
coordinated shock	idiosyncratic shock	coordinated shock	idiosyncratic shock
85.7%	**81.86%**	**56.21%**	**51.07%**

(B)

Table 3: Effect of connectivity on the global stability under coordinated and idiosyncratic shocks for **(A)** (α, β)-heterogeneous ER and SF networks and **(B)** (α, β)-heterogeneous in-arborescence versus (α, β)-heterogeneous SF networks. The number (percentage) shown for a comparison of the type "network A versus network B" indicates the percentage of data points for which the stability of networks of type A was at least as much as that of networks of type B.

			coordinated shock					
			$\Phi = 0.5, \gamma = 0.45$			$\Phi = 0.5, \gamma = 0.40$		
			$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$	$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$
$\|V\| = 50$	homogeneous	in-arborescence	**73%**	**73%**	**73%**	**0%**	**31%**	**59%**
		ER, average degree 3	**89%**	**100%**	**100%**	**43%**	**84%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**44%**	**84%**	**100%**	**25%**	**57%**	**88%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	0%	0%	0%	0%	0%	0%
		ER, average degree 3	0%	0%	1%	0%	0%	0%
		ER, average degree 6	8%	9%	10%	2%	6%	6%
		SF, average degree 3	2%	6%	15%	0%	2%	5%
		SF, average degree 6	18%	23%	30%	9%	10%	11%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	4%	7%	19%	2%	6%	16%
		ER, average degree 6	8%	12%	24%	6%	7%	16%
		SF, average degree 3	2%	6%	22%	0%	2%	18%
		SF, average degree 6	8%	12%	24%	7%	8%	16%
$\|V\| = 100$	homogeneous	in-arborescence	**73%**	**73%**	**73%**	**0%**	**34%**	**73%**
		ER, average degree 3	**66%**	**100%**	**100%**	**25%**	**64%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**29%**	**61%**	**100%**	**20%**	**42%**	**83%**
		SF, average degree 6	**100%**	**100%**	**100%**	**90%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	0%	0%	0%	0%	0%	0%
		ER, average degree 3	0%	0%	6%	0%	0%	1%
		ER, average degree 6	6%	6%	6%	4%	6%	6%
		SF, average degree 3	0%	0%	7%	0%	0%	3%
		SF, average degree 6	6%	10%	15%	6%	6%	6%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	0%	6%	16%	0%	4%	16%
		ER, average degree 6	6%	7%	16%	6%	6%	16%
		SF, average degree 3	0%	2%	14%	0%	1%	13%
		SF, average degree 6	7%	8%	17%	6%	7%	16%
$\|V\| = 300$	homogeneous	in-arborescence	**73%**	**73%**	**73%**	**0%**	**55%**	**73%**
		ER, average degree 3	**71%**	**97%**	**100%**	**22%**	**60%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**22%**	**44%**	**86%**	**18%**	**36%**	**74%**
		SF, average degree 6	**100%**	**100%**	**100%**	**88%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	0%	0%	0%	0%	0%	0%
		ER, average degree 3	0%	0%	6%	0%	0%	1%
		ER, average degree 6	6%	6%	6%	0%	6%	6%
		SF, average degree 3	0%	0%	10%	0%	0%	2%
		SF, average degree 6	6%	6%	16%	6%	6%	6%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	0%	6%	16%	0%	4%	16%
		ER, average degree 6	6%	6%	16%	6%	6%	16%
		SF, average degree 3	0%	0%	13%	0%	0%	12%
		SF, average degree 6	6%	7%	16%	6%	6%	16%

Table 4: Residual instabilities of homogeneous versus heterogeneous networks under coordinated shocks. The percentages shown are the percentages of networks for which $\xi < 0.05$ or $\xi < 0.1$ or $\xi < 0.2$.

			idiosyncratic shock					
			$\Phi = 0.5, \gamma = 0.45$			$\Phi = 0.5, \gamma = 0.40$		
			$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$	$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$
$\|V\| = 50$	homogeneous	in-arborescence	**76%**	**78%**	**82%**	**26%**	**58%**	**75%**
		ER, average degree 3	**99%**	**100%**	**100%**	**43%**	**84%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**42%**	**100%**	**100%**	**23%**	**70%**	**88%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	1%	2%	15%	0%	1%	10%
		ER, average degree 3	0%	2%	16%	0%	1%	8%
		ER, average degree 6	7%	10%	21%	2%	7%	14%
		SF, average degree 3	0%	6%	22%	0%	2%	14%
		SF, average degree 6	8%	19%	34%	4%	12%	21%
	(0.2, 0.6)-heterogeneous	in-arborescence	0%	2%	12%	0%	1%	11%
		ER, average degree 3	7%	14%	22%	4%	9%	18%
		ER, average degree 6	8%	18%	30%	6%	10%	20%
		SF, average degree 3	0%	9%	19%	0%	3%	17%
		SF, average degree 6	8%	12%	24%	4%	7%	18%
$\|V\| = 100$	homogeneous	in-arborescence	**76%**	**78%**	**81%**	**32%**	**62%**	**81%**
		ER, average degree 3	**66%**	**100%**	**100%**	**26%**	**74%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**29%**	**72%**	**100%**	**19%**	**53%**	**88%**
		SF, average degree 6	**100%**	**100%**	**100%**	**88%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	0%	1%	12%	0%	1%	11%
		ER, average degree 3	0%	0%	15%	0%	1%	10%
		ER, average degree 6	6%	7%	16%	6%	6%	10%
		SF, average degree 3	0%	6%	23%	0%	0%	16%
		SF, average degree 6	8%	16%	30%	6%	12%	19%
	(0.2, 0.6)-heterogeneous	in-arborescence	0%	1%	11%	0%	1%	11%
		ER, average degree 3	6%	10%	18%	0%	7%	16%
		ER, average degree 6	7%	12%	22%	6%	8%	17%
		SF, average degree 3	0%	2%	18%	0%	0%	16%
		SF, average degree 6	5%	9%	18%	2%	6%	16%
$\|V\| = 300$	homogeneous	in-arborescence	**76%**	**78%**	**81%**	**36%**	**67%**	**81%**
		ER, average degree 3	**76%**	**100%**	**100%**	**22%**	**73%**	**93%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**26%**	**52%**	**100%**	**20%**	**42%**	**88%**
		SF, average degree 6	**100%**	**100%**	**100%**	**87%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	0%	1%	12%	0%	1%	12%
		ER, average degree 3	0%	2%	16%	0%	0%	5%
		ER, average degree 6	0%	6%	12%	0%	2%	6%
		SF, average degree 3	0%	9%	24%	0%	1%	16%
		SF, average degree 6	7%	16%	28%	6%	9%	19%
	(0.2, 0.6)-heterogeneous	in-arborescence	0%	1%	11%	0%	1%	11%
		ER, average degree 3	6%	8%	16%	0%	2%	16%
		ER, average degree 6	6%	8%	17%	6%	7%	16%
		SF, average degree 3	0%	1%	16%	0%	0%	16%
		SF, average degree 6	2%	6%	16%	2%	5%	16%

Table 5: Residual instabilities of homogeneous versus heterogeneous networks under idiosyncratic shocks. The percentages shown are the percentages of networks for which $\xi < 0.05$ or $\xi < 0.1$ or $\xi < 0.2$.

			coordinated shock					
			$\Phi = 0.6, \gamma = 0.55$			$\Phi = 0.6, \gamma = 0.50$		
			$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$	$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$
$\lvert V\rvert = 50$	homogeneous	in-arborescence	**93%**	**93%**	**93%**	**2%**	**69%**	**93%**
		ER, average degree 3	**97%**	**100%**	**100%**	**64%**	**95%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**56%**	**96%**	**100%**	**44%**	**87%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	0%	0%	1%	0%	0%	0%
		ER, average degree 3	0%	0%	5%	0%	0%	1%
		ER, average degree 6	9%	11%	14%	5%	7%	7%
		SF, average degree 3	5%	9%	19%	0%	3%	8%
		SF, average degree 6	22%	28%	41%	10%	15%	17%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	5%	7%	20%	3%	7%	16%
		ER, average degree 6	11%	14%	26%	7%	7%	19%
		SF, average degree 3	2%	8%	23%	1%	3%	18%
		SF, average degree 6	9%	15%	26%	7%	8%	18%
$\lvert V\rvert = 100$	homogeneous	in-arborescence	**93%**	**93%**	**93%**	**23%**	**84%**	**93%**
		ER, average degree 3	**85%**	**100%**	**100%**	**47%**	**86%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**37%**	**75%**	**100%**	**35%**	**71%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**96%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	0%	0%	0%	0%	0%	0%
		ER, average degree 3	0%	1%	7%	0%	0%	3%
		ER, average degree 6	7%	7%	9%	7%	7%	7%
		SF, average degree 3	0%	1%	9%	0%	0%	5%
		SF, average degree 6	10%	15%	21%	7%	7%	11%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	0%	7%	17%	0%	5%	16%
		ER, average degree 6	7%	7%	19%	7%	7%	16%
		SF, average degree 3	1%	3%	15%	0%	1%	13%
		SF, average degree 6	7%	9%	18%	7%	7%	17%
$\lvert V\rvert = 300$	homogeneous	in-arborescence	**93%**	**93%**	**93%**	**79%**	**93%**	**93%**
		ER, average degree 3	**93%**	**100%**	**100%**	**48%**	**85%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**28%**	**56%**	**97%**	**28%**	**56%**	**97%**
		SF, average degree 6	**100%**	**100%**	**100%**	**93%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	0%	0%	0%	0%	0%	0%
		ER, average degree 3	0%	1%	7%	0%	0%	3%
		ER, average degree 6	7%	7%	7%	3%	7%	7%
		SF, average degree 3	0%	0%	11%	0%	0%	3%
		SF, average degree 6	7%	7%	18%	7%	7%	7%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	1%	7%	17%	1%	5%	16%
		ER, average degree 6	7%	7%	16%	7%	7%	16%
		SF, average degree 3	0%	1%	13%	0%	0%	13%
		SF, average degree 6	7%	7%	16%	7%	7%	16%

Table 6: Residual instabilities of homogeneous versus heterogeneous networks under coordinated shocks. The percentages shown are the percentages of networks for which $\xi < 0.05$ or $\xi < 0.1$ or $\xi < 0.2$.

			coordinated shock					
			$\Phi = 0.7, \gamma = 0.65$			$\Phi = 0.7, \gamma = 0.60$		
			$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$	$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$
$\lvert V\rvert = 50$	homogeneous	in-arborescence	**93%**	**93%**	**93%**	**35%**	**88%**	**93%**
		ER, average degree 3	**97%**	**100%**	**100%**	**81%**	**100%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**56%**	**96%**	**100%**	**52%**	**96%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	0%	1%	7%	0%	0%	0%
		ER, average degree 3	0%	0%	5%	0%	0%	1%
		ER, average degree 6	10%	13%	16%	7%	7%	9%
		SF, average degree 3	7%	12%	25%	1%	4%	9%
		SF, average degree 6	26%	32%	49%	14%	15%	21%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	0%	0%	10%	0%	0%	9%
		ER, average degree 3	5%	8%	21%	3%	7%	17%
		ER, average degree 6	13%	18%	30%	7%	7%	20%
		SF, average degree 3	3%	9%	25%	1%	3%	19%
		SF, average degree 6	9%	16%	28%	8%	9%	19%
$\lvert V\rvert = 100$	homogeneous	in-arborescence	**93%**	**93%**	**93%**	**31%**	**93%**	**93%**
		ER, average degree 3	**85%**	**100%**	**100%**	**62%**	**97%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**37%**	**75%**	**100%**	**37%**	**75%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	0%	0%	0%	0%	0%	0%
		ER, average degree 3	0%	1%	7%	0%	0%	5%
		ER, average degree 6	7%	7%	9%	7%	7%	7%
		SF, average degree 3	0%	2%	15%	0%	0%	5%
		SF, average degree 6	11%	15%	27%	7%	7%	11%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	0%	7%	18%	0%	7%	16%
		ER, average degree 6	7%	7%	19%	7%	7%	16%
		SF, average degree 3	1%	4%	17%	0%	2%	13%
		SF, average degree 6	8%	10%	19%	7%	8%	17%
$\lvert V\rvert = 300$	homogeneous	in-arborescence	**93%**	**93%**	**93%**	**56%**	**93%**	**93%**
		ER, average degree 3	**93%**	**100%**	**100%**	**65%**	**97%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**28%**	**56%**	**97%**	**28%**	**56%**	**97%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	0%	0%	0%	0%	0%	0%
		ER, average degree 3	0%	1%	7%	0%	0%	4%
		ER, average degree 6	7%	7%	7%	6%	7%	7%
		SF, average degree 3	0%	0%	15%	0%	0%	3%
		SF, average degree 6	7%	7%	21%	7%	7%	7%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	1%	7%	17%	0%	7%	16%
		ER, average degree 6	7%	7%	16%	7%	7%	16%
		SF, average degree 3	0%	1%	14%	0%	0%	13%
		SF, average degree 6	7%	8%	16%	7%	7%	16%

Table 7: Residual instabilities of homogeneous versus heterogeneous networks under coordinated shocks. The percentages shown are the percentages of networks for which $\xi < 0.05$ or $\xi < 0.1$ or $\xi < 0.2$.

