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A Political Economy Theory of Fiscal Policy and Unemployment*

Abstract

This paper presents a political economy theory of fiscal policy and unemployment. The underlying economic model is one in which unemployment can arise but can be mitigated by tax cuts and public spending increases. Such policies are fiscally costly, but can be financed by issuing government debt. Policy decisions are made by a legislature consisting of representatives from different political districts. The theory provides a positive account of the simultaneous determination of fiscal policy and unemployment. The most important implication of the theory is that debt and unemployment levels should be positively correlated. Some evidence in support of this prediction is provided using data from a panel of developed countries.

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1 Introduction

An important role for fiscal policy is the mitigation of unemployment and stabilization of the economy.[1] Despite sceptism from some branches of the economics profession, politicians and policy-makers tend to be optimistic about the potential fiscal policy has in this regard. Around the world, countries facing downturns continue to pursue a variety of fiscal strategies, ranging from tax cuts to public works projects. Nonetheless, politicians' willingness to use fiscal policy to aggressively fight unemployment is tempered by high levels of debt. The main political barrier to deficit-financed tax cuts and public spending increases appears to be concern about the long-term burden of high debt.

This extensive practical experience with fiscal policy raises a number of basic positive public finance questions. In general, how do employment concerns impact the setting of taxes and public spending? When will government employ fiscal stimulus plans? What determines the size of these plans and how does this depend upon the economy's debt position? What will be the mix of tax cuts and public spending increases in stimulus plans? What will be the overall effectiveness of fiscal policy in terms of reducing unemployment?

This paper presents a political economy theory of the interaction between fiscal policy and unemployment that sheds light on these questions. The economic model underlying the theory is one in which unemployment can arise but can be mitigated by tax cuts and public spending increases. Such policies are fiscally costly, but can be financed by issuing debt. The political model assumes that policy decisions are made in each period by a legislature consisting of representatives from different political districts. Legislators can transfer revenues back to their districts which creates a political friction. The theory combines the economic and political models to provide a positive account of the simultaneous determination of fiscal policy and unemployment.

The political model underlying the theory follows the approach in our previous work (Battaglini and Coate 2007, 2008). The economic model is novel to this paper. It features a public and private sector.[2] The private sector consists of entrepreneurs who hire workers to produce a private good. The public sector hires workers to produce a public good. Public production is financed by a tax

[1] For an informative discussion of this role see Auerbach, Gale, and Harris (2010).

[2] In this sense, the model is similar to those used in that strand of the macroeconomics literature investigating the aggregate implications of changes in public sector employment, public production, etc. Examples include Ardagna (2007), Economides, Philippoulos, and Vassilatos (2013), Linnemann (2009), and Pappa (2009).

on the private sector. The government can also borrow and lend in the bond market. The private sector is affected by exogenous shocks (oil price hikes, for example) which impact entrepreneurs' demand for labor. Unemployment can arise because of a downwardly rigid wage. In the presence of unemployment, reducing taxes increases private sector hiring, while increasing public production creates public sector jobs. Thus, tax cuts and increases in public production reduce unemployment. However, both actions are costly for the government.

With the available policies, it is possible for the government to completely eliminate unemployment in the long run. However, with political decision making, unemployment will not be eliminated. Under appropriate assumptions, when the private sector experiences negative shocks, unemployment arises. When these shocks occur, government mitigates unemployment with stimulus plans that are financed by increases in debt. These equilibrium stimulus plans typically involve both tax cuts and public production increases. When choosing such plans, the government balances the benefits of reducing unemployment with the costs of distorting the private-public output mix. In normal times, when the private sector is not experiencing negative shocks, the government reduces debt until it reaches a floor level. Even in normal times, the private-public output mix is distorted and unemployment can arise, depending on the economic and political fundamentals. With or without negative shocks, when there is unemployment, it will be higher the larger the government's debt level.

The theory has two unambiguous qualitative implications. The first is that the dynamic pattern of debt is counter-cyclical. This implication also emerges from other theories of fiscal policy, so there is nothing particularly distinctive about it. Some empirical support for this prediction already exists (see, for example, Barro 1986). The second implication is that debt and unemployment levels are positively correlated. We are not aware of any other theoretical work that links debt and unemployment and so we believe this is a novel prediction. While some prior evidence in favor of this prediction exists, the empirical relationship between debt and unemployment has attracted surprisingly little attention.[3] We thus augment existing evidence with an analysis of recent panel data from a group of OECD countries. We also use this data to provide support for another idea suggested by the theory: namely, that the volatility of employment levels should be positively correlated with debt.

[3] Exceptions are Bertola (2011), Fedeli and Forte (2011) and Fedeli, Forte, and Ricchi (2012). We discuss this evidence in Section 4.

While there is a vast theoretical literature on fiscal policy, we are not aware of any work that systematically addresses the positive public finance questions that motivate this paper. Neoclassical theories of fiscal policy, such as the tax smoothing approach, assume frictionless labor markets and thus abstract from unemployment. Traditional Keynesian models incorporate unemployment and allow consideration of the multiplier effects of changes in government spending and taxes. However, these models are static and do not incorporate debt and the costs of debt financing.[4] This limitation also applies to the literature in optimal taxation which has explored how optimal policies are chosen in the presence of involuntary unemployment.[5] The modern new Keynesian literature with its sophisticated dynamic general equilibrium models with sticky prices typically treats fiscal policy as exogenous.[6] Papers in this tradition that do focus on fiscal policy, analyze how government spending shocks impact the economy and quantify the possible magnitude of multiplier effects.[7]

The novelty of our questions and model not withstanding, the basic forces driving the dynamics of debt in our theory are similar to those arising in our previous work on the political determination of fiscal policy in the tax smoothing model (Battaglini and Coate 2008 and Barshegyan, Battaglini, and Coate 2013). In the tax smoothing model the government must finance its spending with distortionary taxes but can use debt to smooth tax rates across periods. The need to smooth is created by shocks to government spending needs as a result of wars or disasters (as in Battaglini and Coate) or by cyclical variation in tax revenue yields due to the business cycle (as in Barshegyan, Battaglini, and Coate). With political determination, debt exhibits a counter-cyclical pattern, going up when the economy experiences negative shocks and back down when it experiences positive shocks. However, even after repeated positive shocks, debt never falls below a floor level. This reflects the fact that after a certain point legislators find it more desirable to transfer revenues back to their districts than to devote them to further debt decummulation. These basic lessons apply in our model of unemployment. This reflects the fact that debt plays a similar economic

[4] For a nice exposition of the traditional Keynesian approach to fiscal policy see Peacock and Shaw (1971). Blinder and Solow (1973) discuss some of the complications associated with debt finance and extend the IS-LM model to try to capture some of these.

[5] This literature includes papers by Bovenberg and van der Ploeg (1996), Dreze (1985), Marchand, Pestieau, and Wibaut (1989), and Roberts (1982).

[6] See, for example, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003).

[7] See, for example, Christiano, Eichenbaum, and Rebelo (2009), Hall (2009), Mertens and Ravn (2010), and Woodford (2010).

role, allowing the government to smooth the distortions arising from a downwardly rigid wage across periods.

Addressing the questions we are interested in requires a simple and tractable dynamic model. In creating such a model, we have made a number of strong assumptions. First, we employ a model without money and therefore abstract from monetary policy. This means that we cannot consider the important issue of whether the government would prefer to use monetary policy to achieve its policy objectives. Second, we obtain unemployment by simply assuming a downwardly rigid wage, as opposed to a more sophisticated micro-founded story.[8] This means that our analysis abstracts from any possible effects of fiscal policy on the underlying friction generating unemployment. Third, the source of cyclical fluctuations in our economy comes from the supply rather than the demand side. In our model, recessions arise because negative shocks to the private sector reduce the demand for labor. Labor market frictions prevent the wage from adjusting and the result is unemployment. This vision differs from the traditional and new Keynesian perspectives that emphasize the importance of shocks to consumer demand.[9] Finally, our model ignores any impact of fiscal policy on capital accumulation.

While these strong assumptions undoubtedly represent limitations of our analysis, we nonetheless feel that our model provides an interesting framework in which to study the interaction between fiscal policy and unemployment. First, the model incorporates the two broad ways in which government can create jobs: *indirectly* by reducing taxes on the private sector, or *directly* through increasing public production. Second, the model allows consideration of two conceptually different types of activist fiscal policy: *balanced-budget policies* wherein tax cuts are financed by public spending decreases or visa versa, and *deficit-financed policies* wherein tax cuts and/or spending increases are financed by increases in public debt. Third, the mechanism by which taxes influence private sector employment in the model is consonant with arguments that are commonplace in the policy arena. For example, the main argument behind objections to eliminating the Bush tax cuts for those making $250,000 and above, was that it would lead small businesses to reduce their hiring during a time of high unemployment. Fourth, the mechanism by which high debt levels are

[8] There is a literature incorporating theories of unemployment into dynamic general equilibrium models (see Gali (1996) for a general discussion). Modelling options include matching and search frictions (Andolfatto 1996), union wage setting (Ardagna 2007), and efficiency wages (Burnside, Eichenbaum, and Fisher 1999).

[9] In the new Keynesian literature demand shocks are created by stochastic discount rates (see, for example, Christiano, Eichenbaum, and Rebelo (2009)).

costly for the economy also captures arguments that are commonly made by politicians and policy-makers. Higher debt levels imply larger service costs which require either greater taxes on the private sector and/or lower public spending. These policies, in turn, have negative consequences for jobs and the economy.

The organization of the remainder of the paper is as follows. Section 2 outlines the model. Section 3 describes equilibrium fiscal policy and unemployment. Section 4 develops and explores the empirical implications of the theory, and Section 5 concludes.

2 Model

The environment We consider an infinite horizon economy in which there are two final goods; a private good x and a public good g. There are two types of citizens; entrepreneurs and workers. Entrepreneurs produce the private good by combining labor l with their own effort. Workers are endowed with 1 unit of labor each period which they supply inelastically. The public good is produced by the government using labor. The economy is divided into m political districts, each a microcosm of the economy as a whole.

There are n_e entrepreneurs and n_w workers where $n_e + n_w = 1$. Each entrepreneur produces with the Leontief production technology $x = A_\theta \min\{l, \epsilon\}$ where ϵ represents the entrepreneur's effort and A_θ is a productivity parameter. The idea underlying this production technology is that when an entrepreneur hires more workers he must put in more effort to manage them. The productivity parameter varies over time, taking on one of two values A_L (low) and A_H (high) where A_L is less than A_H. The probability of high productivity is α. The public good production technology is $g = l$.

A worker who consumes x units of the private good obtains a per period payoff $x + \gamma \ln g$ when the public good level is g. Here, the parameter γ measures the relative value of the public good. Entrepreneurs' per period payoff function is $x + \gamma \ln g - \xi \epsilon^2/2$ where the third term represents the disutility of providing entrepreneurial effort. All individuals discount the future at rate β.

There are markets for the private good and labor. The private good is the numeraire. The wage is denoted ω and the labor market operates under the constraint that the wage cannot go below an exogenous minimum $\underline{\omega}$.[10] This friction is the source of unemployment. There is also

[10] We make this assumption to get a simple and tractable model of unemployment. While $\underline{\omega}$ could be literally interpreted as a statutory minimum wage, what we are really trying to capture are the sort of rigidities identified in

a market for risk-free one period bonds. The assumption that citizens have quasi-linear utility implies that the equilibrium interest rate on these bonds is $\rho = 1/\beta - 1$.

To finance its activities, the government taxes entrepreneurs' incomes at rate τ. It can also borrow and lend in the bond market. Government debt is denoted by b and new borrowing by b'. The government is also able to distribute surplus revenues to citizens via district-specific lump sum transfers. Let s_i denote the transfer going to the residents of district $i \in \{1, ..., m\}$.

Market equilibrium At the beginning of each period, the productivity state is revealed. The government repays existing debt and chooses the tax rate, public good, new borrowing, and transfers. It does this taking into account how its policies impact the market and the need to balance its budget.

To understand how policies impact the market, assume the state is θ, the tax rate is τ, and the public good level is g. Given a wage rate ω, each entrepreneur chooses hiring, the input, and effort, to maximize his utility

$$\max_{(l,\epsilon)}(1-\tau)(A_\theta \min\{l, \epsilon\} - \omega l) - \xi \frac{\epsilon^2}{2}. \tag{1}$$

Obviously, the solution involves $\epsilon = l$. Substituting this into the objective function and maximizing with respect to l reveals that $l = (1-\tau)(A_\theta - \omega)/\xi$. Aggregate labor demand from the private sector is therefore $n_e(1-\tau)(A_\theta - \omega)/\xi$. Labor demand from the public sector is g and labor supply is n_w.[11] Setting demand equal to supply, the market clearing wage is

$$\omega = A_\theta - \xi(\frac{n_w - g}{n_e(1-\tau)}). \tag{2}$$

The minimum wage will bind if this wage is less than $\underline{\omega}$. In this case, the equilibrium wage is $\underline{\omega}$ and the unemployment rate is

$$u = \frac{n_w - g - n_e(1-\tau)(A_\theta - \underline{\omega})/\xi}{n_w}. \tag{3}$$

the survey work of Bewley (1999). The assumption of some type of wage rigidity is common in the macroeconomics literature (see, for example, Blanchard and Gali (2007), Hall (2005), and Michaillat (2012)) and a large empirical literature investigates the extent of wage rigidity in practice (see, for example, Barwell and Schweitzer (2007), Dickens et al (2007), and Holden and Wulfsberg (2009)).

[11] The model assumes that the government pays the same wage as do entrepreneurs and therefore makes no distinction between public and private sector wages. It therefore abstracts from the reality that public and private sector wages are not determined in the same way. For macroeconomic analysis focusing on this distinction see, for example, Fernandez-de-Cordoba, Perez, and Torres (2012).

To sum up, in state θ with government policies τ and g, the equilibrium wage rate is

$$\omega_\theta = \begin{cases} \underline{\omega} \quad \text{if } A_\theta \leq \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}) \\ A_\theta - \xi(\frac{n_w - g}{n_e(1-\tau)}) \quad \text{if } A_\theta > \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}) \end{cases} \tag{4}$$

and the unemployment rate is

$$u_\theta = \begin{cases} \frac{n_w - g - n_e(1-\tau)(A_\theta - \underline{\omega})/\xi}{n_w} \quad \text{if } A_\theta \leq \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}) \\ 0 \quad \text{if } A_\theta > \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}). \end{cases} \tag{5}$$

When the minimum wage is binding, the unemployment rate is increasing in τ. Higher taxes cause entrepreneurs to put in less effort and this reduces private sector demand for workers. The unemployment rate is also decreasing in g because to produce more public goods, the government must hire more workers. When the minimum wage is not binding, the equilibrium wage is decreasing in τ and increasing in g.

Each entrepreneur earns profits of $\pi_\theta = (1-\tau)(A_\theta - \omega_\theta)^2/\xi$. Assuming he receives no government transfers and consumes his profits, an entrepreneur obtains a period payoff of

$$v_{e\theta} = \frac{(A_\theta - \omega_\theta)^2(1-\tau)^2}{2\xi} + \gamma \ln g. \tag{6}$$

Jobs are randomly allocated among workers and so each worker obtains an expected period payoff

$$v_{w\theta} = (1 - u_\theta)\omega_\theta + \gamma \ln g. \tag{7}$$

Again, this assumes that the worker receives no transfers and simply consumes his earnings.

Aggregate output of the private good is $x_\theta = n_e A_\theta (1-\tau)(A_\theta - \omega_\theta)/\xi$. Substituting in the expression for the equilibrium wage, we see that

$$x_\theta = \begin{cases} n_e A_\theta (1-\tau)(A_\theta - \underline{\omega})/\xi \quad \text{if } A_\theta \leq \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}) \\ A_\theta\,(n_w - g) \quad \text{if } A_\theta > \underline{\omega} + \xi(\frac{n_w - g}{n_e(1-\tau)}). \end{cases} \tag{8}$$

Observe that the tax rate has no impact on private sector output when the minimum wage constraint is not binding. This is because labor is inelastically supplied and as a consequence the wage adjusts to ensure full employment. A higher tax rate just leads to an offsetting reduction in the wage rate. However, when there is unemployment, tax hikes reduce private sector output because they lead entrepreneurs to reduce effort. Public good production has no effect on private output when there is unemployment, but reduces it when there is full employment.

The government budget constraint Having understood how markets respond to government policies, we can now formalize the government's budget constraint. Tax revenue is

$$R_\theta(\tau, \omega_\theta) = \tau(n_e \pi_\theta) = \tau n_e (1-\tau)(A_\theta - \omega_\theta)^2/\xi. \tag{9}$$

Total government revenue includes tax revenue and new borrowing and therefore equals $R_\theta(\tau, \omega_\theta)+ b'$. The cost of public good provision and debt repayment is $\omega_\theta g + b(1+\rho)$. The budget surplus available for transfers is therefore

$$B_\theta(\tau, g, b', b, \omega_\theta) = R_\theta(\tau, \omega_\theta) + b' - (\omega_\theta g + b(1+\rho)) \, . \tag{10}$$

The government budget constraint is that this budget surplus be sufficient to fund any transfers made, which requires that

$$B_\theta(\tau, g, b', b, \omega_\theta) \geq \sum_{i=1}^{m} s_i. \tag{11}$$

There is also an upper limit $\overline{b}$ on the amount of debt the government can issue. This limit is motivated by the unwillingness of borrowers to hold bonds that they know will not be repaid. If, in steady state, the government were borrowing an amount b such that the interest payments exceeded the maximum possible tax revenues in the low productivity state; i.e., ρb exceeded $\max_\tau R_L(\tau, \underline{\omega})$, then, if productivity were low, it would be unable to repay the debt *even if it provided no public goods or transfers.* Borrowers would therefore be unwilling to lend more than $\max_\tau R_L(\tau, \underline{\omega})/\rho$. For technical reasons, it is convenient to assume that the upper limit $\overline{b}$ is equal to $\max_\tau R_L(\tau, \underline{\omega})/\rho - \varepsilon$, where $\varepsilon > 0$ can be arbitrarily small.

Political decision-making Government policy decisions are made by a legislature consisting of m representatives, one from each district. Each representative wishes to maximize the aggregate utility of the citizens in his district. In addition to choosing taxes, public goods, and borrowing, the legislature must also divide any budget surplus between the districts. The affirmative votes of m/q representatives are required to enact any legislation, where $q > 1$. Lower values of q mean that more legislators are required to approve legislation and thus represent more inclusive decision-making.

The legislature meets at the beginning of the period after the productivity state θ is known. The decision-making process follows a simple sequential protocol. At stage $j = 1, 2, ...$ of this process, a representative is randomly selected to make a proposal to the floor. A proposal consists of policies (τ, g, b') and district-specific transfers $(s_i)_{i=1}^m$ satisfying the constraints that transfer spending $\sum_i s_i$ does not exceed the budget surplus $B_\theta(\tau, g, b', b, \omega_\theta)$ and new borrowing b' does not exceed the debt limit $\overline{b}$. If the proposal receives the votes of m/q representatives, then it is implemented and the legislature adjourns until the following period. If the proposal does not pass, then the process moves to stage $j+1$, and a representative is selected again to make a new proposal.[12]

3 Equilibrium fiscal policy and unemployment

Following the analysis in Battaglini and Coate (2008), it can be shown that in productivity state θ with initial debt level b, the equilibrium levels of taxation, public good spending, and new borrowing $\{\tau_\theta(b), g_\theta(b), b'_\theta(b)\}$ solve the maximization problem:

$$\max_{(\tau,g,b')} \left\{ \begin{array}{c} qB_\theta(\tau, g, b', b, \omega_\theta) + n_e v_{e\theta} + n_w v_{w\theta} + \beta E V_{\theta'}(b') \\ s.t.\ B_\theta(\tau, g, b', b, \omega_\theta) \geq 0 \ \&\ b' \leq \overline{b} \end{array} \right\}, \tag{12}$$

where $V_{\theta'}(b')$ is equilibrium aggregate lifetime citizen expected utility in state θ' with debt level b'. The equilibrium level of spending on transfers is equal to the budget surplus associated with the policies $\{\tau_\theta(b), g_\theta(b), b'_\theta(b)\}$, which is $B_\theta(\tau_\theta(b), g_\theta(b), b'_\theta(b), b, \omega_\theta)$. The equilibrium value functions $V_H(b)$ and $V_L(b)$ in problem (12) are defined recursively by the equations:

$$V_\theta(b) = B_\theta(\tau_\theta(b), g_\theta(b), b'_\theta(b), b, \omega_\theta) + n_e v_{e\theta} + n_w v_{w\theta} + \beta E V_{\theta'}(b'_\theta(b)) \tag{13}$$

for $\theta \in \{L, H\}$. Representatives' value functions, which reflect only aggregate utility in their respective districts, are equal to $V_H(b)/m$ and $V_L(b)/m$.[13]

A convenient short-hand way of understanding the equilibrium is to imagine that in each period a minimum winning coalition (mwc) of m/q representatives is randomly chosen and that this coalition collectively chooses policies to maximize *its* aggregate utility (as opposed to society's).

[12] This process may either continue indefinitely until a proposal is chosen, or may last for a finite number of stages as in Battaglini and Coate (2008): the analysis is basically the same. In Battaglini and Coate (2008) it is assumed that in the last stage, one representative is randomly picked to choose a policy; this representative is then required to choose a policy that divides the budget surplus evenly between districts.

[13] A political equilibrium amounts to a set of policy functions that solve (12) given the equilibrium value functions, and value functions that satisfy (13) given the equilibrium policies. A political equilibrium is *well-behaved* if the associated value functions $V_L(b)$ and $V_H(b)$ are concave in b. Following the approach in Battaglini and Coate (2008), it can be shown that a well-behaved political equilibrium exists. The analysis will focus on well-behaved equilibria and we will refer to them simply as equilibria. More explanation of this characterization of equilibrium and a discussion of the existence of an equilibrium can be found in our working paper, Battaglini and Coate (2011).