			coordinated shock					
			$\Phi = 0.8, \gamma = 0.75$			$\Phi = 0.8, \gamma = 0.70$		
			$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$	$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$
$\|V\| = 50$	homogeneous	in-arborescence	**0%**	**0%**	**93%**	**0%**	**0%**	**93%**
		ER, average degree 3	**97%**	**100%**	**100%**	**89%**	**100%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**56%**	**96%**	**100%**	**54%**	**96%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	1%	7%	7%	0%	0%	0%
		ER, average degree 3	0%	1%	6%	0%	0%	1%
		ER, average degree 6	12%	15%	19%	7%	7%	9%
		SF, average degree 3	9%	15%	32%	1%	5%	11%
		SF, average degree 6	27%	37%	60%	14%	19%	21%
	(0.2, 0.6)-heterogeneous	in-arborescence	0%	0%	10%	0%	0%	9%
		ER, average degree 3	5%	11%	22%	4%	7%	17%
		ER, average degree 6	14%	19%	30%	7%	11%	21%
		SF, average degree 3	3%	11%	27%	1%	4%	19%
		SF, average degree 6	11%	19%	31%	8%	9%	21%
$\|V\| = 100$	homogeneous	in-arborescence	**93%**	**93%**	**93%**	**51%**	**93%**	**93%**
		ER, average degree 3	**85%**	**100%**	**100%**	**71%**	**100%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**37%**	**75%**	**100%**	**37%**	**75%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	0%	0%	1%	0%	0%	0%
		ER, average degree 3	0%	1%	7%	0%	0%	5%
		ER, average degree 6	7%	9%	10%	7%	7%	7%
		SF, average degree 3	0%	3%	20%	0%	0%	6%
		SF, average degree 6	11%	19%	33%	7%	11%	11%
	(0.2, 0.6)-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	0%	7%	19%	0%	7%	17%
		ER, average degree 6	7%	8%	21%	7%	7%	17%
		SF, average degree 3	1%	5%	18%	0%	2%	14%
		SF, average degree 6	8%	10%	23%	7%	8%	17%
$\|V\| = 300$	homogeneous	in-arborescence	**93%**	**93%**	**93%**	**76%**	**93%**	**93%**
		ER, average degree 3	**93%**	**100%**	**100%**	**76%**	**99%**	**100%**
		ER, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
		SF, average degree 3	**28%**	**56%**	**97%**	**28%**	**56%**	**97%**
		SF, average degree 6	**100%**	**100%**	**100%**	**99%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	0%	0%	0%	0%	0%	0%
		ER, average degree 3	0%	1%	7%	0%	1%	5%
		ER, average degree 6	7%	7%	7%	7%	7%	7%
		SF, average degree 3	0%	1%	16%	0%	0%	6%
		SF, average degree 6	7%	11%	22%	7%	7%	8%
	(0.2, 0.6)-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	1%	7%	17%	1%	7%	16%
		ER, average degree 6	7%	7%	16%	7%	7%	16%
		SF, average degree 3	0%	1%	14%	0%	1%	13%
		SF, average degree 6	7%	8%	17%	7%	7%	16%

Table 8: Residual instabilities of homogeneous versus heterogeneous networks under coordinated shocks. The percentages shown are the percentages of networks for which $\xi < 0.05$ or $\xi < 0.1$ or $\xi < 0.2$.

			coordinated shock					
			$\Phi = 0.9, \gamma = 0.85$			$\Phi = 0.9, \gamma = 0.80$		
			$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$	$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$
$\lvert V \rvert = 50$	homogeneous	in-arborescence	**93%**	**93%**	**93%**	**81%**	**93%**	**93%**
		ER, average degree 3	**98%**	**100%**	**100%**	**95%**	**100%**	**100%**
		ER, average degree 6	**0%**	**0%**	**0%**	**100%**	**100%**	**100%**
		SF, average degree 3	**56%**	**96%**	**100%**	**56%**	**96%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	7%	7%	7%	0%	0%	0%
		ER, average degree 3	0%	1%	7%	0%	0%	1%
		ER, average degree 6	13%	18%	22%	7%	9%	10%
		SF, average degree 3	11%	17%	37%	3%	5%	13%
		SF, average degree 6	31%	41%	66%	18%	19%	25%
	(0.2, 0.6)-heterogeneous	in-arborescence	0%	0%	15%	0%	0%	9%
		ER, average degree 3	5%	11%	25%	5%	7%	19%
		ER, average degree 6	14%	19%	34%	7%	12%	23%
		SF, average degree 3	4%	12%	28%	2%	6%	21%
		SF, average degree 6	17%	21%	33%	8%	11%	23%
$\lvert V \rvert = 100$	homogeneous	in-arborescence	**93%**	**93%**	**93%**	**75%**	**93%**	**93%**
		ER, average degree 3	**85%**	**100%**	**100%**	**78%**	**100%**	**100%**
		ER, average degree 6	**0%**	**0%**	**0%**	**100%**	**100%**	**100%**
		SF, average degree 3	**37%**	**75%**	**100%**	**37%**	**75%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	0%	0%	1%	0%	0%	0%
		ER, average degree 3	0%	1%	7%	0%	0%	6%
		ER, average degree 6	7%	9%	10%	7%	7%	7%
		SF, average degree 3	1%	4%	25%	0%	1%	7%
		SF, average degree 6	14%	22%	41%	7%	11%	15%
	(0.2, 0.6)-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	0%	7%	21%	0%	7%	17%
		ER, average degree 6	7%	8%	23%	7%	7%	17%
		SF, average degree 3	1%	5%	19%	1%	3%	15%
		SF, average degree 6	9%	11%	23%	7%	9%	17%
$\lvert V \rvert = 300$	homogeneous	in-arborescence	**93%**	**93%**	**93%**	**93%**	**93%**	**93%**
		ER, average degree 3	**0%**	**0%**	**100%**	**85%**	**99%**	**100%**
		ER, average degree 6	**0%**	**0%**	**0%**	**100%**	**100%**	**100%**
		SF, average degree 3	**28%**	**56%**	**97%**	**28%**	**56%**	**97%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	0%	0%	0%	0%	0%	0%
		ER, average degree 3	0%	1%	7%	0%	1%	6%
		ER, average degree 6	7%	7%	7%	7%	7%	7%
		SF, average degree 3	0%	1%	17%	0%	0%	10%
		SF, average degree 6	7%	11%	24%	7%	7%	10%
	(0.2, 0.6)-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	1%	7%	17%	1%	7%	16%
		ER, average degree 6	7%	7%	17%	7%	7%	16%
		SF, average degree 3	0%	1%	14%	0%	1%	13%
		SF, average degree 6	7%	8%	17%	7%	7%	16%

Table 9: Residual instabilities of homogeneous versus heterogeneous networks under coordinated shocks. The percentages shown are the percentages of networks for which $\xi < 0.05$ or $\xi < 0.1$ or $\xi < 0.2$.

			idiosyncratic shock					
			$\Phi = 0.6, \gamma = 0.55$			$\Phi = 0.6, \gamma = 0.50$		
			$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$	$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$
$\lvert V\rvert = 50$	homogeneous	in-arborescence	**94%**	**95%**	**95%**	**53%**	**87%**	**95%**
		ER, average degree 3	**100%**	**100%**	**100%**	**62%**	**95%**	**100%**
		ER, average degree 6	**57%**	**58%**	**60%**	**57%**	**57%**	**57%**
		SF, average degree 3	**54%**	**100%**	**100%**	**45%**	**90%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	2%	4%	16%	1%	3%	11%
		ER, average degree 3	0%	1%	14%	0%	0%	5%
		ER, average degree 6	8%	11%	23%	5%	7%	12%
		SF, average degree 3	0%	7%	24%	0%	2%	11%
		SF, average degree 6	9%	21%	39%	7%	15%	25%
	(0.2, 0.6)-heterogeneous	in-arborescence	1%	3%	13%	1%	1%	11%
		ER, average degree 3	7%	14%	25%	5%	10%	19%
		ER, average degree 6	11%	21%	34%	7%	11%	23%
		SF, average degree 3	1%	11%	21%	0%	9%	18%
		SF, average degree 6	10%	15%	27%	7%	9%	19%
$\lvert V\rvert = 100$	homogeneous	in-arborescence	**94%**	**94%**	**95%**	**69%**	**92%**	**95%**
		ER, average degree 3	**77%**	**100%**	**100%**	**45%**	**87%**	**100%**
		ER, average degree 6	**57%**	**57%**	**57%**	**57%**	**57%**	**57%**
		SF, average degree 3	**36%**	**81%**	**100%**	**34%**	**75%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**95%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	1%	2%	13%	1%	1%	11%
		ER, average degree 3	1%	2%	17%	0%	1%	13%
		ER, average degree 6	7%	7%	18%	7%	7%	15%
		SF, average degree 3	0%	9%	25%	0%	3%	17%
		SF, average degree 6	11%	19%	32%	7%	12%	22%
	(0.2, 0.6)-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	5%	9%	19%	1%	8%	17%
		ER, average degree 6	8%	13%	23%	7%	9%	19%
		SF, average degree 3	0%	3%	19%	0%	1%	17%
		SF, average degree 6	7%	9%	20%	7%	8%	17%
$\lvert V\rvert = 300$	homogeneous	in-arborescence	**94%**	**95%**	**95%**	**93%**	**95%**	**96%**
		ER, average degree 3	**96%**	**100%**	**100%**	**48%**	**89%**	**100%**
		ER, average degree 6	**57%**	**57%**	**57%**	**57%**	**57%**	**57%**
		SF, average degree 3	**34%**	**70%**	**100%**	**31%**	**67%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**93%**	**100%**	**100%**
	(0.1, 0.95)-heterogeneous	in-arborescence	1%	1%	12%	1%	1%	11%
		ER, average degree 3	0%	4%	16%	0%	1%	10%
		ER, average degree 6	7%	7%	16%	5%	7%	12%
		SF, average degree 3	0%	8%	25%	0%	3%	19%
		SF, average degree 6	9%	17%	29%	7%	11%	21%
	(0.2, 0.6)-heterogeneous	in-arborescence	0%	0%	9%	0%	0%	9%
		ER, average degree 3	7%	8%	17%	3%	7%	16%
		ER, average degree 6	7%	9%	18%	7%	7%	17%
		SF, average degree 3	0%	1%	17%	0%	0%	16%
		SF, average degree 6	7%	8%	17%	7%	7%	16%

Table 10: Residual instabilities of homogeneous versus heterogeneous networks under coordinated shocks. The percentages shown are the percentages of networks for which $\xi < 0.05$ or $\xi < 0.1$ or $\xi < 0.2$.

			idiosyncratic shock					
			$\Phi = 0.7, \gamma = 0.65$			$\Phi = 0.7, \gamma = 0.60$		
			$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$	$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$
$\lvert V\rvert = 50$	homogeneous	in-arborescence	**94%**	**95%**	**96%**	**73%**	**94%**	**95%**
		ER, average degree 3	**100%**	**100%**	**100%**	**81%**	**100%**	**100%**
		ER, average degree 6	**57%**	**59%**	**60%**	**57%**	**57%**	**57%**
		SF, average degree 3	**55%**	**100%**	**100%**	**55%**	**100%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	3%	7%	17%	1%	2%	12%
		ER, average degree 3	1%	5%	21%	0%	3%	13%
		ER, average degree 6	10%	18%	28%	7%	9%	19%
		SF, average degree 3	7%	9%	26%	0%	3%	19%
		SF, average degree 6	13%	24%	41%	9%	16%	27%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	1%	3%	14%	1%	1%	11%
		ER, average degree 3	7%	15%	27%	7%	10%	20%
		ER, average degree 6	13%	23%	35%	7%	13%	25%
		SF, average degree 3	1%	11%	23%	0%	6%	19%
		SF, average degree 6	11%	17%	29%	7%	9%	20%
$\lvert V\rvert = 100$	homogeneous	in-arborescence	**94%**	**95%**	**95%**	**79%**	**95%**	**95%**
		ER, average degree 3	**77%**	**100%**	**100%**	**57%**	**97%**	**100%**
		ER, average degree 6	**57%**	**57%**	**57%**	**57%**	**57%**	**57%**
		SF, average degree 3	**37%**	**78%**	**100%**	**35%**	**76%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	1%	3%	13%	1%	1%	12%
		ER, average degree 3	0%	4%	17%	0%	1%	14%
		ER, average degree 6	7%	9%	19%	7%	7%	16%
		SF, average degree 3	0%	10%	25%	0%	3%	19%
		SF, average degree 6	9%	18%	33%	8%	15%	25%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	1%	1%	11%	1%	1%	11%
		ER, average degree 3	6%	11%	21%	3%	9%	17%
		ER, average degree 6	7%	15%	25%	7%	9%	19%
		SF, average degree 3	0%	3%	19%	0%	1%	17%
		SF, average degree 6	7%	10%	21%	7%	8%	17%
$\lvert V\rvert = 300$	homogeneous	in-arborescence	**94%**	**95%**	**95%**	**85%**	**95%**	**95%**
		ER, average degree 3	**96%**	**100%**	**100%**	**67%**	**98%**	**100%**
		ER, average degree 6	**57%**	**57%**	**57%**	**57%**	**57%**	**57%**
		SF, average degree 3	**33%**	**71%**	**100%**	**33%**	**69%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**99%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	1%	1%	12%	1%	1%	12%
		ER, average degree 3	1%	5%	16%	0%	2%	13%
		ER, average degree 6	7%	7%	16%	6%	7%	15%
		SF, average degree 3	0%	9%	25%	0%	3%	19%
		SF, average degree 6	7%	17%	31%	7%	13%	21%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	1%	1%	11%	1%	1%	11%
		ER, average degree 3	7%	8%	17%	3%	7%	17%
		ER, average degree 6	7%	9%	19%	7%	8%	17%
		SF, average degree 3	0%	3%	17%	0%	1%	16%
		SF, average degree 6	7%	8%	17%	7%	7%	16%

Table 11: Residual instabilities of homogeneous versus heterogeneous networks under coordinated shocks. The percentages shown are the percentages of networks for which $\xi < 0.05$ or $\xi < 0.1$ or $\xi < 0.2$.