Problem (12) reflects the coalition's maximization problem. Recall that $v_{e\theta}$ and $v_{w\theta}$ denote, respectively, entrepreneur and worker per period payoffs net of transfers. Thus, if q were equal to 1 so that legislation required unanimous approval, the objective function in (12) would exactly equal aggregate societal utility. In this case (12) would correspond to the planner's problem for this economy. Since q exceeds 1, problem (12) differs from a planning problem in that extra weight is put on the surplus available for transfers. This extra weight reflects the fact that transfers are shared only among coalition members. Because membership in the mwc is random, all representatives are ex ante identical and have a common value function given by (13) (divided by $1/m$). In what follows, we will use this way of understanding the equilibrium and speak *as if* a randomly drawn mwc is choosing policy in each period.

The equilibrium policies are characterized by solving problem (12). It will prove instructive to break down the analysis of this problem into two parts. First, we study the associated static problem. Thus, we fix new borrowing b' and assume that the mwc faces an exogenous revenue requirement equal to $b' - (1+\rho)b$. Then, we endogenize the revenue requirement by studying the choice of debt.

3.1 The static problem

The static problem for the mwc is to choose a tax rate τ and a level of public good g to maximize its collective utility given that revenues net of public production costs must cover a revenue requirement r and that net revenues in excess of r finance transfers to the districts of coalition members. Using the definition of the budget surplus function in (10) and the assumption that $r = b' - (1+\rho)b$, the mwc's static problem can be posed as:

$$\max_{(\tau,g)} \left\{ \begin{array}{c} q\left(R_\theta(\tau,\omega_\theta) - \omega_\theta g - r\right) + n_e v_{e\theta} + n_w v_{w\theta} \\ s.t.\ R_\theta(\tau,\omega_\theta) - \omega_\theta g \geq r \end{array} \right\}. \tag{14}$$

Since the difference between new borrowing and debt repayment (i.e., $b' - (1+\rho)b$) could in principle be positive or negative, the revenue requirement r can be positive or negative.

The first point to note about the problem is that the mwc will always set taxes sufficiently high so that the equilibrium wage equals $\underline{\omega}$. As noted earlier, taxes are non-distortionary when the wage exceeds $\underline{\omega}$ and the mwc has the ability to target transfers to its members. Thus, if the wage exceeded $\underline{\omega}$, there would be an increase in the mwc's collective utility if it raised taxes and

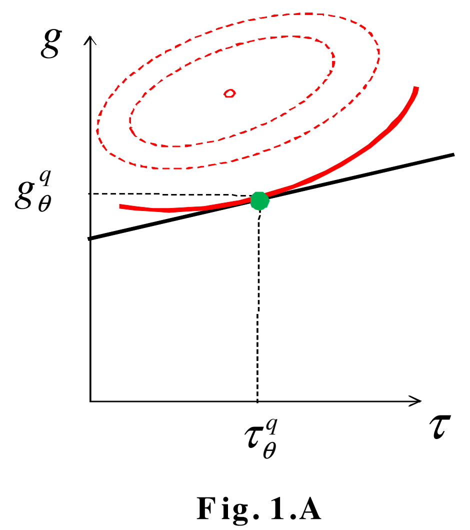

Figure 1:

used the additional tax revenues to fund transfers. Combining this observation with equations (6), (7), and (8), allows us to write problem (14) as:

$$\max_{(\tau,g)} \left\{ \begin{array}{c} x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g + (q-1)\left(R_\theta(\tau,\underline{\omega}) - \underline{\omega} g\right) - qr \\ s.t.\ R_\theta(\tau,\underline{\omega}) - \underline{\omega} g \geq r \ \&\ g + \frac{x_\theta(\tau)}{A_\theta} \leq n_w \end{array} \right\}, \tag{15}$$

where $x_\theta(\tau)$ is the output of the private good when the tax rate is τ and the wage rate is $\underline{\omega}$ (see the top line of (8)).

Problem (15) has a simple interpretation. The objective function is the mwc's collective surplus.[14] The first inequality is the *budget constraint*: it requires that the mwc have sufficient net revenues to meet the revenue requirement under the assumption that the wage is $\underline{\omega}$. The second inequality is the *resource constraint*: it requires that the demand for labor at wage $\underline{\omega}$ is less than or equal to the number of workers n_w. This constraint ensures that the equilibrium wage is indeed $\underline{\omega}$.

A diagrammatic approach will be helpful in explaining the solution to problem (15). Without loss of generality, we assume that r is less than or equal to the maximum possible tax revenue

[14] The expression for the surplus generated by $x_\theta(\tau)$ (the first two terms) reflects the fact that the surplus associated with the private good consists of the consumption benefits it generates less the costs associated with the entrepreneurial effort necessary to produce it.

which is $R_\theta(1/2, \underline{\omega})$.[15] We also assume that unemployment would result if the government faced the maximal revenue requirement.[16] To understand our diagrammatic approach, consider first Fig. 1.A and 1.B, where we ignore the budget constraint. The tax rate is measured on the horizontal axis and the public good on the vertical. In both figures, the upward sloping line is the frontier of the resource constraint. Using the expression for $x_\theta(\tau)$ from (8), this line is described by

$$g = n_w - n_e(1-\tau)(A_\theta - \underline{\omega})/\xi. \tag{16}$$

At points along this line, there is full employment at the wage $\underline{\omega}$ and we therefore refer to it as the *full-employment line.* The resource constraint implies that policies must be on or below this line and points below are associated with unemployment. The other curves in the figures represents the mwc's *indifference curves.* Each curve satisfies for some target utility level U, the equation

$$x_\theta(\tau) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g + (q-1)\left(R_\theta(\tau, \underline{\omega}) - \underline{\omega} g\right) = U. \tag{17}$$

As illustrated, the mwc's preferences exhibit an interior satiation point in (τ, g) space. Two cases are possible. The first, represented in Figure 1.A, is where the satiation point is outside the resource constraint. In this case the optimal policies for the mwc ignoring the budget constraint (hereafter referred to as the *unconstrained optimal policies* and denoted $(\tau_\theta^q, g_\theta^q)$) lie at the point of tangency between the indifference curve and the full employment line. The second case, represented in Figure 1.B, is where the satiation point is inside the full employment line. In this case, the unconstrained optimal policies $(\tau_\theta^q, g_\theta^q)$ are just the satiation point. As we show in the Appendix, the mwc's satiation point lies inside the full employment line if and only if q is greater than q_θ^*, where q_θ^* is defined by:

$$n_e\left[\frac{q(A_\theta - \underline{\omega}) + \underline{\omega}}{\xi(2q-1)}\right] + \frac{\gamma}{(q-1)\,\underline{\omega}} = n_w. \tag{18}$$

Intuitively, higher values of q increase the mwc's preference for surplus revenues. Surplus revenues are increased by higher taxes and lower public good spending. When q exceeds q_θ^*, the mwc's

[15] The revenue maximizing tax rate is 1/2 and the maximum revenue requirement is $n_e A_\theta(A_\theta - \underline{w})/4\xi$. Of course, if r were higher than this level, the problem would have no solution. In the dynamic model, however, this case will never arise.

[16] If the government faces the maximal revenue requirement it will set the tax rate equal to 1/2 and provide no public good. Private sector employment will be $n_e(A_\theta - \underline{\omega})/2\xi$ and there will be no public sector employment. Thus, this assumption amounts to the requirement that n_w exceeds $n_e(A_\theta - \underline{\omega})/2\xi$.

preferred tax rate is sufficiently high and its preferred public good level sufficiently low, that unemployment arises.[17]

To complete the description of problem (15), we need to add to this diagrammatic representation the government's budget constraint. The frontier of the budget constraint associated with revenue requirement r is given by

$$g = \frac{R_\theta(\tau, \underline{\omega})}{\underline{\omega}} - \frac{r}{\underline{\omega}}. \tag{19}$$

We refer to this as the *budget line.* The budget constraint requires that policies must be on or below this line and points below are associated with positive transfers. Each budget line is hump shaped, with peak at $\tau = 1/2$. Increasing the revenue requirement shifts down the budget line but does not change the slope. Figure 2 illustrates the budget line associated with three different revenue requirements. The feasible set of (τ, g) pairs for the mwc's problem are those that lie below both the budget and full employment lines. This set is represented by the gray areas in Figure 2. Observe that the feasible set is (weakly) convex which makes the problem well-behaved. As the revenue requirement is raised, the set of policies for which full employment results shrinks (compare Fig. 2.A and Fig. 2.B). For sufficiently high revenue requirements it is not possible to achieve full employment (as in Fig. 2.C).

With this diagrammatic apparatus in place, we can now explain the mwc's optimal policies. We start with the case illustrated in Fig. 1.A in which the satiation point is outside the resource constraint (i.e., q is less than q_θ^*). Define r_θ^q to be the revenue requirement equal to $R_\theta(\tau_\theta^q, \underline{\omega}) - \underline{\omega} g_\theta^q$. When r is below r_θ^q the budget constraint is not binding and the mwc will choose the unconstrained optimal policies $(\tau_\theta^q, g_\theta^q)$ and use the surplus revenues $r_\theta^q - r$ to finance transfers to their districts. This situation is illustrated in Fig. 3.A.

When r is higher than r_θ^q, the mwc's unconstrained optimal tax rate and public good level are too expensive. To meet its revenue requirement, the mwc must reduce public good provision and/or raise taxes. In making this decision, the mwc balances the costs of two types of distortions: unemployment and having a sub-optimal mix of public and private outputs. Starting from the position of full employment and its preferred output mix, the costs of distorting the output mix are second order, while the costs of unemployment are first order. As a result, for revenue requirements only slightly higher than r_θ^q, the mwc will preserve full employment by appropriate adjustment of

[17] Indeed, as q increases to infinity, the satiation point converges to $(1/2, 0)$, the policies that maximize the revenue available for transfers. By assumption, at this point, there is unemployment.

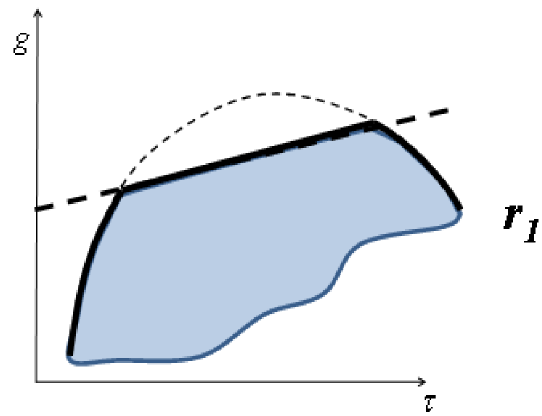

Fig. 2.A

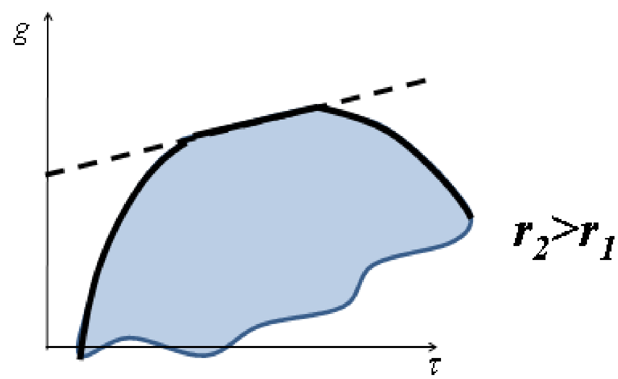

Fig. 2.B

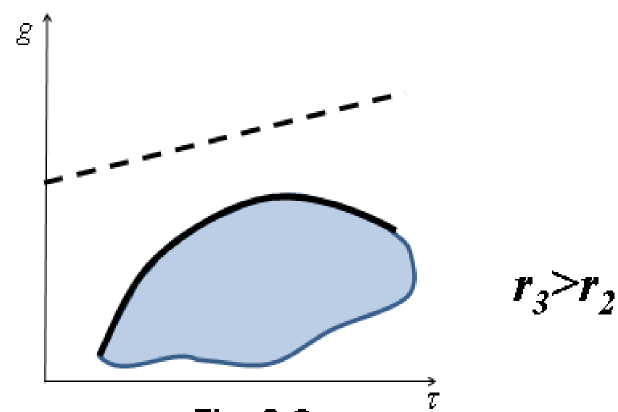

Fig. 2.C

Figure 2:

the output mix. The situation is illustrated in Fig. 3.B and Fig. 3.C.