			idiosyncratic shock					
			$\Phi = 0.8, \gamma = 0.75$			$\Phi = 0.8, \gamma = 0.70$		
			$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$	$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$
$\lvert V\rvert = 50$	homogeneous	in-arborescence	**94%**	**95%**	**95%**	**83%**	**94%**	**95%**
		ER, average degree 3	**100%**	**100%**	**100%**	**89%**	**100%**	**100%**
		ER, average degree 6	**57%**	**60%**	**63%**	**57%**	**57%**	**58%**
		SF, average degree 3	**55%**	**100%**	**100%**	**52%**	**100%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	4%	9%	21%	1%	2%	12%
		ER, average degree 3	0%	7%	21%	0%	1%	15%
		ER, average degree 6	10%	21%	31%	7%	9%	17%
		SF, average degree 3	1%	11%	27%	0%	5%	20%
		SF, average degree 6	9%	23%	43%	9%	17%	31%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	2%	5%	15%	1%	1%	12%
		ER, average degree 3	8%	18%	29%	7%	11%	21%
		ER, average degree 6	15%	25%	38%	7%	15%	27%
		SF, average degree 3	1%	12%	23%	0%	7%	19%
		SF, average degree 6	12%	21%	33%	7%	10%	22%
$\lvert V\rvert = 100$	homogeneous	in-arborescence	**94%**	**95%**	**95%**	**84%**	**94%**	**95%**
		ER, average degree 3	**78%**	**100%**	**100%**	**68%**	**100%**	**100%**
		ER, average degree 6	**57%**	**57%**	**59%**	**57%**	**57%**	**57%**
		SF, average degree 3	**37%**	**79%**	**100%**	**35%**	**81%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	1%	3%	13%	1%	1%	13%
		ER, average degree 3	0%	4%	17%	0%	3%	15%
		ER, average degree 6	7%	9%	21%	7%	7%	16%
		SF, average degree 3	0%	9%	25%	0%	4%	19%
		SF, average degree 6	10%	19%	35%	7%	17%	25%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	1%	2%	13%	1%	1%	11%
		ER, average degree 3	5%	12%	21%	5%	9%	19%
		ER, average degree 6	9%	16%	28%	7%	11%	20%
		SF, average degree 3	0%	4%	19%	0%	1%	17%
		SF, average degree 6	8%	11%	23%	7%	8%	17%
$\lvert V\rvert = 300$	homogeneous	in-arborescence	**94%**	**95%**	**96%**	**92%**	**95%**	**95%**
		ER, average degree 3	**96%**	**100%**	**100%**	**73%**	**100%**	**100%**
		ER, average degree 6	**57%**	**57%**	**57%**	**57%**	**57%**	**57%**
		SF, average degree 3	**33%**	**67%**	**100%**	**31%**	**69%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	1%	1%	12%	1%	1%	12%
		ER, average degree 3	1%	5%	16%	1%	3%	16%
		ER, average degree 6	7%	7%	17%	7%	7%	15%
		SF, average degree 3	0%	9%	25%	0%	7%	22%
		SF, average degree 6	9%	17%	31%	7%	13%	25%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	1%	1%	11%	1%	1%	11%
		ER, average degree 3	7%	8%	18%	6%	7%	17%
		ER, average degree 6	7%	9%	20%	7%	8%	17%
		SF, average degree 3	0%	2%	17%	0%	1%	17%
		SF, average degree 6	7%	8%	17%	7%	7%	17%

Table 12: Residual instabilities of homogeneous versus heterogeneous networks under coordinated shocks. The percentages shown are the percentages of networks for which $\xi < 0.05$ or $\xi < 0.1$ or $\xi < 0.2$.

			idiosyncratic shock					
			$\Phi = 0.9, \gamma = 0.85$			$\Phi = 0.9, \gamma = 0.80$		
			$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$	$\xi < 0.05$	$\xi < 0.1$	$\xi < 0.2$
$\lvert V\rvert = 50$	homogeneous	in-arborescence	**94%**	**95%**	**95%**	**93%**	**94%**	**95%**
		ER, average degree 3	**100%**	**100%**	**100%**	**96%**	**100%**	**100%**
		ER, average degree 6	**5%**	**5%**	**8%**	**57%**	**57%**	**59%**
		SF, average degree 3	**55%**	**100%**	**100%**	**53%**	**100%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	9%	11%	21%	1%	3%	14%
		ER, average degree 3	1%	9%	23%	0%	2%	14%
		ER, average degree 6	11%	21%	31%	7%	10%	22%
		SF, average degree 3	1%	11%	27%	0%	7%	20%
		SF, average degree 6	12%	23%	44%	9%	20%	35%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	2%	4%	15%	1%	2%	12%
		ER, average degree 3	8%	17%	31%	7%	12%	22%
		ER, average degree 6	17%	25%	39%	9%	17%	29%
		SF, average degree 3	1%	12%	25%	0%	8%	19%
		SF, average degree 6	17%	23%	35%	9%	13%	23%
$\lvert V\rvert = 100$	homogeneous	in-arborescence	**94%**	**95%**	**95%**	**92%**	**95%**	**95%**
		ER, average degree 3	**77%**	**100%**	**100%**	**74%**	**100%**	**100%**
		ER, average degree 6	**3%**	**3%**	**5%**	**57%**	**57%**	**57%**
		SF, average degree 3	**34%**	**81%**	**100%**	**37%**	**81%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	1%	3%	14%	1%	1%	12%
		ER, average degree 3	0%	6%	19%	0%	1%	16%
		ER, average degree 6	7%	13%	22%	7%	7%	17%
		SF, average degree 3	0%	9%	25%	0%	6%	19%
		SF, average degree 6	11%	21%	37%	7%	17%	29%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	1%	2%	13%	1%	1%	11%
		ER, average degree 3	6%	13%	23%	5%	9%	19%
		ER, average degree 6	9%	19%	31%	7%	11%	21%
		SF, average degree 3	0%	5%	21%	0%	2%	17%
		SF, average degree 6	9%	14%	25%	7%	8%	18%
$\lvert V\rvert = 300$	homogeneous	in-arborescence	**94%**	**95%**	**95%**	**94%**	**95%**	**95%**
		ER, average degree 3	**0%**	**0%**	**100%**	**85%**	**100%**	**100%**
		ER, average degree 6	**3%**	**3%**	**3%**	**57%**	**57%**	**57%**
		SF, average degree 3	**30%**	**69%**	**100%**	**31%**	**69%**	**100%**
		SF, average degree 6	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**
	$(0.1, 0.95)$-heterogeneous	in-arborescence	1%	1%	12%	1%	1%	11%
		ER, average degree 3	0%	5%	17%	0%	3%	16%
		ER, average degree 6	7%	8%	17%	7%	7%	16%
		SF, average degree 3	0%	9%	25%	0%	7%	22%
		SF, average degree 6	10%	17%	33%	6%	15%	25%
	$(0.2, 0.6)$-heterogeneous	in-arborescence	1%	1%	11%	1%	1%	11%
		ER, average degree 3	7%	9%	18%	6%	7%	17%
		ER, average degree 6	7%	10%	21%	7%	9%	17%
		SF, average degree 3	0%	1%	17%	0%	1%	17%
		SF, average degree 6	7%	9%	17%	7%	7%	16%

Table 13: Residual instabilities of homogeneous versus heterogeneous networks under coordinated shocks. The percentages shown are the percentages of networks for which $\xi < 0.05$ or $\xi < 0.1$ or $\xi < 0.2$.

ER average degree 3 versus average degree 6		SF average degree 3 versus average degree 6	
coordinated shock	idiosyncratic shock	coordinated shock	idiosyncratic shock
97.43%	**97.05%**	**98.89%**	**98.29%**

Table 14: Effect of connectivity on the global stability for homogeneous networks under coordinated and idiosyncratic shocks. The number (percentage) shown for a comparison of the type "network A versus network B" indicates the percentage of data points for which the stability of networks of type A was at least as much as that of networks of type B.

(α, β)-heterogeneous networks

In-arborescence		ER average degree 3		ER average degree 6		SF average degree 3		SF average degree 6	
$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.2$
$\beta = 0.95$	$\beta = 0.6$	$\beta = 0.95$	$\beta = 0.6$	$\beta = 0.95$	$\beta = 0.6$	$\beta = 0.95$	$\beta = 0.6$	$\beta = 0.95$	$\beta = 0.6$
56.64%	**57.27%**	**89.66%**	**90.97%**	**98.99%**	**98.04%**	**93.16%**	**64.13%**	**94.44%**	**66.48%**

homogeneous networks

In-arborescence	ER average degree 3	ER average degree 6	SF average degree 3	SF average degree 6
84.62%	**74.97%**	**78.59%**	**81.15%**	**54.80%**

Table 15: Comparisons of strengths of coordinated versus idiosyncratic shocks. The numbers (percentages) indicate the percentage of total number of data points (combinations of parameters Φ, γ, E and κ) for that network type that resulted in ξ_c being at least ξ_r, where ξ_c and ξ_r are the vulnerability indices under coordinated and idiosyncratic shocks, respectively.