The nature of this adjustment depends on the location of the unconstrained optimal policies $(\tau_\theta^q, g_\theta^q)$. The mwc can achieve full employment by choosing any tax rate in the range $[\tau_\theta^-(r), \tau_\theta^+(r)]$ with associated level of public good given by (16), where $\tau_\theta^-(r)$ and $\tau_\theta^+(r)$ are defined by the left and right intersections of the budget and full employment lines. When τ_θ^q lies to the left of the interval $[\tau_\theta^-(r), \tau_\theta^+(r)]$, as in Fig. 3.B, the optimal policies are the tax rate $\tau_\theta^-(r)$ with associated public good level $g_\theta^-(r)$. This policy combination involves the least distortion in the output mix consistent with achieving full employment. When τ_θ^q lies to the right of the interval $[\tau_\theta^-(r), \tau_\theta^+(r)]$, as in Fig. 3.C, the optimal policies are the tax rate $\tau_\theta^+(r)$ with associated public good level $g_\theta^+(r)$. Notice that in the former case, the mwc distorts the output mix *towards* public production and in the latter it distorts *away* from public production. These cases also generate different comparative static implications. In the former case, as the revenue requirement increases, taxes and public production increase; in the latter, taxes and public production decrease. Intuitively, in the former case, the mwc generates a fiscal surplus by raising taxes on the private sector and hiring the displaced workers in the public sector. In the latter, it generates a fiscal surplus by laying off public sector workers and using tax cuts to incentivize the private sector to hire them.

For revenue requirements significantly higher than r_θ^q, maintaining full employment requires a larger distortion in the output mix. Indeed, as already noted, for sufficiently high revenue requirements maintaining full employment is not possible. Eventually, therefore, increased fiscal pressure must result in unemployment. In the Appendix, we prove that for any q less than q_θ^* there is a cut point r_θ^* strictly higher than r_θ^q at which the mwc abandons the effort to maintain full employment. For revenue requirements higher than r_θ^*, the resource constraint is not binding and the optimal policies are at the tangency of the budget line and the indifference curve. These policies are denoted by $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$ and are illustrated in Fig. 3.D. As the revenue requirement climbs above r_θ^*, the tax rate increases and the public production level decreases, so $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$ moves to the South-East. Unemployment also increases.

When the mwc's satiation point lies inside the full employment line as illustrated in Fig. 1.B (i.e., q exceeds q_θ^*), the analysis is analogous. When r is sufficiently small, the unconstrained optimal policies $(\tau_\theta^q, g_\theta^q)$ coincide with the mwc's satiation point, as illustrated in Fig. 4.A. As in the previous case, we can define a threshold r_θ^q such that this arises if and only if r is less than or equal to r_θ^q. This threshold can be shown to be smaller than r_θ^*. When r is higher than r_θ^q the

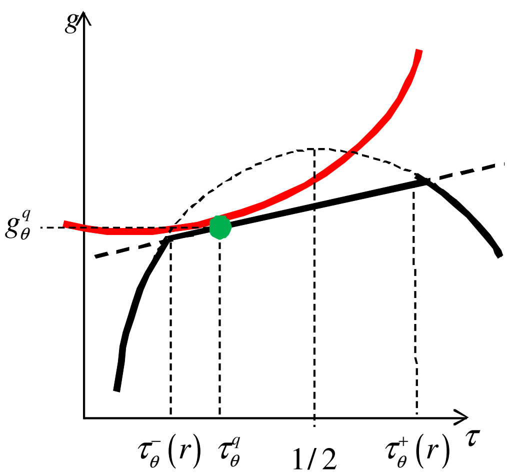

Fig. 3.A

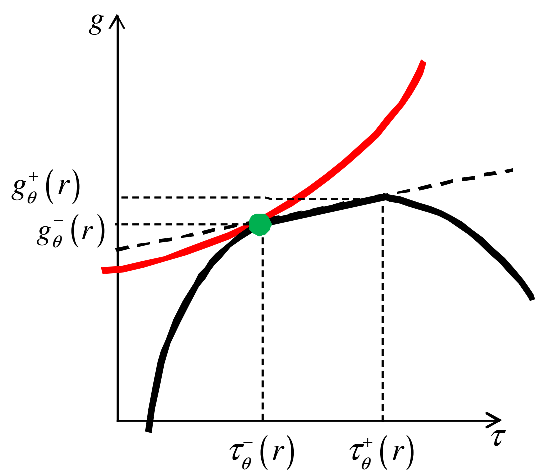

Fig. 3.B

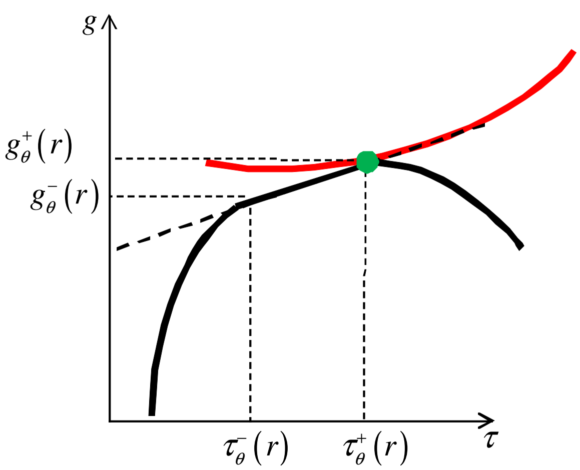

Fig. 3.C

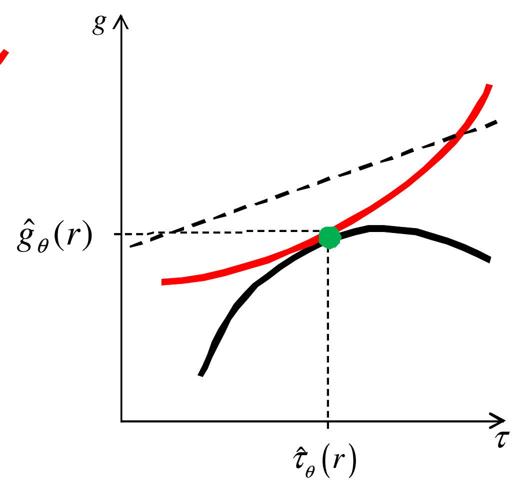

Fig. 3.D

Figure 3:

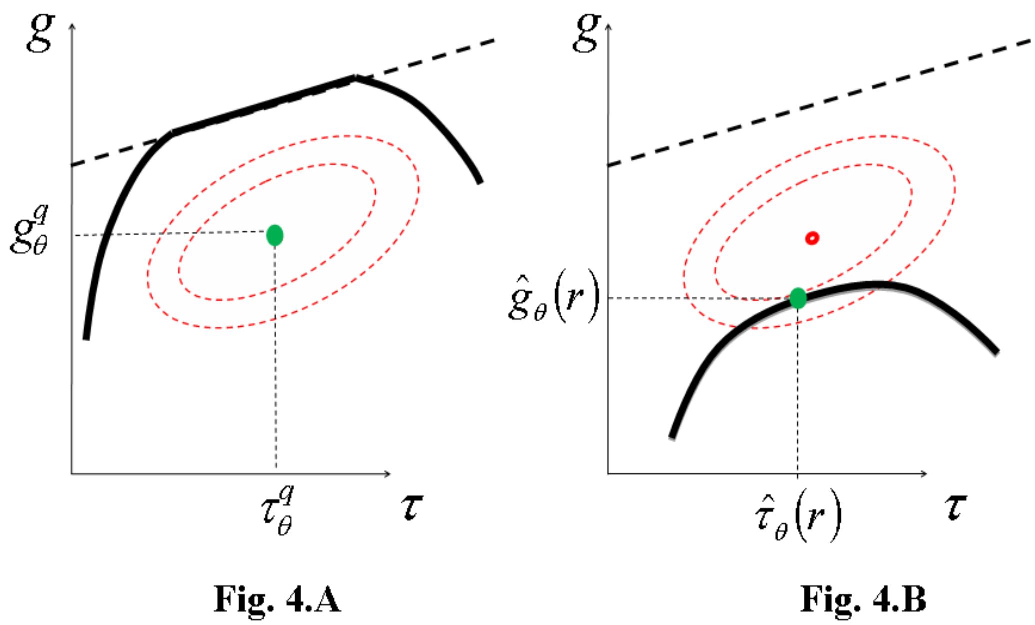

Figure 4:

optimal policy is $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$; i.e. the point of tangency between the indifference curve and the budget line, as illustrated in Fig. 4.B.

Summarizing this discussion, we have the following description of the solution to the static problem.

Proposition 1 *There exists a revenue requirement r_θ^* such that the solution to problem (14) has the following properties.*

- *If $r \leq r_\theta^q$, the optimal policies are $(\tau_\theta^q, g_\theta^q)$. When $q < q_\theta^*$, these policies involve full employment. When $q > q_\theta^*$, these policies involve unemployment.*

- *If $r \in (r_\theta^q, \max\{r_\theta^q, r_\theta^*\}]$, the optimal policies involve full employment. If $\tau_\theta^q < \tau_\theta^-(r)$, the optimal policies are $(\tau_\theta^-(r), g_\theta^-(r))$, and, if $\tau_\theta^q > \tau_\theta^+(r)$, the optimal policies are $(\tau_\theta^+(r), g_\theta^+(r))$. This case only arises when $q < q_\theta^*$, since $r_\theta^q > r_\theta^*$ when $q > q_\theta^*$.*

- *If $r > \max\{r_\theta^q, r_\theta^*\}$, the optimal policies are $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$ and involve unemployment. In this range, as r increases, unemployment increases.*

3.2 The choice of debt

We now bring debt back into the picture. Intuitively, debt should be helpful since it allows the mwc to transfer revenues from good times in which high productivity creates robust private sector profits and labor demand, to bad times in which low productivity results in a depressed private sector. In bad times, the revenues transferred will reduce fiscal pressure and permit policy changes which reduce unemployment and distortions in the output mix. The benefits from these changes will exceed the costs associated with raising revenue in good times because the distortions created by tax increases and public good reductions are lower in good times. Indeed, as noted earlier, taxation is non-distortionary when the minimum wage constraint is not binding.

Define $r_\theta(b) = (1+\rho)b - b'_\theta(b)$ to be the revenue requirement implied by the equilibrium policies in state θ with initial debt level b. Letting $(\tau^e_\theta(r), g^e_\theta(r))$ denote the static equilibrium policies described in Proposition 1, it is clear that $(\tau_\theta(b), g_\theta(b))$ will equal $(\tau^e_\theta(r_\theta(b)), g^e_\theta(r_\theta(b)))$. The issue is thus to identify the range of revenue requirements that can arise in equilibrium and this requires understanding the behavior of debt.

To focus the analysis on the natural case of interest, we assume that with no debt a planner that wanted to maximize aggregate societal utility would be able to finance its unconstrained optimal policies without borrowing in the high but not the low state. More precisely, we make:

Assumption 1

$$R_H(\tau^1_H, \underline{\omega}) - \underline{\omega} g^1_H > 0 > R_L(\tau^1_L, \underline{\omega}) - \underline{\omega} g^1_L.$$

Here $\left(\tau^1_\theta, g^1_\theta\right)$ are the unconstrained optimal policies associated with problem (15) when $q = 1$. As discussed above, when q equals 1, the equilibrium policies maximize aggregate utility. Recalling the definition of r^q_θ, the critical revenue requirement delineating the first and second cases of Proposition 1, Assumption 1 implies that r^1_L is negative and r^1_H is positive.[18]

Given the equilibrium policy functions, for any initial debt level b, we can define $H(b, b')$ to be the probability that next period's debt level will be less than b'. Given a distribution $\psi_{t-1}(b)$ of debt at time $t-1$, the distribution at time t is $\psi_t(b') = \int_b H(b, b')d\psi_{t-1}(b)$. A distribution $\psi^*(b')$ is said to be an *invariant distribution* if $\psi^*(b') = \int_b H(b, b')d\psi^*(b)$. If it exists, the invariant

[18] In terms of the fundamental parameters of the model this assumption can be shown to be equivalent to:

$$n_e A_L - \xi n_w < n_e\left(\underline{\omega} + \frac{\gamma}{n_w}\right) - \frac{\xi\gamma}{\underline{\omega} + \frac{\gamma}{n_w}} < n_e A_H - \xi n_w.$$

distribution describes the steady state of the government's debt distribution. We now have:

Proposition 2 *There exists a debt level $b^q \in (r_L^q/\rho, \overline{b})$ such that the equilibrium debt distribution converges to a unique, non-degenerate, invariant distribution with full support on $[b^q, \overline{b}]$. The dynamic pattern of debt is counter-cyclical: the government expands debt when private sector productivity is low and contracts debt when productivity is high until it reaches the floor level b^q.*

The logic underlying the counter-cyclical behavior of debt is straightforward. As just explained, in this economy, debt allows government to transfer revenues from good times to bad times which permits the smoothing of distortions. However, in good times, the mwc will not reduce debt below the floor level b^q. Intuitively, once the debt level has reached b^q, the mwc prefers to divert surplus revenues to transfers rather than to paying down more debt. As noted in the introduction, this finding is analogous to the results of Battaglini and Coate (2008) and Barshegyan, Battaglini, and Coate (2013) for the tax smoothing model. The debt level b^q depends on the fundamentals of the economy and can be characterized following the approach in Battaglini and Coate (2008), but these details are not central to our mission here.[19] For now, we will simply assume that the underlying parameters are such that the floor level is positive, which seems the empirically relevant case.

Higher debt levels translate into higher revenue requirements for the government. Thus, Proposition 2 implies that the range of revenue requirements arising in equilibrium in state θ are $[r_\theta(b^q), r_\theta(\overline{b})]$. By comparing these ranges with the thresholds in Proposition 1, we obtain the following result.

Proposition 3 *The following is true in the long run.*

- *If $q > q_L^*$, there is always unemployment in both states. Unemployment is weakly increasing in the economy's debt level, strictly so in the low productivity state and in the high productivity state for sufficiently high debt levels.*

- *If $q \in (q_H^*, q_L^*)$, there is always unemployment in the low productivity state. In the high productivity state, there is full employment for low debt levels and unemployment for high debt levels. Unemployment is weakly increasing in the economy's debt level, strictly so in the low productivity state and in the high productivity state for sufficiently high debt levels.*

[19] The formal characterization of the debt level b^q is provided in the proof of Proposition 2.

- *If $q < q_H^*$, in the low productivity state, there is full employment for low debt levels and unemployment for high debt levels. In the high productivity state, there is full employment for low debt levels and either full employment or unemployment for high debt levels. Unemployment is weakly increasing in the economy's debt level, strictly so in the low productivity state for sufficiently high debt levels.*

To illustrate the workings of the model, consider the case in which q is between q_H^* and q_L^*. In this case, there is always unemployment in low productivity states but full employment in high states for suitably low debt levels. The government mitigates unemployment in low productivity states by issuing debt. If the economy's debt level is low enough, then a return to the high state will restore full employment. If the economy is in the low productivity state for a sufficiently long period of time, however, debt will get sufficiently high that unemployment will persist even when the high state returns. When the high state returns, the legislators reduce debt. If the high state persists, debt will fall below the level at which full employment is achieved. Debt will eventually fall to the floor level b^q, at which point the mwc will divert surplus revenues to transfers rather than debt reduction.

3.3 Equilibrium stimulus plans

Proposition 2 tells us that in the steady state of the political equilibrium, when private sector productivity is low, the government expands debt and the funds are used to mitigate unemployment.[20] The government therefore employs fiscal stimulus plans, as conventionally defined. By studying the size of these stimulus plans and the changes in policy they finance, we obtain a positive theory of fiscal stimulus. More specifically, in the low productivity state, we can interpret $\rho b - r_L(b)$ as the magnitude of the stimulus, since this measures the amount of additional resources obtained by the government to finance fiscal policy changes (i.e., the debt increase $b_L'(b) - b$). An understanding of how the stimulus funds are used can be obtained by comparing the equilibrium tax and public good policies with the policies that would be optimal if the debt level were held constant.

The use of stimulus funds The simplest case to consider is when the stimulus package does not completely eliminate unemployment. From Proposition 3, this must be the case when q exceeds

[20] Even when q is less than q_H^* and the economy's debt is low, Assumption 1 implies that there will be unemployment prior to government stimulus if the floor debt level b^q is positive.

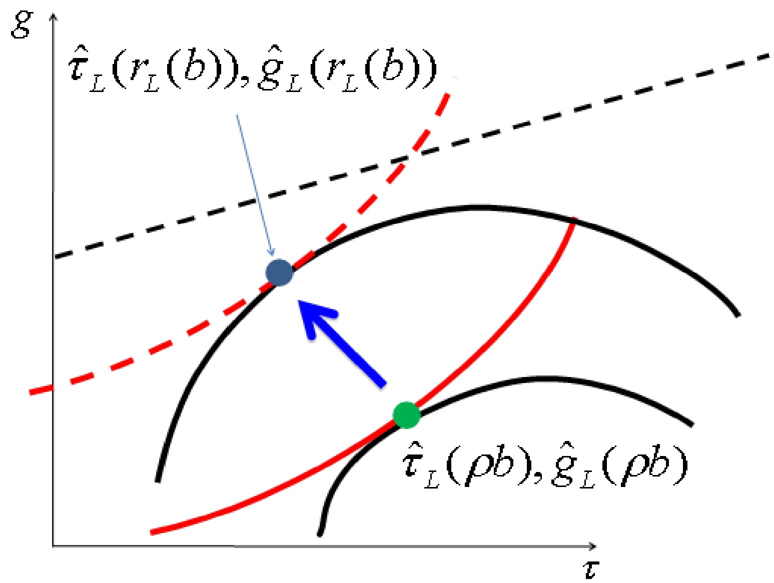

Figure 5:

q_L^*. When q is less than q_L^*, unemployment will remain post-stimulus whenever the economy's debt level is sufficiently high (i.e., $r_L(b) > r_L^*$). Drawing on the analysis in Section 3.1, Fig. 5 illustrates what happens in this case. From Proposition 1, the policies that would be chosen if the debt level were held constant are $(\widehat{\tau}_L(\rho b), \widehat{g}_L(\rho b))$. The reduction in the revenue requirement made possible by the stimulus funds, shifts the budget line up and permits a new policy choice $(\widehat{\tau}_L(r_L(b)), \widehat{g}_L(r_L(b)))$. As discussed in Section 3.1, in the unemployment range, the tax rate is increasing in the revenue requirement and public production is decreasing. Thus, we know that $\widehat{\tau}_L(r_L(b))$ is less than $\widehat{\tau}_L(\rho b)$ and that $\widehat{g}_L(r_L(b))$ exceeds $\widehat{g}_L(\rho b)$, implying that stimulus funds will be used for *both* tax cuts and increases in public production.[21]

Effectiveness and multipliers In terms of the effectiveness of equilibrium stimulus plans, the equilibrium policies will not typically minimize unemployment. The unemployment minimizing

[21] It should be stressed that the purpose of the tax cuts is to incentivize the private sector to hire more workers. This is logically distinct from the idea that tax cuts return purchasing power to citizens and stimulate demand, thereby creating jobs. Both types of arguments for tax cuts arise in the policy debate and it is important to keep them distinct. Similarly, the purpose of the increase in government spending is to hire more public sector workers, not to increase transfers to citizens. Notice that while the model allows the government to use stimulus funds to increase transfers, it chooses not to do so. Such transfers would have no aggregate stimulative effect because they must be paid for by future taxation. Taylor (2011) argues that the 2009 American Recovery and Reinvestment Act largely consisted of increases in transfers. Moreover, he argues that these transfer increases had little impact on household consumption since they were saved.

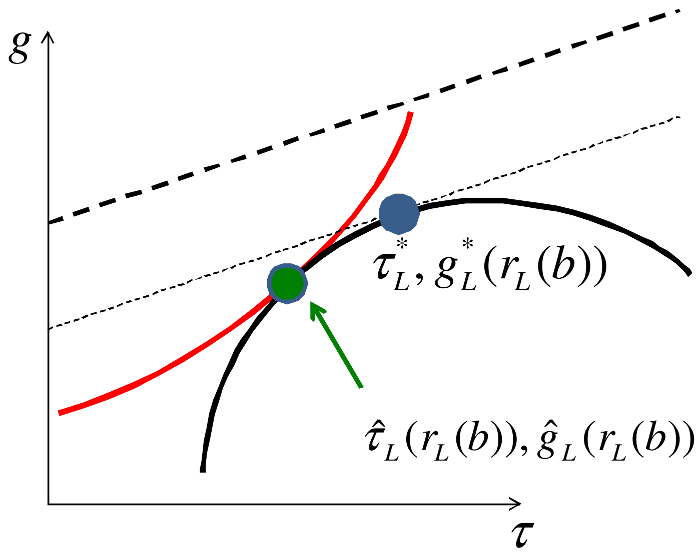

Figure 6:

policies when the revenue requirement is r involve the tax rate τ_L^* at which the slope of the budget line is equal to the slope of the full employment line with associated public good level $g_L^*(r)$ given by (19) (see Fig. 6).[22] If $\widehat{\tau}_L(r_L(b))$ is less than τ_L^* (as in Fig. 6), then reducing the tax cut slightly and using the revenues to finance a slightly larger public production increase will produce a bigger reduction in unemployment. Conversely, if $\widehat{\tau}_L(r_L(b))$ exceeds τ_L^* then reducing the public production increase and using the revenues to finance a slightly larger tax cut will produce a bigger reduction in unemployment. Both situations are possible depending on the parameters and the economy's debt level.[23] In the former case, legislators hold back from increasing taxes because, even though more jobs are created, the lost private output is more valuable than the additional public output. In the latter case, legislators hold back from reducing public production for the opposite reason.