		$\Phi = 0.5$										$\Phi = 0.8$									
		ER average degree 6, $\kappa =$					SF average degree 6, $\kappa =$					ER average degree 6, $\kappa =$					SF average degree 6, $\kappa =$				
E/I	$\lvert V \rvert$	0.1	0.2	0.3	0.4	0.5	0.1	0.2	0.3	0.4	0.5	0.1	0.2	0.3	0.4	0.5	0.1	0.2	0.3	0.4	0.5
	50	**0.79**	**0.77**	**0.79**	**0.83**	**0.81**	**0.99**	**0.98**	**0.97**	**0.95**	**0.93**	**0.81**	**0.77**	**0.77**	0.74	0.7	**0.95**	**0.9**	**0.85**	**0.78**	0.72
0.25	100	**0.8**	**0.79**	**0.81**	**0.77**	**0.77**	**0.99**	**0.97**	**0.95**	**0.92**	**0.9**	**0.84**	**0.81**	**0.79**	**0.76**	0.72	**0.95**	**0.86**	**0.79**	0.71	0.66
	300	**0.93**	**0.89**	**0.86**	**0.83**	**0.81**	**0.98**	**0.96**	**0.93**	**0.9**	**0.88**	**0.83**	**0.8**	**0.79**	**0.76**	0.71	**0.89**	**0.79**	0.71	0.63	0.59
	50	**0.79**	**0.77**	**0.79**	**0.83**	**0.81**	**0.99**	**0.98**	**0.97**	**0.95**	**0.93**	**0.83**	**0.78**	**0.78**	0.74	0.68	**0.95**	**0.89**	**0.84**	**0.78**	0.72
0.5	100	**0.8**	**0.79**	**0.81**	**0.77**	**0.77**	**0.99**	**0.97**	**0.95**	**0.92**	**0.9**	**0.83**	**0.81**	**0.79**	**0.76**	0.71	**0.94**	**0.85**	**0.78**	0.71	0.65
	300	**0.93**	**0.89**	**0.86**	**0.83**	**0.81**	**0.98**	**0.96**	**0.93**	**0.9**	**0.88**	**0.83**	**0.81**	**0.79**	**0.76**	0.7	**0.88**	**0.78**	0.7	0.63	0.61
	50	**0.79**	**0.77**	**0.79**	**0.83**	**0.81**	**0.99**	**0.98**	**0.97**	**0.95**	**0.93**	**0.84**	**0.78**	**0.77**	**0.73**	0.67	**0.95**	**0.89**	**0.83**	**0.77**	0.71
0.75	100	**0.8**	**0.79**	**0.81**	**0.77**	**0.77**	**0.99**	**0.97**	**0.95**	**0.92**	**0.9**	**0.83**	**0.82**	**0.8**	**0.76**	0.71	**0.94**	**0.85**	**0.77**	0.7	0.65
	300	**0.93**	**0.89**	**0.86**	**0.83**	**0.81**	**0.98**	**0.96**	**0.93**	**0.9**	**0.88**	**0.83**	**0.8**	**0.78**	0.74	0.69	**0.9**	**0.77**	0.69	0.63	0.61
	50	**0.78**	**0.77**	**0.78**	**0.79**	**0.79**	**0.99**	**0.96**	**0.93**	**0.88**	**0.82**	**0.83**	**0.79**	**0.78**	0.72	0.67	**0.95**	**0.89**	**0.83**	**0.76**	0.69
1	100	**0.8**	**0.8**	**0.8**	**0.8**	**0.78**	**0.98**	**0.93**	**0.87**	**0.81**	**0.75**	**0.83**	**0.81**	**0.8**	**0.76**	0.7	**0.95**	**0.83**	0.74	0.66	0.61
	300	**0.87**	**0.82**	**0.83**	**0.81**	**0.77**	**0.97**	**0.91**	**0.87**	**0.81**	**0.75**	**0.82**	**0.79**	**0.78**	0.74	0.69	**0.86**	0.71	0.61	0.57	0.55
	50	**0.78**	**0.77**	**0.78**	**0.79**	**0.79**	**0.99**	**0.96**	**0.93**	**0.88**	**0.82**	**0.84**	**0.79**	**0.79**	0.73	0.67	**0.95**	**0.89**	**0.83**	**0.76**	0.69
1.25	100	**0.8**	**0.8**	**0.8**	**0.8**	**0.78**	**0.98**	**0.93**	**0.87**	**0.81**	**0.75**	**0.83**	**0.82**	**0.8**	**0.76**	0.7	**0.95**	**0.83**	0.74	0.67	0.61
	300	**0.87**	**0.82**	**0.83**	**0.81**	**0.77**	**0.97**	**0.91**	**0.87**	**0.81**	**0.75**	**0.83**	**0.8**	**0.78**	0.74	0.68	**0.86**	0.7	0.63	0.59	0.56
	50	**0.78**	**0.77**	**0.78**	**0.79**	**0.79**	**0.99**	**0.96**	**0.93**	**0.88**	**0.82**	**0.84**	**0.8**	**0.79**	0.72	0.65	**0.94**	**0.89**	**0.83**	**0.76**	0.69
1.5	100	**0.8**	**0.8**	**0.8**	**0.8**	**0.78**	**0.98**	**0.93**	**0.87**	**0.81**	**0.75**	**0.83**	**0.81**	**0.79**	0.74	0.68	**0.94**	**0.81**	0.72	0.67	0.62
	300	**0.87**	**0.82**	**0.83**	**0.81**	**0.77**	**0.97**	**0.91**	**0.87**	**0.81**	**0.75**	**0.82**	**0.8**	**0.77**	0.73	0.67	**0.85**	0.69	0.62	0.59	0.56
	50	**0.78**	**0.77**	**0.78**	**0.79**	**0.79**	**0.99**	**0.96**	**0.93**	**0.88**	**0.82**	**0.83**	**0.78**	**0.78**	0.72	0.65	**0.94**	**0.89**	**0.82**	**0.76**	0.69
1.75	100	**0.8**	**0.8**	**0.8**	**0.8**	**0.78**	**0.98**	**0.93**	**0.87**	**0.81**	**0.75**	**0.83**	**0.8**	**0.78**	0.74	0.66	**0.93**	**0.81**	0.73	0.67	0.63
	300	**0.87**	**0.82**	**0.83**	**0.81**	**0.77**	**0.97**	**0.91**	**0.87**	**0.81**	**0.75**	**0.82**	**0.8**	**0.77**	0.72	0.66	**0.83**	0.68	0.61	0.6	0.59
	50	**0.79**	**0.77**	**0.76**	**0.77**	**0.76**	**0.99**	**0.95**	**0.89**	**0.83**	**0.77**	**0.83**	**0.79**	**0.78**	0.72	0.65	**0.94**	**0.89**	**0.8**	0.72	0.64
2	100	**0.8**	**0.84**	**0.8**	**0.78**	**0.76**	**0.98**	**0.89**	**0.81**	0.71	0.69	**0.82**	**0.79**	**0.77**	0.72	0.64	**0.93**	**0.78**	0.69	0.65	0.6
	300	**0.85**	**0.83**	**0.8**	**0.78**	**0.75**	**0.95**	**0.88**	**0.79**	0.72	0.61	**0.81**	**0.79**	**0.77**	0.71	0.65	**0.81**	0.64	0.59	0.58	0.57
	50	**0.79**	**0.77**	**0.76**	**0.77**	**0.76**	**0.99**	**0.95**	**0.89**	**0.83**	**0.77**	**0.83**	**0.79**	**0.78**	0.72	0.65	**0.94**	**0.88**	**0.8**	0.72	0.63
2.25	100	**0.8**	**0.84**	**0.8**	**0.78**	**0.76**	**0.98**	**0.89**	**0.81**	0.71	0.69	**0.83**	**0.8**	**0.77**	0.7	0.62	**0.93**	**0.78**	0.7	0.65	0.6
	300	**0.85**	**0.83**	**0.8**	**0.78**	**0.75**	**0.95**	**0.88**	**0.79**	0.72	0.61	**0.82**	**0.8**	**0.77**	0.71	0.64	**0.8**	0.65	0.6	0.6	0.56
	50	**0.79**	**0.77**	**0.76**	**0.77**	**0.76**	**0.99**	**0.95**	**0.89**	**0.83**	**0.77**	**0.82**	**0.78**	**0.78**	0.71	0.64	**0.94**	**0.87**	**0.77**	0.69	0.61
2.5	100	**0.8**	**0.84**	**0.8**	**0.78**	**0.76**	**0.98**	**0.89**	**0.81**	0.71	0.69	**0.81**	**0.8**	**0.76**	0.69	0.6	**0.92**	**0.77**	0.69	0.63	0.58
	300	**0.85**	**0.83**	**0.8**	**0.78**	**0.75**	**0.95**	**0.88**	**0.79**	0.72	0.61	**0.82**	**0.8**	**0.76**	0.69	0.62	**0.79**	0.63	0.59	0.59	0.56
	50	**0.79**	**0.77**	**0.76**	**0.77**	**0.76**	**0.99**	**0.95**	**0.89**	**0.83**	**0.77**	**0.8**	**0.77**	**0.77**	0.69	0.61	**0.94**	**0.87**	**0.76**	0.66	0.59
2.75	100	**0.8**	**0.84**	**0.8**	**0.78**	**0.76**	**0.98**	**0.89**	**0.81**	0.71	0.69	**0.81**	**0.79**	**0.75**	0.68	0.6	**0.92**	**0.77**	0.69	0.63	0.58
	300	**0.85**	**0.83**	**0.8**	**0.78**	**0.75**	**0.95**	**0.88**	**0.79**	0.72	0.61	**0.81**	**0.8**	**0.75**	0.69	0.61	**0.78**	0.63	0.6	0.6	0.56
	50	**0.78**	**0.79**	**0.79**	**0.76**	0.72	**0.98**	**0.94**	**0.86**	**0.77**	0.7	**0.8**	**0.78**	**0.76**	0.69	0.59	**0.94**	**0.87**	**0.76**	0.66	0.58
3	100	**0.79**	**0.81**	**0.8**	**0.77**	0.73	**0.98**	**0.87**	0.73	0.68	0.63	**0.8**	**0.78**	0.74	0.66	0.57	**0.92**	**0.77**	0.69	0.63	0.56
	300	**0.87**	**0.8**	**0.78**	**0.75**	0.71	**0.94**	**0.84**	0.72	0.61	0.58	**0.81**	**0.79**	**0.75**	0.68	0.6	**0.77**	0.62	0.61	0.6	0.55
	50	**0.78**	**0.79**	**0.79**	**0.76**	0.72	**0.98**	**0.94**	**0.86**	**0.77**	0.7	**0.8**	**0.79**	**0.76**	0.68	0.57	**0.94**	**0.88**	**0.75**	0.65	0.55
3.25	100	**0.79**	**0.81**	**0.8**	**0.77**	0.73	**0.98**	**0.87**	0.73	0.68	0.63	**0.81**	**0.79**	0.74	0.65	0.55	**0.92**	**0.77**	0.69	0.62	0.53
	300	**0.87**	**0.8**	**0.78**	**0.75**	0.71	**0.94**	**0.84**	0.72	0.61	0.58	**0.81**	**0.79**	0.74	0.67	0.58	**0.77**	0.61	0.61	0.59	0.52
	50	**0.78**	**0.79**	**0.79**	**0.76**	0.72	**0.98**	**0.94**	**0.86**	**0.77**	0.7	**0.81**	**0.78**	**0.75**	0.67	0.55	**0.94**	**0.87**	**0.75**	0.64	0.54
3.5	100	**0.79**	**0.81**	**0.8**	**0.77**	0.73	**0.98**	**0.87**	0.73	0.68	0.63	**0.81**	**0.79**	0.73	0.64	0.54	**0.92**	**0.76**	0.69	0.62	0.52
	300	**0.87**	**0.8**	**0.78**	**0.75**	0.71	**0.94**	**0.84**	0.72	0.61	0.58	**0.81**	**0.78**	0.73	0.65	0.56	**0.76**	0.61	0.61	0.59	0.52

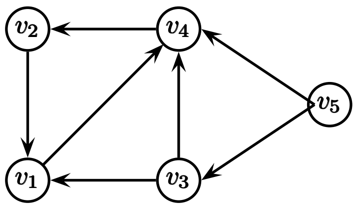
Table 16: Values of $\Lambda(n, \Phi, E/I, \kappa) = \dfrac{\max\limits_{0.05 \leq \gamma \leq 0.2} \{\xi\} - \min\limits_{0.05 \leq \gamma \leq 0.2} \{\xi\}}{\max\limits_{\text{entire range of } \gamma} \{\xi\} - \min\limits_{\text{entire range of } \gamma} \{\xi\}}$ for homogeneous dense ER and SF networks under coordinated shocks. Entries that are at least 2×0.375 are shown in **boldface**.

				κ								
				0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Coordinated shock	$\lvert V\rvert = 50$	$\Phi = 0.5$	$\gamma = 0.25$	**0.87**	**0.86**	**0.84**	**0.83**	**0.83**	**0.83**	**0.83**	**0.83**	**0.83**
			$\gamma = 0.3$	**1.00**	**1.00**	**0.99**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
		$\Phi = 0.6$	$\gamma = 0.3$	0.27	**0.49**	**0.52**	**0.52**	**0.52**	**0.52**	**0.52**	**0.52**	**0.52**
			$\gamma = 0.35$	0.27	**0.51**	**0.37**	**0.37**	**0.37**	**0.37**	**0.37**	**0.37**	**0.37**
		$\Phi = 0.7$	$\gamma = 0.35$	0.27	**0.48**	**0.59**	**0.64**	**0.65**	**0.65**	**0.65**	**0.65**	**0.65**
			$\gamma = 0.4$	0.30	**0.34**	**0.32**	0.29	0.29	0.29	0.29	0.29	0.29
		$\Phi = 0.8$	$\gamma = 0.4$	0.24	**0.42**	**0.47**	**0.47**	**0.47**	**0.47**	**0.47**	**0.47**	**0.47**
			$\gamma = 0.45$	**0.33**	**0.45**	**0.43**	**0.41**	**0.40**	**0.40**	**0.40**	**0.40**	**0.40**
		$\Phi = 0.9$	$\gamma = 0.45$	0.24	**0.47**	**0.54**	**0.56**	**0.56**	**0.56**	**0.56**	**0.56**	**0.56**
			$\gamma = 0.5$	0.28	**0.38**	**0.37**	**0.34**	**0.32**	**0.32**	**0.32**	**0.32**	**0.32**
	$\lvert V\rvert = 100$	$\Phi = 0.5$	$\gamma = 0.25$	**0.95**	**0.94**	**0.94**	**0.93**	**0.93**	**0.93**	**0.93**	**0.93**	**0.93**
			$\gamma = 0.3$	**0.98**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
		$\Phi = 0.6$	$\gamma = 0.3$	**0.39**	0.27	**0.33**	0.31	0.29	0.29	0.29	0.29	0.29
			$\gamma = 0.35$	**0.32**	0.24	0.28	0.30	**0.32**	**0.32**	**0.32**	**0.32**	**0.32**
		$\Phi = 0.7$	$\gamma = 0.35$	**0.39**	**0.57**	**0.67**	**0.69**	**0.69**	**0.69**	**0.69**	**0.69**	**0.69**
			$\gamma = 0.4$	**0.36**	**0.44**	**0.42**	**0.42**	**0.42**	**0.42**	**0.42**	**0.42**	**0.42**
		$\Phi = 0.8$	$\gamma = 0.4$	**0.36**	**0.49**	**0.54**	**0.53**	**0.53**	**0.53**	**0.53**	**0.53**	**0.53**
			$\gamma = 0.45$	**0.37**	**0.46**	**0.47**	**0.48**	**0.48**	**0.48**	**0.48**	**0.48**	**0.48**
		$\Phi = 0.9$	$\gamma = 0.45$	**0.39**	**0.56**	**0.66**	**0.67**	**0.67**	**0.67**	**0.67**	**0.67**	**0.67**
			$\gamma = 0.5$	**0.37**	**0.47**	**0.48**	**0.47**	**0.47**	**0.47**	**0.47**	**0.47**	**0.47**
	$\lvert V\rvert = 300$	$\Phi = 0.5$	$\gamma = 0.25$	**0.93**	**0.94**	**0.94**	**0.93**	**0.93**	**0.93**	**0.93**	**0.93**	**0.93**
			$\gamma = 0.3$	**0.99**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
		$\Phi = 0.6$	$\gamma = 0.3$	**0.39**	0.27	0.29	0.25	0.25	0.25	0.25	0.25	0.25
			$\gamma = 0.35$	0.31	0.14	0.16	0.14	0.14	0.14	0.14	0.14	0.14
		$\Phi = 0.7$	$\gamma = 0.35$	**0.40**	**0.53**	**0.61**	**0.62**	**0.62**	**0.62**	**0.62**	**0.62**	**0.62**
			$\gamma = 0.4$	**0.34**	**0.36**	**0.34**	**0.33**	**0.33**	**0.33**	**0.33**	**0.33**	**0.33**
		$\Phi = 0.8$	$\gamma = 0.4$	**0.38**	**0.47**	**0.50**	**0.50**	**0.50**	**0.50**	**0.50**	**0.50**	**0.50**
			$\gamma = 0.45$	**0.39**	**0.43**	**0.42**	**0.40**	**0.40**	**0.40**	**0.40**	**0.40**	**0.40**
		$\Phi = 0.9$	$\gamma = 0.45$	**0.40**	**0.53**	**0.60**	**0.61**	**0.61**	**0.61**	**0.61**	**0.61**	**0.61**
			$\gamma = 0.5$	**0.39**	**0.44**	**0.44**	**0.43**	**0.43**	**0.43**	**0.43**	**0.43**	**0.43**
Idiosyncratic shock	$\lvert V\rvert = 50$	$\Phi = 0.5$	$\gamma = 0.25$	**0.95**	**0.95**	**0.95**	**0.76**	**0.77**	**0.37**	0.08	0.11	0.25
			$\gamma = 0.3$	**0.95**	**0.95**	**0.93**	**0.99**	**0.76**	**0.33**	0.07	0.23	0.29
		$\Phi = 0.6$	$\gamma = 0.3$	0.08	**0.37**	**0.32**	**0.47**	**0.48**	0.21	**0.35**	**0.39**	0.25
			$\gamma = 0.35$	0.08	0.12	0.27	0.30	**0.38**	0.27	0.24	0.13	0.16
		$\Phi = 0.7$	$\gamma = 0.35$	0.08	0.15	0.12	0.04	0.26	0.13	0.07	0.11	0.15
			$\gamma = 0.4$	0.04	0.06	0.03	0.25	0.17	0.06	0.09	0.06	0.08
		$\Phi = 0.8$	$\gamma = 0.4$	0.05	0.09	0.13	0.21	0.12	0.10	0.08	0.14	0.10
			$\gamma = 0.45$	0.02	0.17	0.08	0.27	0.17	0.19	0.05	0.04	0.08
		$\Phi = 0.9$	$\gamma = 0.45$	0.09	0.07	0.20	0.21	0.03	0.12	0.02	0.17	0.17
			$\gamma = 0.5$	0.08	0.04	0.13	0.24	0.12	0.10	0.10	0.06	0.10
	$\lvert V\rvert = 100$	$\Phi = 0.5$	$\gamma = 0.25$	**0.98**	**0.96**	**0.98**	**0.96**	**0.67**	**0.37**	0.16	0.03	0.16
			$\gamma = 0.3$	**0.99**	**1.00**	**0.95**	**0.99**	**0.71**	0.31	0.07	0.11	0.26
		$\Phi = 0.6$	$\gamma = 0.3$	0.04	0.15	0.3	**0.44**	0.03	0.08	0.22	0.16	0.06
			$\gamma = 0.35$	0.03	0.04	0.29	0.30	0.17	**0.33**	0.06	0.10	0.10
		$\Phi = 0.7$	$\gamma = 0.35$	0.02	0.11	0.09	0.23	0.13	0.01	0.10	0.05	0.06
			$\gamma = 0.4$	0.02	0.1	0.16	0.11	0.12	0.13	0.05	0.03	0.05
		$\Phi = 0.8$	$\gamma = 0.4$	0.03	0.03	0.07	0.07	0.07	0.04	0.06	0.02	0.08
			$\gamma = 0.45$	0.03	0.06	0.14	0.17	0.09	0.13	0.08	0.03	0.08
		$\Phi = 0.9$	$\gamma = 0.45$	0.04	0.07	0.07	0.10	0.14	0.10	0.09	0.08	0.07
			$\gamma = 0.5$	0.05	0.15	0.10	0.07	0.08	0.13	0.02	0.06	0.04
	$\lvert V\rvert = 300$	$\Phi = 0.5$	$\gamma = 0.25$	**1.00**	**0.99**	**0.98**	**1.00**	**0.69**	**0.36**	0.16	0.07	0.16
			$\gamma = 0.3$	**1.00**	**0.98**	**0.96**	**1.00**	**0.69**	**0.36**	0.12	0.06	0.23
		$\Phi = 0.6$	$\gamma = 0.3$	0.03	0.12	0.03	0.21	0.12	0.14	0.12	0.08	0.14
			$\gamma = 0.35$	0.04	0.03	0.01	0.15	0.08	0.04	0.07	0.09	0.04
		$\Phi = 0.7$	$\gamma = 0.35$	0.06	0.08	0.13	0.17	0.14	0.05	0.07	0.02	0.08
			$\gamma = 0.4$	0.03	0.06	0.09	0.11	0.10	0.08	0.02	0.02	0.05
		$\Phi = 0.8$	$\gamma = 0.4$	0.04	0.09	0.12	0.14	0.12	0.11	0.04	0.05	0.08
			$\gamma = 0.45$	0.04	0.17	0.11	0.15	0.17	0.09	0.04	0.04	0.05
		$\Phi = 0.9$	$\gamma = 0.45$	0.05	0.08	0.11	0.18	0.14	0.02	0.03	0.04	0.07
			$\gamma = 0.5$	0.05	0.07	0.10	0.21	0.11	0.09	0.04	0.01	0.05

Table 17: Values of $\Delta(n,\Phi,\gamma,\kappa) = \dfrac{\max\limits_{0.5\leq E/I\leq 1}\{\xi\} - \min\limits_{0.5\leq E/I\leq 1}\{\xi\}}{\max\limits_{\text{entire range of } E}\{\xi\} - \min\limits_{\text{entire range of } E}\{\xi\}}$ for homogeneous in-arborescence networks. Entries that are at least 2×0.16 are shown in **boldface black**. Entries that are at least $\frac{3}{2}\times 0.16$ are shown in **boldface gray**.

APPENDIX

A Illustration of calculations of parameters for a sample financial network shown in Fig. 3



n = number of nodes = 5
m = number of edges = 7
$\gamma = 0.1$
I = total inter-bank exposure = m = 7
E = total external asset = $2I$ = 14

Figure 3: A banking network.

(a) Homogeneous version of the network

- w = weight of every edge = $I/m = 1$.
- $E_{v_1} = E_{v_2} = E_{v_3} = E_{v_4} = E_{v_5} = 14/5$.
- $\iota_{v_1} = \iota_{v_2} = \iota_{v_4} = 1, \iota_{v_3} = \iota_{v_5} = 2$.
- $b_{v_1} = \text{in–degree}(v_1) = 2, b_{v_2} = \text{in–degree}(v_2) = 1, b_{v_3} = \text{in–degree}(v_3) = 1, b_{v_4} = \text{in–degree}(v_4) = 3, b_{v_5} = \text{in–degree}(v_5) = 0$.
- $e_{v_1} = (b_{v_1} - \iota_{v_1}) + E_{v_1} = 3.8, e_{v_2} = (b_{v_2} - \iota_{v_2}) + E_{v_2} = 2.8, e_{v_3} = (b_{v_3} - \iota_{v_3}) + E_{v_3} = 1.8, e_{v_4} = (b_{v_4} - \iota_{v_4}) + E_{v_4} = 4.8, e_{v_5} = (b_{v_5} - \iota_{v_5}) + E_{v_5} = 0.8$.
- $a_{v_1} = b_{v_1} + E_{v_1} = 4.8, a_{v_2} = b_{v_2} + E_{v_2} = 3.8, a_{v_3} = b_{v_3} + E_{v_3} = 3.8, a_{v_4} = b_{v_4} + E_{v_4} = 5.8, a_{v_5} = b_{v_5} + E_{v_5} = 2.8$.
- $c_{v_1} = \gamma\, a_{v_1} = 0.48$, $c_{v_2} = \gamma\, a_{v_2} = 0.38$, $c_{v_3} = \gamma\, a_{v_3} = 0.38$, $c_{v_4} = \gamma\, a_{v_4} = 0.58$, $c_{v_5} = \gamma\, a_{v_5} = 0.28$.

(b) Heterogeneous version of the network

Suppose that 95% of E is distributed (equally) on the two banks v_1 and v_2, and the rest 5% of E is distributed (equally) on the remaining three banks. Thus:

$$E_{v_1} = \alpha_{v_1}\left(E - \sum_{j=1}^{5}\left(b_{v_j} - \iota_{v_j}\right)\right) = 6.65, \quad E_{v_2} = \alpha_{v_2}\left(E - \sum_{j=1}^{5}\left(b_{v_j} - \iota_{v_j}\right)\right) = 6.65, \quad E_{v_3} = \alpha_{v_3}\left(E - \sum_{j=1}^{5}\left(b_{v_j} - \iota_{v_j}\right)\right) \approx 0.233$$

$$E_{v_4} = \alpha_{v_4}\left(E - \sum_{j=1}^{5}\left(b_{v_j} - \iota_{v_j}\right)\right) \approx 0.233, \quad E_{v_5} = \alpha_{v_5}\left(E - \sum_{j=1}^{5}\left(b_{v_j} - \iota_{v_j}\right)\right) \approx 0.233$$

Suppose that 95% of I is distributed (equally) on the three edges $f_1 = (v_2, v_1), f_2 = (v_1, v_4), f_3 = (v_4, v_2)$, and the remaining 5% of I is distributed (equally) on the remaining four edges $f_4 = (v_3, v_1), f_5 = (v_3, v_4), f_6 = (v_5, v_4), f_7 = (v_5, v_3)$. Then,

$$w(f_1) = w(f_2) = w(f_3) = \frac{0.95\, I}{3} \approx 2.216, \quad w(f_4) = w(f_5) = w(f_6) = w(f_7) = \frac{0.05\, I}{4} = 0.08725$$

$$\text{for bank } v_1\text{:}\quad \begin{array}{l} b_{v_1} = w(f_1) + w(f_4) \approx 2.30325,\ \ \iota_{v_1} = w(f_2) = 2.216,\ \ e_{v_1} = (b_{v_1} - \iota_{v_1}) + E_{v_1} \approx 6.7365 \\ a_{v_1} = b_{v_1} + E_{v_1} = 8.9525,\ \ c_{v_1} = \gamma\, a_{v_1} = 0.8925 \end{array}$$

$$\text{for bank } v_2\text{:}\quad \begin{array}{l} b_{v_2} = w(f_3) \approx 2.216,\ \ \iota_{v_2} = w(f_1) \approx 2.216,\ \ e_{v_2} = (b_{v_2} - \iota_{v_2}) + E_{v_2} = 6.65 \\ a_{v_2} = b_{v_2} + E_{v_2} \approx 8.866,\ \ c_{v_2} = \gamma\, a_{v_2} \approx 0.8666 \end{array}$$

$$\text{for bank } v_3\text{:}\quad \begin{array}{l} b_{v_3} = w(f_7) = 0.08725,\ \ \iota_{v_3} = w(f_4) + w(f_5) = 0.1745,\ \ e_{v_3} = (b_{v_3} - \iota_{v_3}) + E_{v_3} \approx 0.14575 \\ a_{v_3} = b_{v_3} + E_{v_3} \approx 0.32025,\ \ c_{v_3} = \gamma\, a_{v_3} \approx 0.032035 \end{array}$$

$$\text{for bank } v_4\text{:}\quad \begin{array}{l} b_{v_4} = w(f_2) + w(f_5) + w(f_6) \approx 2.39050,\ \ \iota_{v_4} = w(f_3) \approx 2.216,\ \ e_{v_4} = (b_{v_4} - \iota_{v_4}) + E_{v_4} \approx 0.4075 \\ a_{v_4} = b_{v_4} + E_{v_4} \approx 2.6235,\ \ c_{v_4} = \gamma\, a_{v_4} \approx 0.26235 \end{array}$$

$$\text{for bank } v_5\text{:}\quad \begin{array}{l} b_{v_5} = 0,\ \ \iota_{v_5} = w(f_6) + w(f_7) = 0.1745,\ \ e_{v_5} = (b_{v_5} - \iota_{v_5}) + E_{v_5} \approx 0.0585 \\ a_{v_5} = b_{v_5} + E_{v_5} \approx 0.233,\ \ c_{v_5} = \gamma\, a_{v_5} \approx 0.0233 \end{array}$$

Supplementary color figures S1—S11

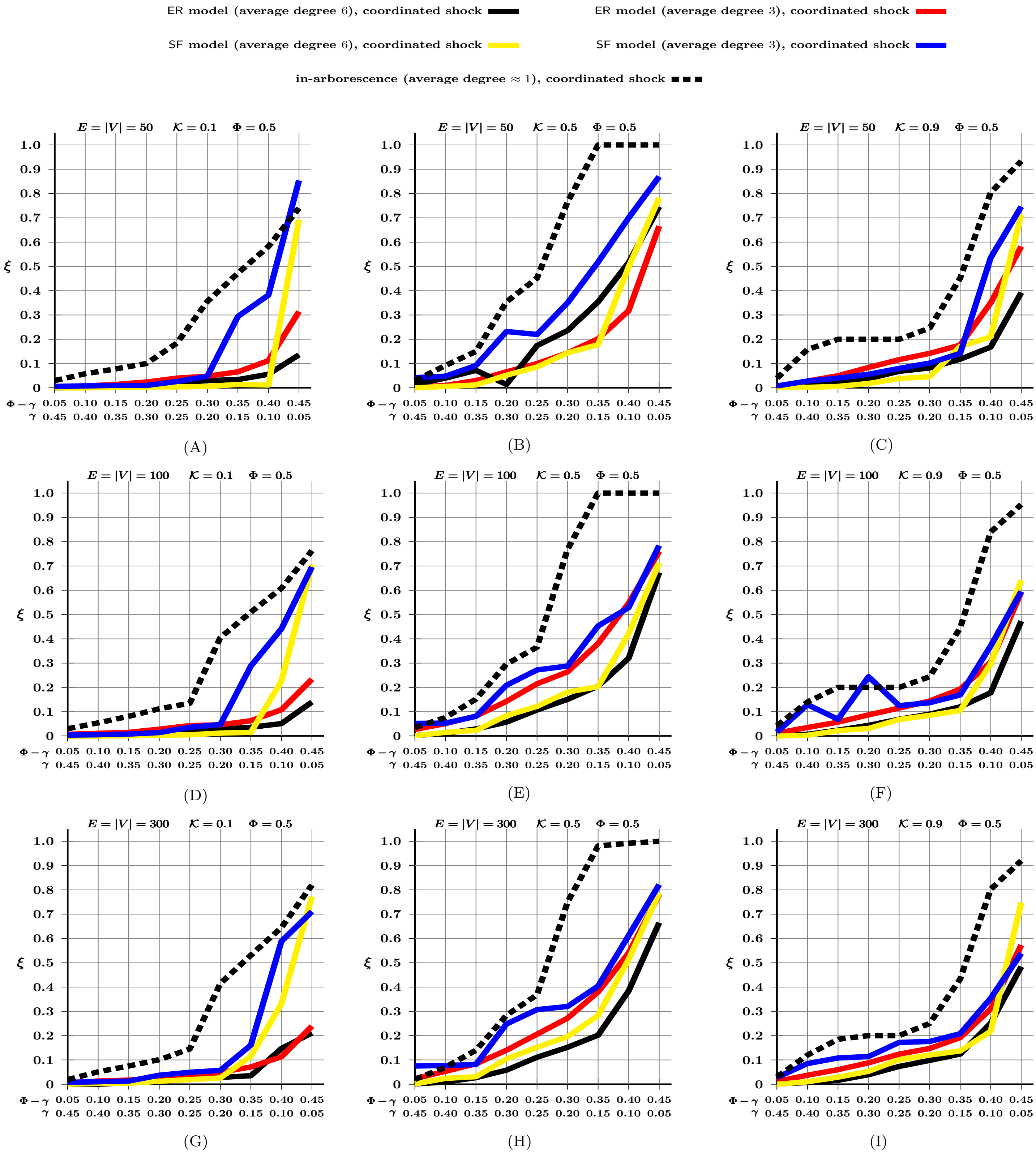


Figure S1: (drawn in color) Effect of variations of equity to asset ratio (with respect to shock) on the vulnerability index ξ for homogeneous networks. Lower values of ξ imply higher global stability of a network.