[22] In the Appendix, we show that $\tau_\theta^* = (A_\theta - 2\underline{\omega})/2(A_\theta - \underline{\omega})$. This discussion assumes that $g_\theta^*(r) = \frac{R_\theta(\tau_\theta^*, \underline{w})}{\underline{w}} - \frac{r}{\underline{w}}$ is non-negative. If this is not the case, the unemployment minimizing tax rate is such that $R_\theta(\tau, \underline{w}) = r$ and the associated public good level is 0.

[23] If condition (20) of Section 3.4 is not satisfied, the equilibrium tax rate is greater than τ_θ^*. If condition (20) is satisfied, matters depend on the revenue requirement. For sufficiently high revenue requirements, the equilibrium tax rate in the low productivity state must again be greater than τ_L^*. This is because as r_L approaches $r_L(\overline{b})$, the equilibrium tax rate approaches the revenue maximizing level 1/2, which exceeds τ_L^*. However, in either state for sufficiently low revenue requirements, the equilibrium tax rate can be less than τ_θ^*.

One way of thinking about these results is in terms of multipliers. It is commonplace in the empirical literature to try to evaluate the multipliers associated with different stimulus measures.[24] The multiplier associated with a particular stimulus measure is defined to be the change in GDP divided by the budgetary cost of the measure. In our model, measuring GDP is more problematic than in the typical macroeconomic model because output is produced by both the private and public sectors, and there is no obvious way to value public sector output. Perhaps the simplest approach is to define GDP as equalling private sector output plus the cost of public production. With this definition, when there is unemployment, the public production multiplier is 1 and the tax cut multiplier is approximately $A_L/\left(1-2\widehat{\tau}_L(r_L(b))\right)(A_L-\underline{\omega})$.[25] The tax cut multiplier will exceed the public production multiplier if $\widehat{\tau}_L(r_L(b))$ exceeds τ_L^* and be less than the public production multiplier if $\widehat{\tau}_L(r_L(b))$ is less than τ_L^*. The analysis illustrates why we should not expect the government to choose policies in such a way as to equate multipliers across instruments. Tax cuts and public production increases have different implications for the mix of public and private outputs. A further important point to note is that the tax multiplier is highly non-linear.[26] Tax cuts will be more effective the larger is the tax rate and the tax rate will be higher the larger the economy's initial debt level.

When unemployment is eliminated When the stimulus package eliminates unemployment, as would be the case when q exceeds q_L^* and the economy's debt level is low $(r_L(b) < r_L^*)$, matters are more complicated. This is because of the non-monotonic behavior of policies in the second case identified in Proposition 1. In particular, we will not necessarily get both tax cuts and an increase in public production. Fig. 7.A illustrates a case in which the stimulus package involves not only using all the stimulus funds to fund tax cuts but also reducing public production to supplement the stimulus funds. Fig. 7.B illustrates a case in which the stimulus package involves increases in both taxes and public production. The model is therefore consistent with a variety of possible

[24] Papers trying to measure the multiplier impacts of different policies include Alesina and Ardagna (2010), Barro and Redlick (2011), Blanchard and Perotti (2002), Mountford and Uhlig (2009), Nakamura and Steinsson (2011), Ramey (2011a), Romer and Romer (2010), Serrato and Wingender (2011), and Shoag (2010). A central issue in this literature is the relative size of tax cut and public spending multipliers. For overviews and discussion of the literature see Auerbach, Gale, and Harris (2010), Parker (2011), and Ramey (2011b).

[25] This definition of GDP becomes more problematic when there is full employment and the minimum wage constraint is not binding. This is because τ and g impact the wage and hence the costs of public production. Perhaps the key point to note concerning multipliers when the minimum wage constraint is not binding, is that the level of employment is independent of τ and g.

[26] The importance of non-linearities and the difficulties this creates for measurement is a theme of Parker (2011).

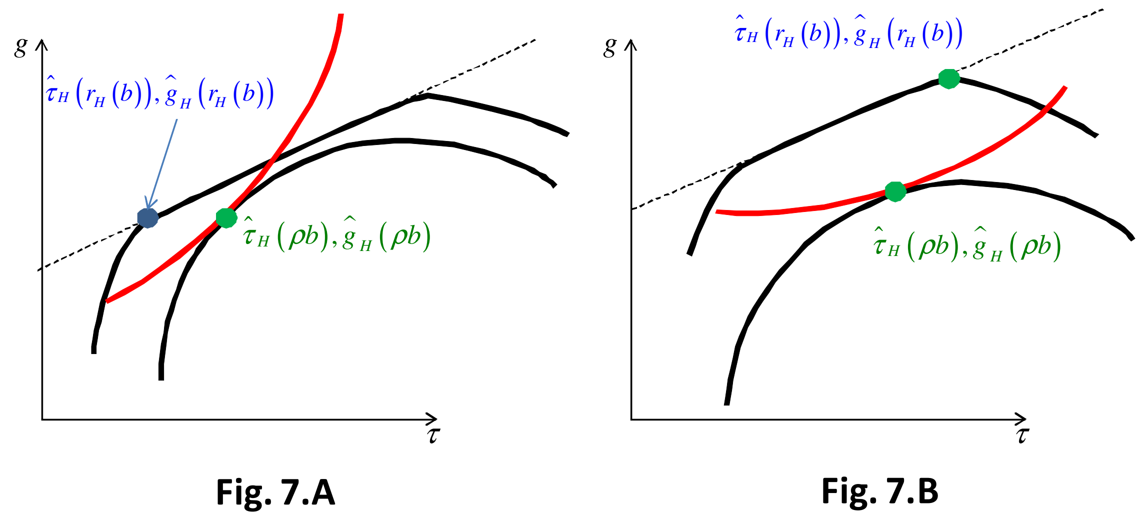

Figure 7:

stimulus plans.

The magnitude of stimulus It is interesting to understand how the magnitude of the stimulus as measured by $\rho b - r_L(b)$ depends on the initial debt level b. Note first that as b approaches its maximum level $\bar{b}$, the size of the stimulus must converge to zero. Interpreting the distance $\bar{b} - b$ as the economy's *fiscal space*, this result is simply saying that when the economy's fiscal space becomes very small (as a result, say, of a sequence of negative shocks or less inclusive political decision-making), its efforts to fight further negative shocks with fiscal policy will necessarily be limited.[27] We conjecture that, more generally, the magnitude of the stimulus as measured by $\rho b - r_L(b)$ will depend negatively on the initial debt level b. We also expect that as a result of this, an economy will experience higher increases in unemployment as a result of negative shocks when it has a higher debt level. This in turn suggests that employment levels in an economy will be more volatile when that economy is more indebted. We will return to this idea in Section 4.

3.4 Political distortions

There are two types of distortions that can arise in our economy. The first is unemployment: some of the available workforce is not utilized. The second is an inefficient output mix: the

[27] For more on the concept of fiscal space and an attempt to measure it see Ostroy, Ghosh, Kim and Qureshi (2010).

workforce is not allocated optimally between private and public production. If policies are chosen by a planner seeking to maximize aggregate societal utility, it can be shown that there will be no distortions in the long run.[28] The way in which the government achieves this first best outcome is by accumulating bond holdings. In the long run, in every period the government hires sufficient public sector workers to provide the Samuelson level of the public good and sets taxes so that the private sector has the incentive to hire the remaining workers. Under Assumption 1, when the private sector is experiencing negative shocks, these taxes are sufficiently low that tax revenues fall short of the costs of public good provision. The earnings from government bond holdings are then used to finance this shortfall. This result parallels similar results for the tax smoothing model (Aiyagari et al 2002, Battaglini and Coate 2008, and Barshegyan, Battaglini, and Coate 2013).[29]

As we have already seen, in political equilibrium, unemployment can arise in the long run. It should also be noted that, even when there is full employment, the output mix will be distorted. This means that either the public sector is too large or too small. The direction of the distortion turns out to depend on the underlying parameters of the economy in a relatively simple way. In the Appendix, we show that the output mix is distorted towards public production when

$$n_e < \frac{1 - 2\gamma/A_\theta}{1 + A_\theta/2\xi}. \tag{20}$$

Otherwise, it is distorted towards the private good. Condition (20) is more likely to hold the smaller is the number of entrepreneurs n_e and the larger is the economy's preference for public goods γ.

To gain intuition for this result, recall that the first best output mix can be found by solving problem (15) with $q = 1$ and no revenue requirement. In the solution, the government chooses the Samuelson level of the public good and adjusts the tax rate to get the private sector to employ the remaining workers. Relative to this problem, the mwc puts more weight on raising revenue (either

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[28] An extensive analysis of the benevolent government solution can be found in our working paper, Battaglini and Coate (2011).

[29] In the tax smoothing model, the government eventually accumulates sufficient assets so that it can finance government spending needs at first best levels without distortionary taxation. Thus, there are no distortions in the long run. When the need for revenue is low, the government not only pays down the debt that was issued in times of high revenue need, it also reduces the base debt level. Gradually, over time, it starts to accumulate a stock of assets. It only stops accumulating when the interest earnings from assets are sufficient to completely eliminate the need for distortionary taxation. While the nature of the distortions are very different in this model, the same forces are operative.

because it wants revenues for transfers or because it needs to meet the revenue requirement). If the mwc is choosing policies that yield full employment, then relative to the first best, it is choosing tax rates and public production that keep employment constant but generate more revenue. Keeping employment constant requires that if taxes are raised, any private sector workers laid off are employed in the public sector. Conversely, if public production is reduced, entrepreneurs must be incentivized to hire the displaced public sector workers. Clearly, if entrepreneurs can be induced to hire more workers for only a very small tax cut, then it makes sense to reduce public production. The savings from reducing public production will exceed the loss in tax revenues. The employment response for any given tax cut will be greater, the higher are the first best taxes. Accordingly, when socially optimal taxes are high, reducing public production will be the optimal way to distort the output mix. Socially optimal taxes will be high when the first best public good level is high (high γ) and when the size of the private sector is large (high n_e).

4 Empirical implications and some evidence

Our theory has two unambiguous qualitative implications. The first is that the dynamic pattern of debt is counter-cyclical. More precisely, increases in debt should be positively correlated with reductions in output and visa versa. This follows from Proposition 2. This implication also emerges from tax smoothing models and simple Keynesian theories of fiscal policy, so there is nothing particularly distinctive about it. Empirical support for this prediction for U.S. debt is provided by Barro (1986).