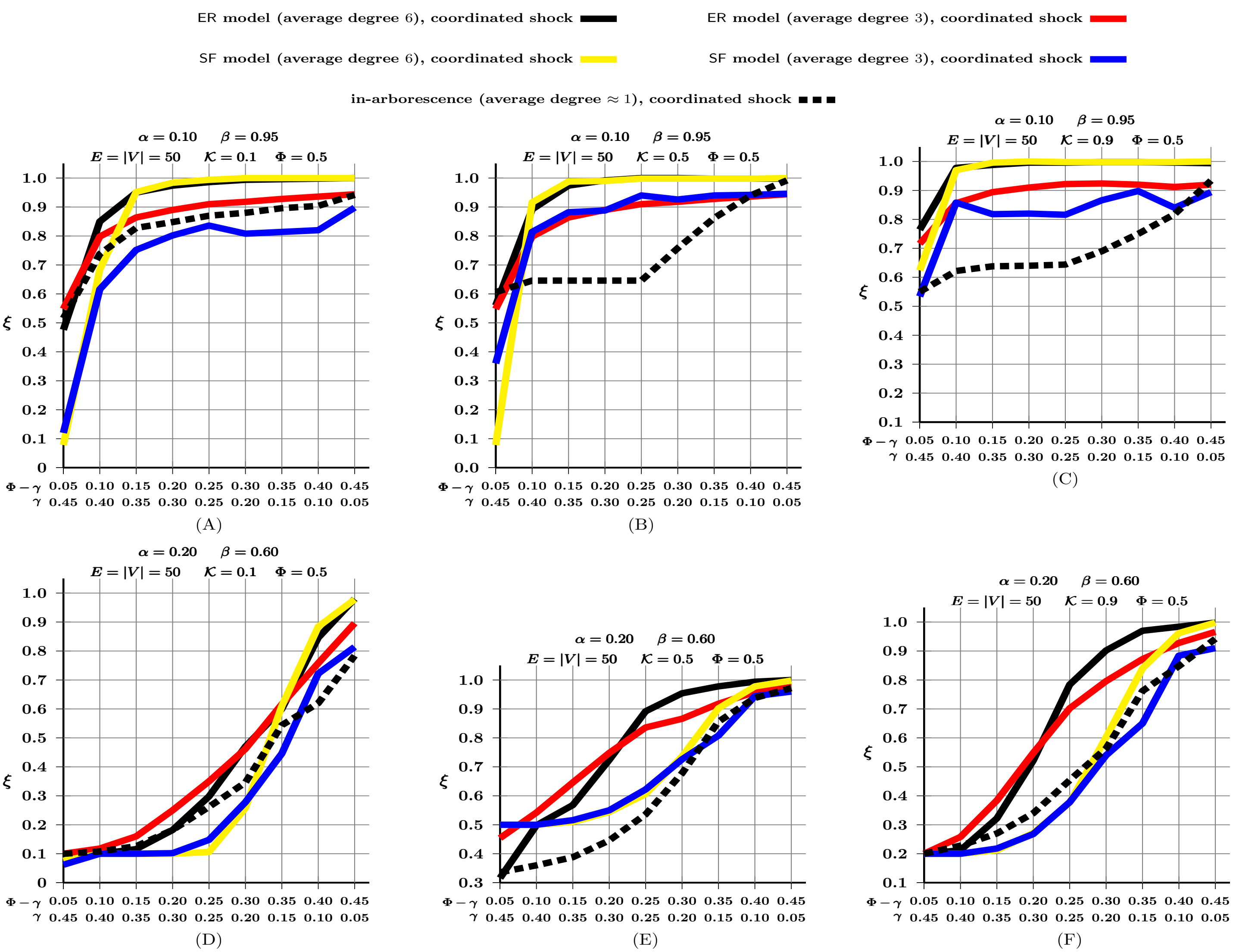


Figure S2: (drawn in color) Effect of variations of equity to asset ratio (with respect to shock) on the vulnerability index ξ for (α, β)-heterogeneous networks. Lower values of ξ imply higher global stability of a network.

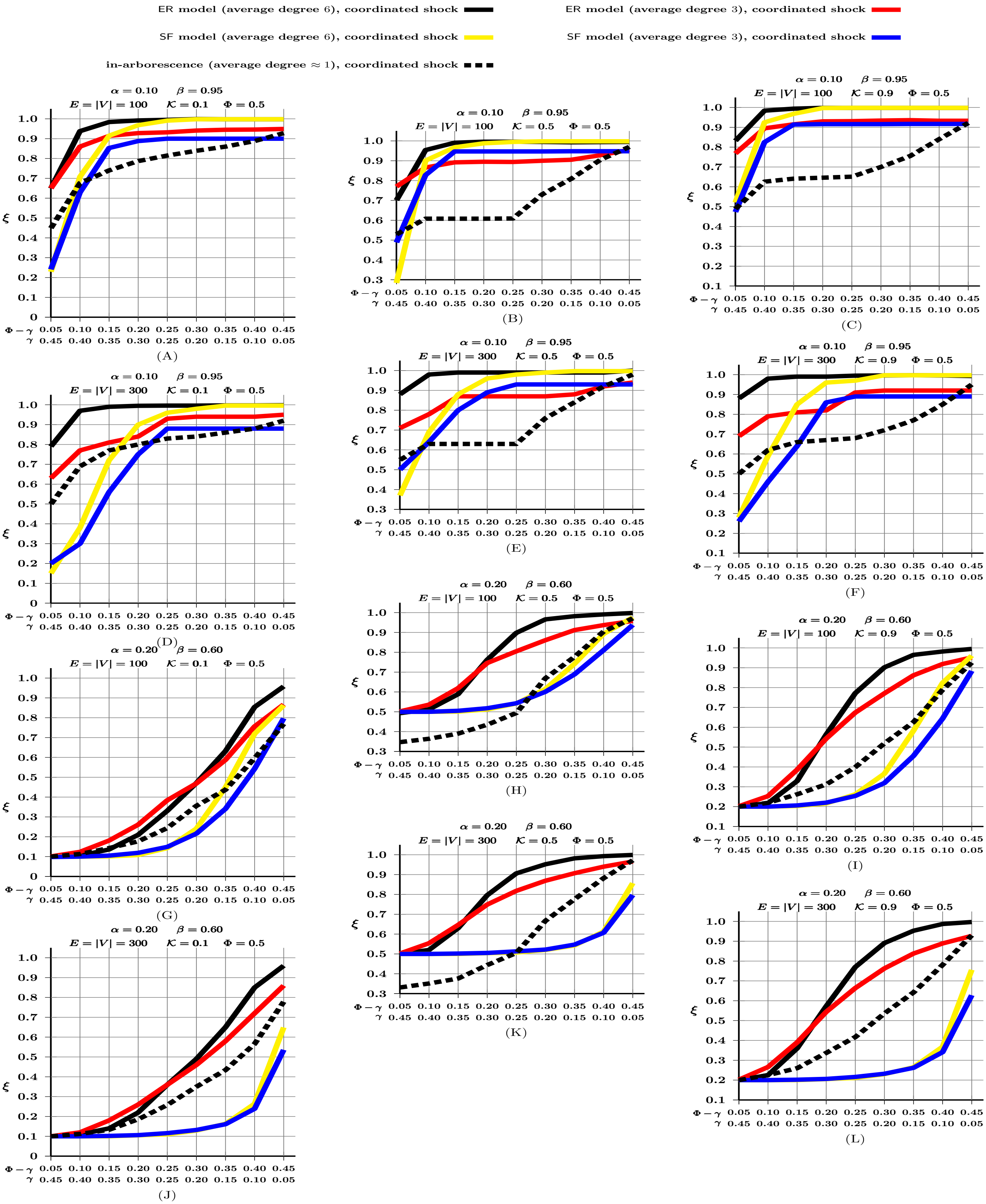


Figure S3: (drawn in color) Effect of variations of equity to asset ratio (with respect to shock) on the vulnerability index ξ for (α, β)-heterogeneous networks. Lower values of ξ imply higher global stability of a network.

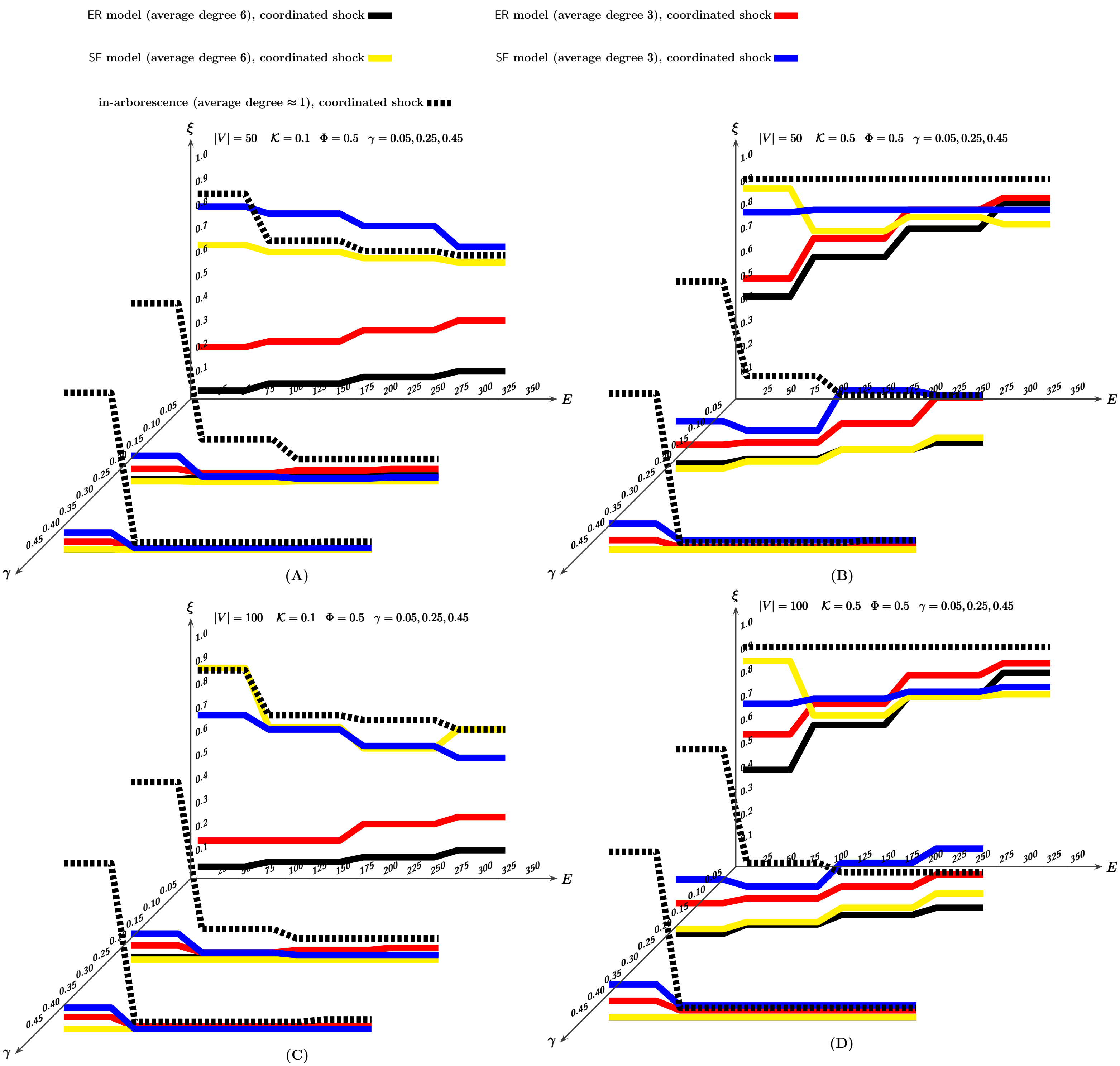


Figure S4: (drawn in color) Effect of variations of the total external to internal asset ratio E/I on the vulnerability index ξ for homogeneous networks. Lower values of ξ imply higher global stability of a network.

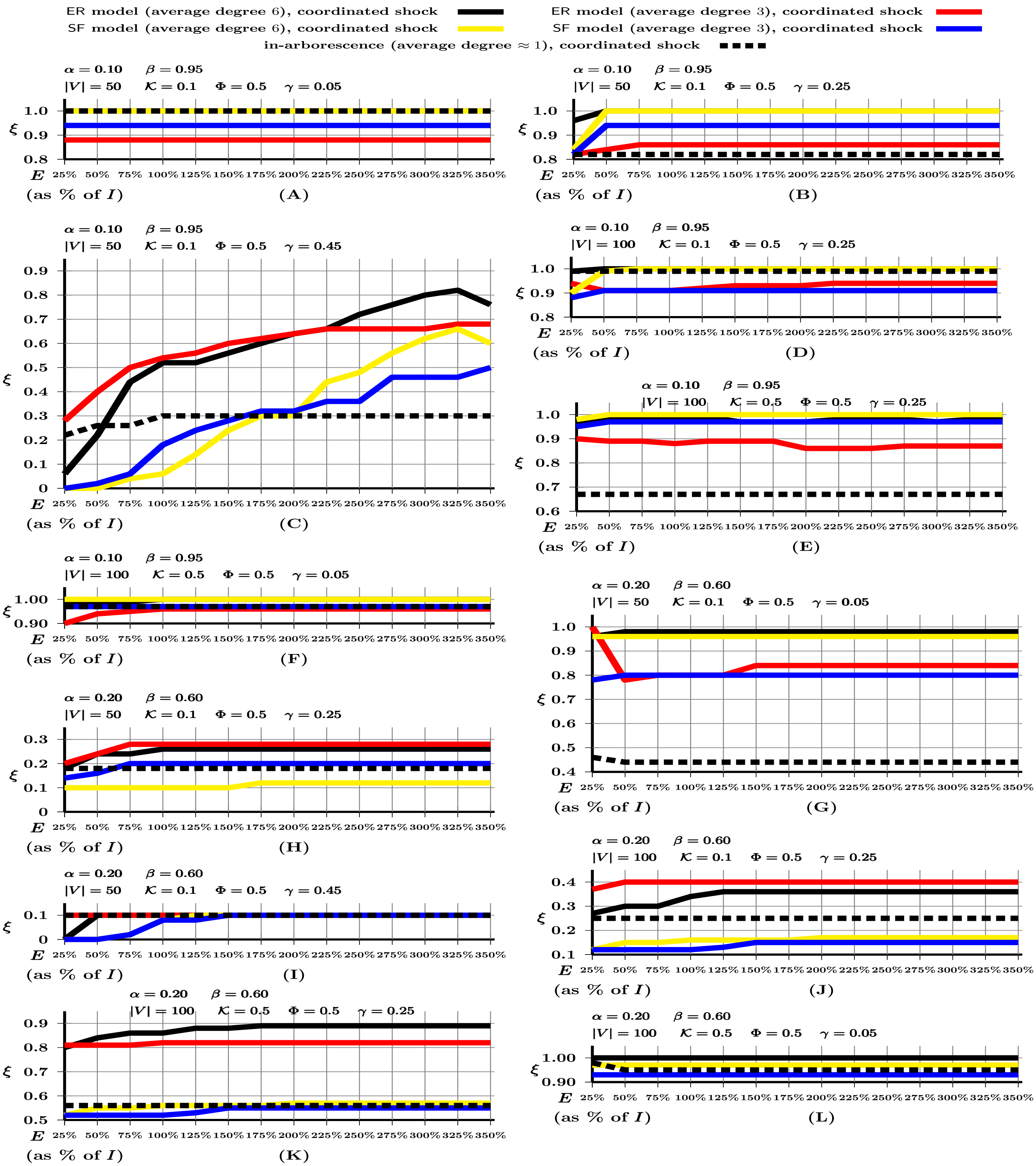


Figure S5: (drawn in color) Effect of variations of the total external to internal asset ratio E/I on the vulnerability index ξ for (α, β)-heterogeneous networks. Lower values of ξ imply higher global stability of a network.

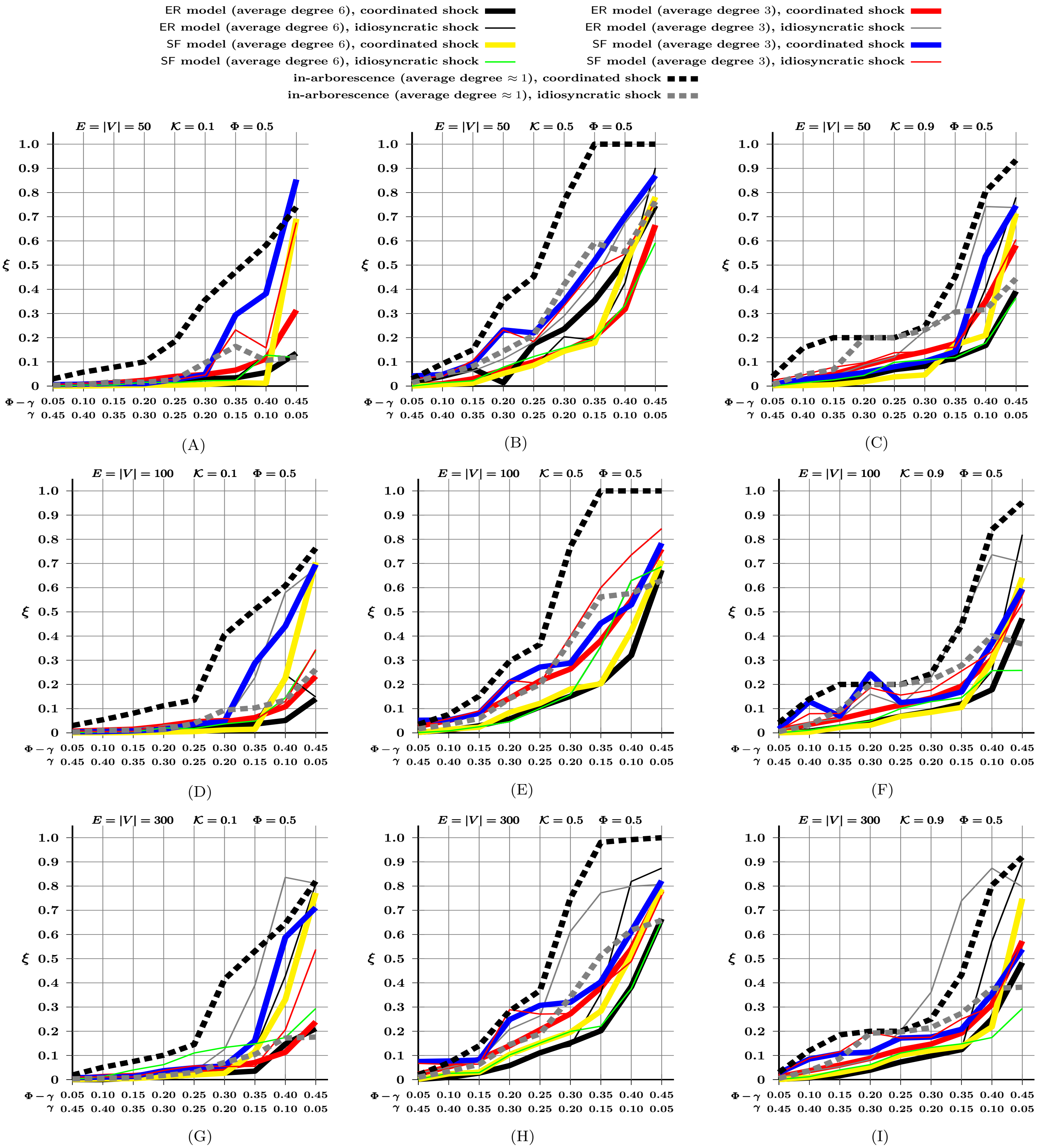


Figure S6: (drawn in color) Effect of variations of equity to asset ratio (with respect to shock) on the vulnerability index ξ for homogeneous networks. Lower values of ξ imply higher global stability of a network.

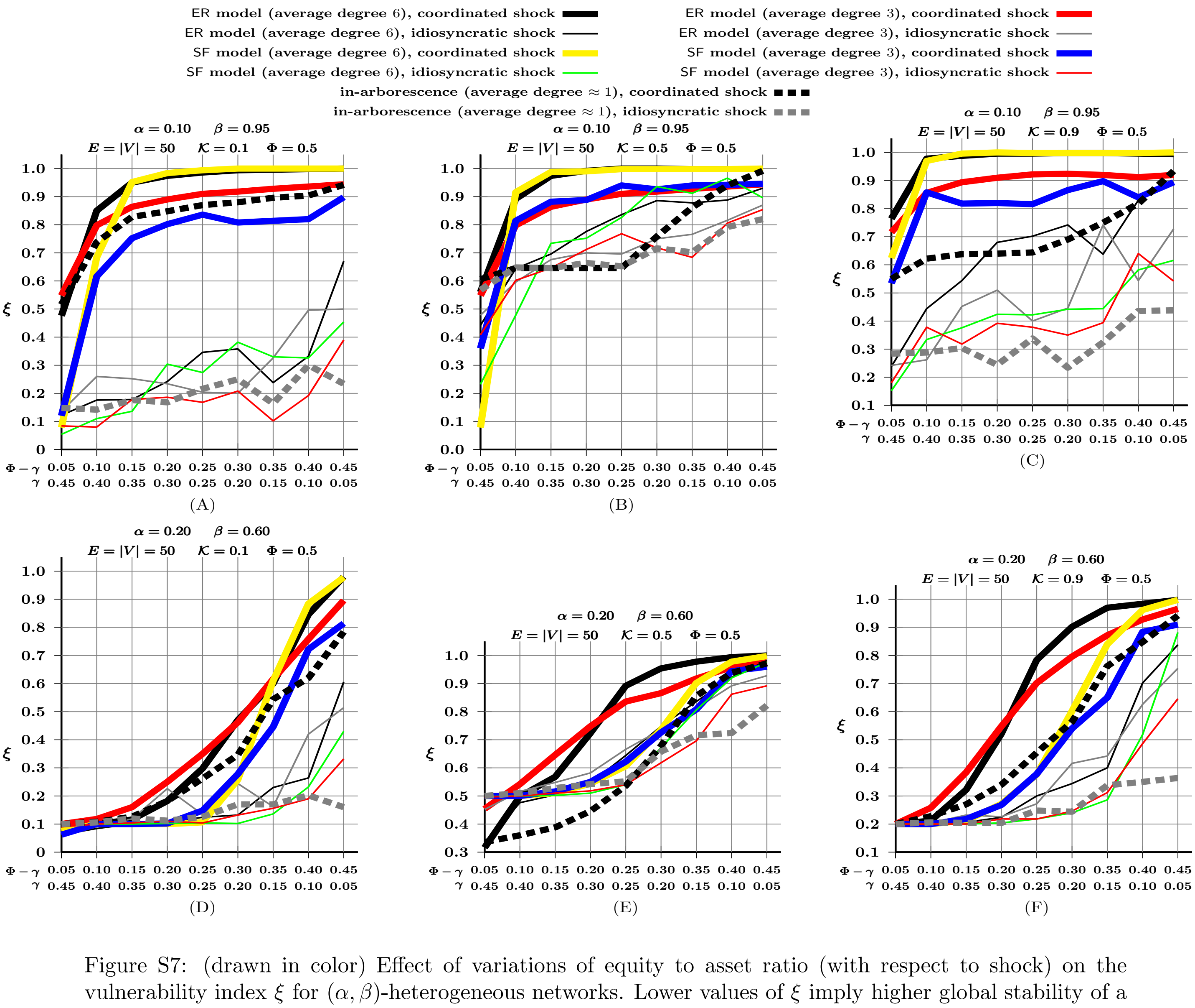


Figure S7: (drawn in color) Effect of variations of equity to asset ratio (with respect to shock) on the vulnerability index ξ for (α, β)-heterogeneous networks. Lower values of ξ imply higher global stability of a network.

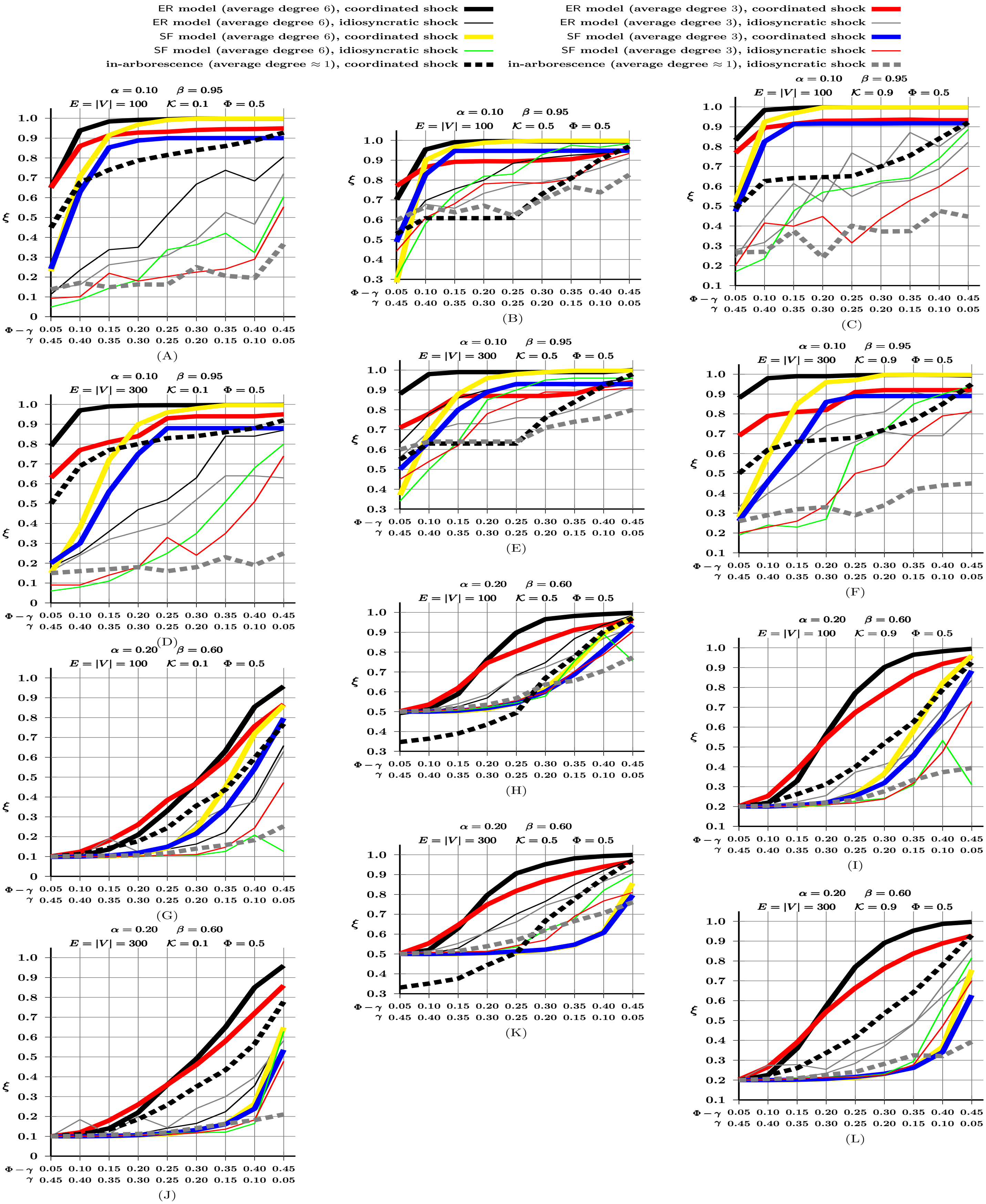


Figure S8: (drawn in color) Effect of variations of equity to asset ratio (with respect to shock) on the vulnerability index ξ for (α, β)-heterogeneous networks. Lower values of ξ imply higher global stability of a network.