The second implication is that debt and unemployment levels should be positively correlated. This follows from Proposition 3. Since we are not aware of any other theoretical work that links debt and unemployment, we believe this is a novel prediction. Assessing its validity is not immediate because the empirical literature does not appear to have extensively analyzed the relationship. A positive correlation between public debt and unemployment has been noted by Bertola (2011) using a panel of OECD countries from 1980 up to 2003.[30] In Figure 8 and Table 1 we have augmented Bertola's analysis considering panel data from 2006 to 2010 and controlling for economically relevant variables.[31] Each point in Figure 8 corresponds to an OECD country

[30] A positive correlation between debt and unemployment is also found by Fedeli and Forte (2011) and Fedeli, Forte, and Ricchi (2012) in a cointegration analysis of OECD data from 1970 to 2009.

[31] Our focus on a short panel is motivated by the desire to avoid non-stationarity problems along the time dimension.

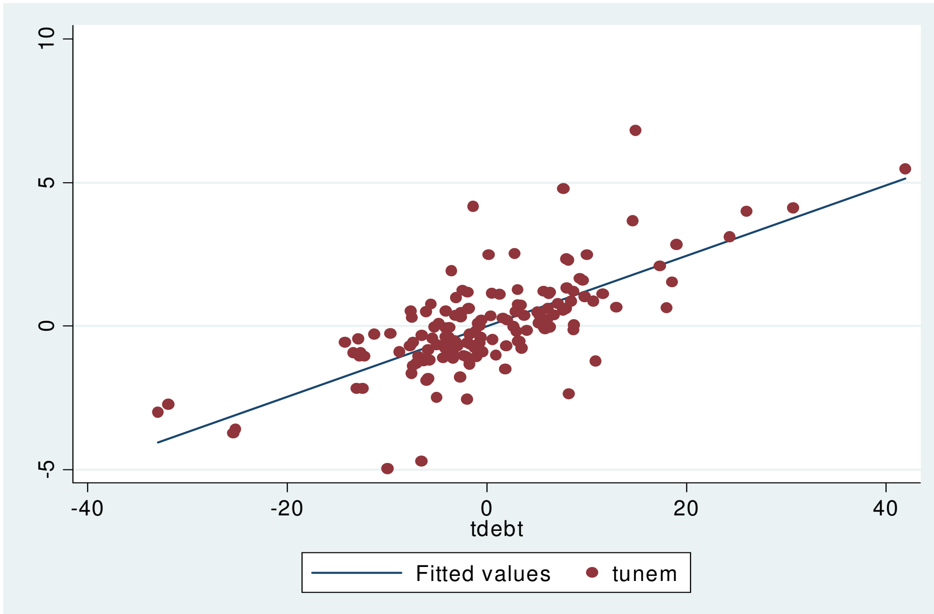

Figure 8:

in a given year in the five year period 2006 to 2010.[32] The height of a point on the vertical axis measures the country's unemployment rate in that year less its average rate over the five year period. The length of a point on the horizontal axis measures the country's debt/GDP ratio at the beginning of that year less its average ratio. The Figure reveals a strong positive correlation. In Table 1, the level of unemployment in period t is regressed on the level of debt at the beginning of the period and a selection of controls.[33] Country and year fixed effects are included. Column 1 presents the basic results. Column 2 controls for interest rates and Column 3 controls for both interest rates and for non linear effects including a variable equal to the debt/GDP ratio if it is larger than 90%. As shown, in each specification, the effect of debt on unemployment is positive and highly significant. The variable for the 90% threshold is not significant.

A further idea concerning the relationship between debt and unemployment suggested by the

[32] The panel includes 30 countries. Turkey, Mexico, Luxemburg and Estonia were dropped from the regression because values of the debt/GDP variable for the relevant years are missing in the OECD database.

[33] In Table 1 the variable *debt* is the debt/GDP ratio, *debt_plus_90* is a variable equal to *debt* if *debt* is larger than 90%, *l_dependency* is the % of working age population, *l_popgrowth* is the annual % rate of population growth, *l_open* measures imports plus exports in goods and services as a % of GDP, *l_bonds* is the interest rate on 10 year bonds.

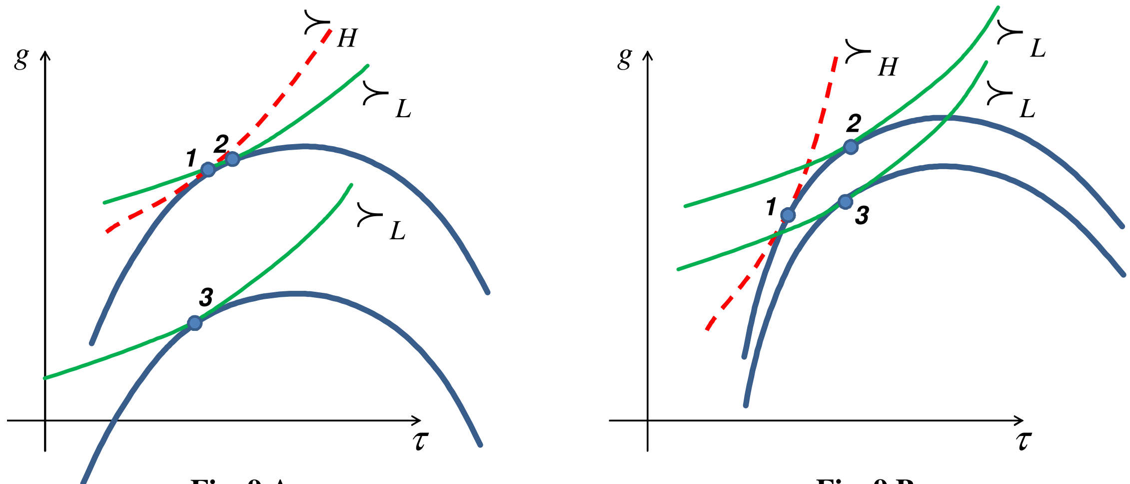

Fig. 9.A **Fig. 9.B**

Figure 9:

theory is that unemployment in a country might be more responsive to shocks when that country has higher debt. As discussed in Section 3.3, the logic of the model suggests that, with lower debt, the country will be able to better self insure against shocks. This in turn suggests that any given negative shock is likely to result in a bigger bump up in unemployment.[34] Some preliminary support for this idea is presented in columns 4-6 of Table 1 that document a positive and significant correlation between the absolute value of the change in unemployment rate between period t and period $t-1$ in a country on the level of debt at the beginning of period $t-1$.

A final point worth noting is that the model has no robust implications for the cyclical behavior of taxes and public spending.[35] Depending on the parameters, when the economy experiences a negative shock, public spending could increase or decrease, and tax rates could increase or decrease. To illustrate, consider a situation in which there is unemployment pre and post-shock.

[34] While we do not have a general proof of this proposition, it is clearly true under the assumption that q is less than q_H^* when comparing debt levels at either ends of the equilibrium distribution. Proposition 3 tells us that there will be full employment in both states at low levels of debt and therefore unemployment will be invariant with respect to shocks. On the other hand, at the other end of the debt distribution, there will be unemployment in the low productivity state. Moreover, we know that, for a given debt level, if there is unemployment in the low state, unemployment must be lower in the high state (this needs proving). This implies that unemployment will be responsive to shocks at high debt levels.

[35] There is an extensive literature on the cyclical behavior of public spending and taxes. See, for example, Alesina, Campante, and Tabellini (2008), Barro (1986), Barshegyan, Battaglini, and Coate (2013), Furceri and Karras (2011), Gavin and Perotti (1997), Lane (2003), and Talvi and Vegh (2005). In light of the variety of empirical correlations found in the literature, the fact that the model predicts no clear pattern of behavior is perhaps a virtue.

Two effects are at work. First, when private sector productivity decreases the mwc's indifference curve becomes flatter, so if the budget line did not change, τ and g would increase (this is represented by the move from point 1 to point 2 in Figure 9). Intuitively, the marginal cost of raising taxes is lower because the private sector is less productive and therefore taxation results in a lower output response. It therefore becomes optimal to increase the size of the public sector. The reduction in private sector productivity, however, does impact the budget line. Specifically, it both shifts downward and becomes flatter. Intuitively, any given tax raises less revenue and any given increase in taxes results in a smaller revenue increase. Although the downward shift is partially compensated by an increase in debt, the combination of the downward shift and the flattening makes the net effect on taxes and public spending ambiguous. This is illustrated in Figure 9. When the economy experiences a positive shock, taxes and public spending move from point 1 to point 3. In Fig. 9.A, public spending and taxes decrease and in Fig. 9.B, public spending and taxes increase. Matters are only made more complicated if there is full employment pre-shock.

5 Conclusion

This paper has presented a political economy theory of the interaction between fiscal policy and unemployment. For appropriate parameterizations, unemployment will arise when the private sector experiences negative shocks. To mitigate this unemployment, the government will employ debt-financed fiscal stimulus plans, which will generally involve both tax cuts and public production increases. When the private sector is healthy, the government will contract debt until it reaches a floor level. Depending on the extent of political frictions, unemployment can arise even in normal times. At all times, unemployment levels are increasing in the economy's debt level. Even when there is full employment, the mix of public and private output is distorted.

There are many different directions in which the ideas presented here might usefully be developed. In terms of the basic model, it would be desirable to incorporate a richer model of unemployment into the analysis. The search theoretic approach of Michaillat (2012) would seem promising in this regard since it allows for both rationing unemployment (as in this paper) and frictional unemployment. This would permit us to move beyond the sharp distinction between full employment and unemployment, which would be helpful for developing empirical predictions. With respect to political decision-making, it would be interesting to introduce class conflict into the analysis. The current model limits the conflict among citizens to disagreements concerning the allocation of transfers between districts. This is made possible by assuming that each legislator behaves so as to maximize the aggregate utility of the citizens in his district. Alternatively, we could assume that legislators either represent workers or entrepreneurs in their districts. This would introduce an additional conflict over policies in the sense that workers prefer policies that keep wages and employment high, while entrepreneurs prefer policies which keep profits high. Such class conflict may have important implications for the choice of fiscal policy. Finally, it would be interesting to introduce money into the model and explore how monetary policy interacts with fiscal policy and unemployment. Comparing the control of monetary policy by legislators and a central bank would be of particular interest.

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6 Appendix

6.1 Proof of Proposition 1

As argued in the text, Problem (14) is equivalent to Problem (15). The Lagrangian for Problem (15) is

$$L = x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g + (\lambda + q - 1)\left(R_\theta(\tau, \underline{\omega}) - \underline{\omega} g - r\right) + \mu \left(n_w - g - \frac{x_\theta(\tau)}{A_\theta}\right).$$

Thus, λ is the multiplier on the budget constraint and μ is the multiplier on the resource constraint. Using the expressions for $x_\theta(\tau)$ and $R_\theta(\tau, \underline{\omega})$ in (8) and (9), the first order conditions with respect to g and τ are

$$\frac{\gamma}{g} = (\lambda + q - 1)\underline{\omega} + \mu, \tag{21}$$

and

$$(\lambda + q - 1)(1 - 2\tau)(A_\theta - \underline{\omega}) = \tau A_\theta + (1 - \tau)\underline{\omega} - \mu. \tag{22}$$

6.1.1 $r \leq r_\theta^q$

We deal first with the case in which the revenue requirement r is less than or equal to r_θ^q. Recall that by definition, $r_\theta^q = R_\theta(\tau_\theta^q, \underline{\omega}) - \underline{\omega} g_\theta^q$, where $(\tau_\theta^q, g_\theta^q)$ are the optimal policies for the mwc ignoring the budget constraint (the unconstrained optimal policies). If $r \leq r_\theta^q$, the budget constraint will not be binding and the optimal policies will clearly be $(\tau_\theta^q, g_\theta^q)$. We need to show that when $q < q_\theta^*$, these policies involve full employment and when $q > q_\theta^*$ they involve unemployment.