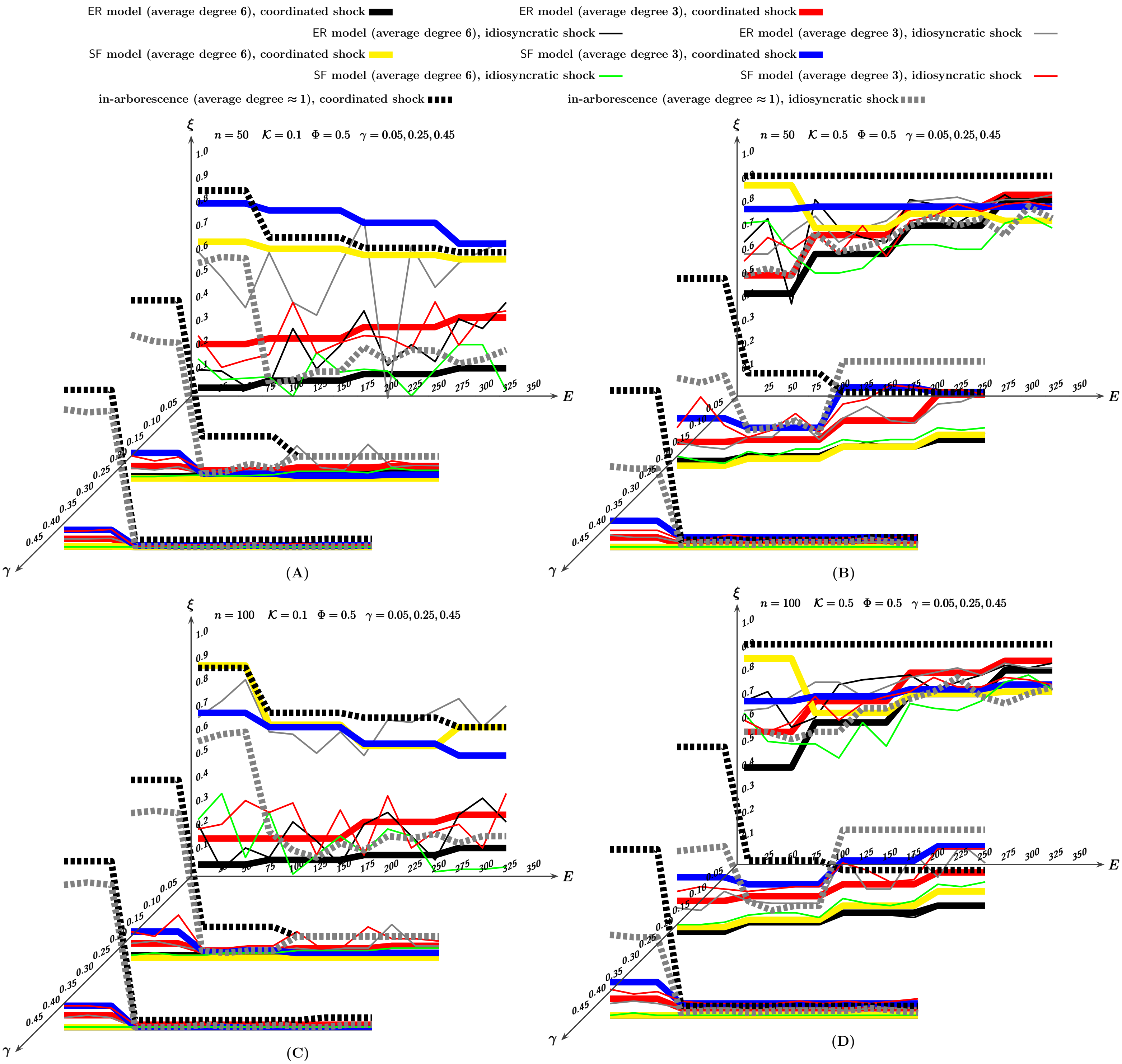


Figure S9: (drawn in color) Effect of variations of the total external to internal asset ratio $^{E}/_{I}$ on the vulnerability index ξ for homogeneous networks. Lower values of ξ imply higher global stability of a network.

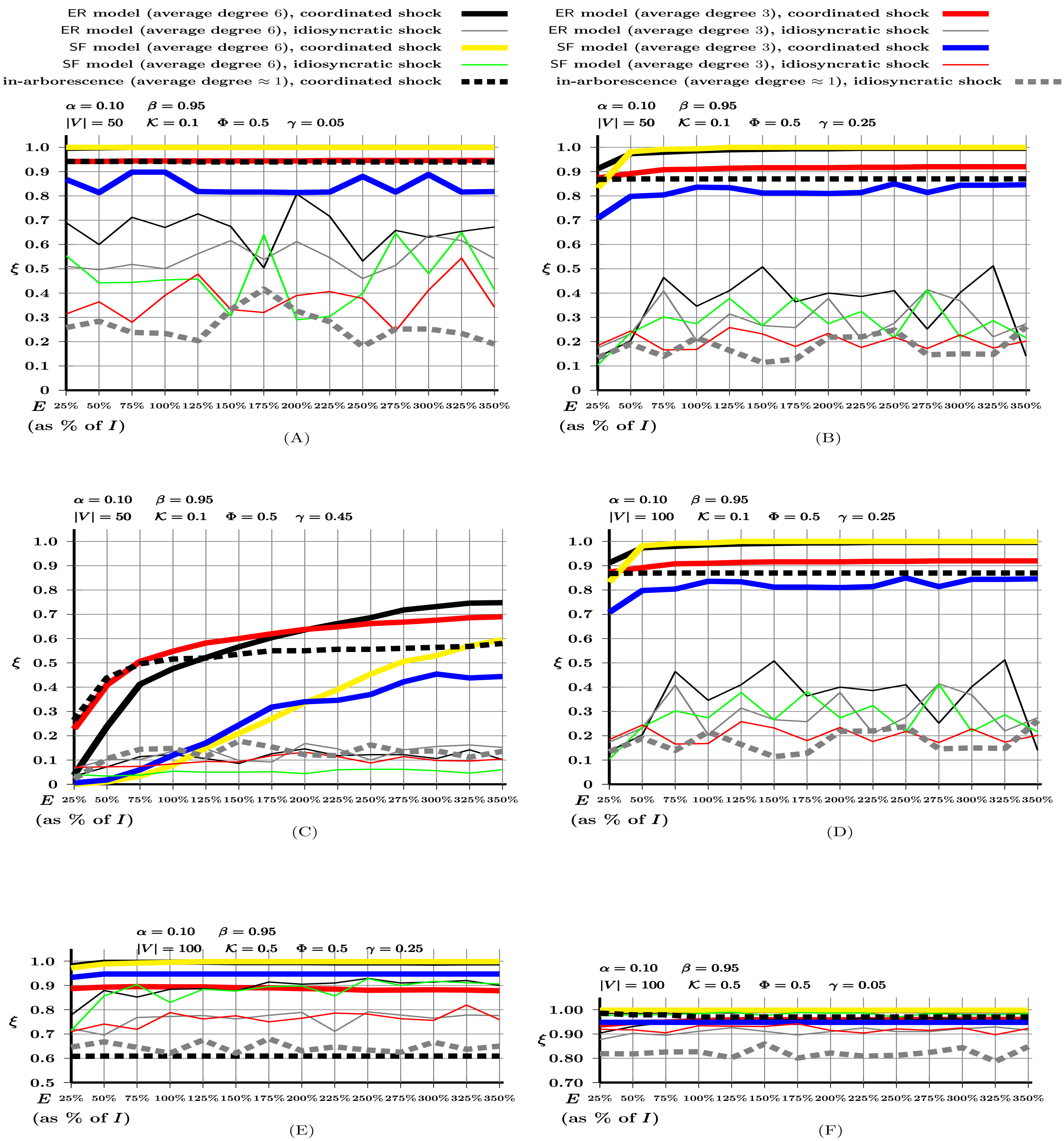


Figure S10: (drawn in color) Effect of variations of the total external to internal asset ratio E/I on the vulnerability index ξ for (α, β)-heterogeneous networks. Lower values of ξ imply higher global stability of a network.

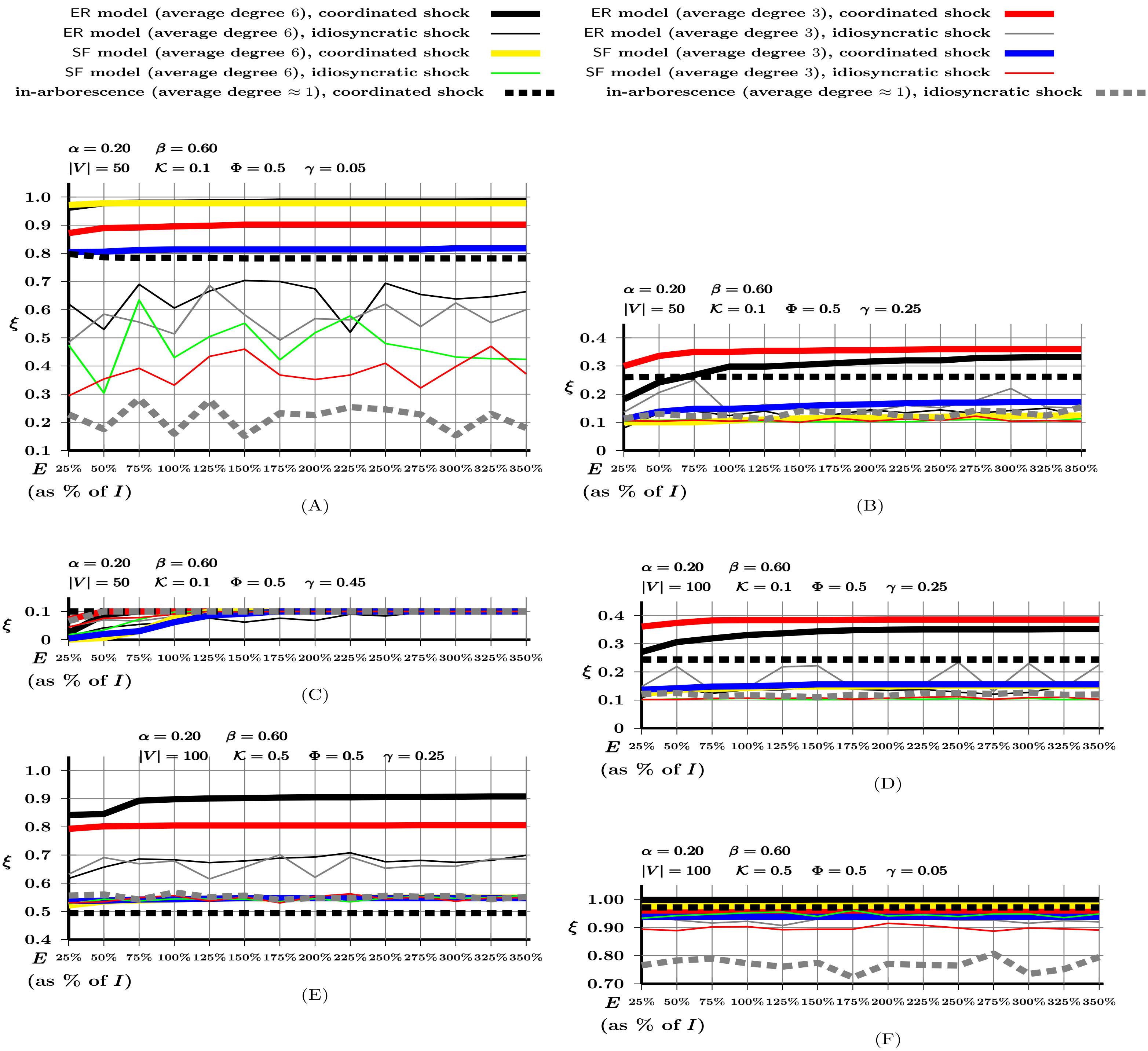


Figure S11: (drawn in color) Effect of variations of the total external to internal asset ratio E/I on the vulnerability index ξ for (α, β)-heterogeneous networks. Lower values of ξ imply higher global stability of a network.