The policies $(\tau_\theta^q, g_\theta^q)$ can be obtained from the first order conditions by setting λ equal to zero. There are two cases depending on whether the resource constraint binds. If μ equals zero, (21) and (22) imply that the solution is given by

$$(\tau_\theta^q, g_\theta^q) = \left(\frac{(q-1)\, A_\theta - q\underline{\omega}}{(A_\theta - \underline{\omega})\,(2q-1)}, \frac{\gamma}{(q-1)\,\underline{\omega}}\right). \tag{23}$$

It follows that the resource constraint binds if at these values of $(\tau_\theta^q, g_\theta^q)$, it is the case that $g_\theta^q + \frac{x_\theta(\tau_\theta^q)}{A}$ exceeds n_w. This implies:

$$n_e \left[\frac{q(A_\theta - \underline{w}) + \underline{w}}{\xi(2q-1)}\right] + \frac{\gamma}{(q-1)\,\underline{w}} > n_w. \tag{24}$$

Note that the left hand side of (24) is decreasing in q, so the inequality is satisfied if and only if q is less than q_θ^*, where q_θ^* is defined by (18). If μ is positive, (21) implies that $\mu = \frac{\gamma}{g} - (q-1)\underline{\omega}$.

Substituting this expression for μ into (22), we obtain

$$\tau A_\theta + (1-\tau)\underline{\omega} = \frac{\gamma}{g} + (q-1)\left[(1-2\tau)(A_\theta - \underline{\omega}) - \underline{\omega}\right]. \tag{25}$$

Combining this equation with the resource constraint and solving we find that the solution is

$$(\tau_\theta^q, g_\theta^q) = \left(1 - \frac{\xi\,(n_w - g_\theta^q)}{n_e(A_\theta - \underline{\omega})}, \frac{\sqrt{(qA_\theta n_e - (2q-1)\xi n_w)^2 + (2q-1)4\xi n_e \gamma} - (qA_\theta n_e - (2q-1)\xi n_w)}{(2q-1)2\xi}\right). \tag{26}$$

We conclude from all this that when q is less than q_θ^*, the optimal policies are described by (26) and involve full employment. When q exceeds q_θ^*, these policies are described by (23) and involve unemployment.

6.1.2 $r > r_\theta^q$ and $q < q_\theta^*$

We deal next with the case in which the revenue requirement r exceeds r_θ^q and q is less than the threshold q_θ^*. When $r > r_\theta^q$, the budget constraint will be binding. We need to show that in this case there exists a revenue requirement r_θ^* such that if $r < r_\theta^*$ the optimal policies are as described in the second part of the Proposition and if $r > r_\theta^*$ they are as described in the third part.

Assume first that both the budget constraint and the resource constraints are binding, so that $\lambda > 0$ and $\mu > 0$. In this case, we have that

$$R_\theta(\tau, \underline{\omega}) - \underline{\omega} g = r, \tag{27}$$

and

$$g + \frac{x_\theta(\tau)}{A_\theta} = n_w. \tag{28}$$

Substituting (28) into (27), we obtain

$$R_\theta(\tau, \underline{\omega}) - \underline{\omega}\left(n_w - \frac{x_\theta(\tau)}{A_\theta}\right) = r. \tag{29}$$

Assuming that (29) has a solution, it will have two solutions $\tau_\theta^-(r)$ and $\tau_\theta^+(r)$, which correspond to the points illustrated in Fig. 3.B and Fig 3.C. The associated public good levels, $g_\theta^-(r)$ and $g_\theta^+(r)$, are then obtained from (28).

It remains to describe when (29) has a solution and also whether $\left(\tau_\theta^-(r), g_\theta^-(r)\right)$ or $\left(\tau_\theta^+(r), g_\theta^+(r)\right)$ provide a higher value of the objective function. In order for (29) to have a solution, the budget line must lie above the resource constraint for some range of taxes. Let τ_θ^* denote the tax rate at which the slope of the budget line is equal to the slope of the full employment line and let $g^*_\theta(r)$ denote the level of public good that satisfies the budget constraint (27) given this tax rate. Now define r^{**}_θ to be that revenue requirement at which the point $(\tau^*_\theta, g^*_\theta(r))$ is tangent to the full employment line. Then, (29) has a solution if and only if $r \leq r^{**}_\theta$. Moreover, $\tau^-_\theta(r^{**}_\theta) = \tau^+_\theta(r^{**}_\theta) = \tau^*_\theta$.

Turning to the issue of whether $\left(\tau^-_\theta(r), g^-_\theta(r)\right)$ or $\left(\tau^+_\theta(r), g^+_\theta(r)\right)$ provide a higher value of the objective function, assume that $r \in (r^q_\theta, r^{**}_\theta)$. There are two possibilities: i) $\tau^q_\theta < \tau^-_\theta(r)$ in which case the unconstrained optimal policy $(\tau^q_\theta, g^q_\theta)$ lies to the left of $\left(\tau^-_\theta(r), g^-_\theta(r)\right)$, and ii) $\tau^q_\theta > \tau^+_\theta(r)$ in which case the unconstrained optimal policy $(\tau^q_\theta, g^q_\theta)$ lies to the right of $\left(\tau^+_\theta(r), g^+_\theta(r)\right)$. Since the objective function is concave, in case i) the optimal choice is $\left(\tau^-_\theta(r), g^-_\theta(r)\right)$ and in case ii), the optimal policy choice is $\left(\tau^+_\theta(r), g^+_\theta(r)\right)$. We conclude that a necessary condition for both the budget and resource constraints to be binding, is that $r \in (r^q_\theta, r^{**}_\theta]$ and that, when this happens, the solution is $\left(\tau^-_\theta(r), g^-_\theta(r)\right)$ if $\tau^q_\theta < \tau^-_\theta(r)$ and $\left(\tau^+_\theta(r), g^+_\theta(r)\right)$ if $\tau^q_\theta > \tau^+_\theta(r)$. We will return to provide a necessary and sufficient condition for both constraints to bind shortly.

Suppose now that only the budget constraint binds, so that $\lambda > 0$ and $\mu = 0$. Substituting (21) into (22), we obtain

$$g = \frac{\gamma(1-2\tau)(A_\theta - \underline{\omega})}{\underline{\omega}\left(\tau(A_\theta - \underline{\omega}) + \underline{\omega}\right)}. \tag{30}$$

Substituting (30) into the budget constraint (27), we obtain:

$$\tau n_e(1-\tau)(A_\theta - \underline{\omega})^2/\xi - \left(\frac{\gamma(1-2\tau)(A_\theta - \underline{\omega})}{\tau(A_\theta - \underline{\omega}) + \underline{\omega}}\right) = r \tag{31}$$

This equation has a unique solution $\widehat{\tau}_\theta(r)$ in the relevant range for τ, i.e. $[0, 1/2]$. Since the right hand side of (31) is always increasing for τ less than $1/2$, $\widehat{\tau}_\theta(r)$ is increasing in r. The associated value of g, $\widehat{g}_\theta(r)$, is obtained from (30). Since the right hand side of (30), is decreasing in τ, $\widehat{g}_\theta(r)$ is decreasing in r. Furthermore, note that unemployment is an increasing function of τ and a decreasing function of g, so it is increasing in r as well. Now define the revenue requirement r^*_θ to be such that the resource constraint is satisfied with equality at the policies $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$; that is, which satisfies

$$\widehat{g}_\theta(r^*_\theta) = n_w - n_e(1 - \widehat{\tau}_\theta(r^*_\theta))(A_\theta - \underline{\omega})/\xi. \tag{32}$$

It is clear that this revenue requirement exists, is unique, and satisfies $r^*_\theta \in (r^q_\theta, r^{**}_\theta]$. Moreover, if $r < r^*_\theta$, then it must be the case that the resource constraint binds while if $r > r^*_\theta$, then the resource constraint does not bind. Thus, assuming that $q < q^*_\theta$, if $r \in (r^q_\theta, r^*_\theta]$ the optimal policies are indeed as described in the second part of the proposition and if $r > r_\theta^*$ they are as described in the third part.

6.1.3 $r > r_\theta^q$ and $q > q_\theta^*$

Finally, we deal with the case in which the revenue requirement r exceeds r_θ^q and q exceeds the threshold q_θ^*. We need to show that in this case $r_\theta^q > r_\theta^*$. In addition, we need to show that the optimal policies are $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$, involve unemployment, and that unemployment is increasing in r.

For the first part, note that

$$(\tau_\theta^q, g_\theta^q) = (\widehat{\tau}_\theta(r_\theta^q), \widehat{g}_\theta(r_\theta^q)).$$

To see this, consider the first order conditions (21) into (22) when $\lambda = 0$ and $\mu = 0$. Substituting (21) into (22), we obtain

$$g_\theta^q = \frac{\gamma(1 - 2\tau_\theta^q)(A_\theta - \underline{\omega})}{\underline{\omega}\left(\tau_\theta^q(A_\theta - \underline{\omega}) + \underline{\omega}\right)}. \tag{33}$$

By definition,

$$R_\theta(\tau_\theta^q, \underline{\omega}) - \underline{\omega} g_\theta^q = r_\theta^q. \tag{34}$$

Substituting (33) into (34), we obtain:

$$\tau_\theta^q n_e(1 - \tau_\theta^q)(A_\theta - \underline{\omega})^2/\xi - \left(\frac{\gamma(1 - 2\tau_\theta^q)(A_\theta - \underline{\omega})}{\tau_\theta^q(A_\theta - \underline{\omega}) + \underline{\omega}}\right) = r_\theta^q. \tag{35}$$

Comparing (33) and (35) with (30) and (31) yields the result. We can now prove that $r_\theta^q > r_\theta^*$. Since $q > q_\theta^*$, we know that there is unemployment at $(\tau_\theta^q, g_\theta^q)$. Thus, we have know that

$$\widehat{g}_\theta(r_\theta^q) < n_w - n_e(1 - \widehat{\tau}_\theta(r_\theta^q))(A_\theta - \underline{\omega})/\xi.$$

Given the definition of r_θ^* in (32) this implies that $r_\theta^q > r_\theta^*$.

For the second part, note that if $r > r_\theta^q$ the budget constraint must bind. Moreover, since $r_\theta^q > r_\theta^*$, it must be the case that $r > r_\theta^*$, in which case it follows from the earlier analysis that the resource constraint does not bind. As already shown, when the budget constraint binds and the resource constraint does not the optimal policies are $(\widehat{\tau}_\theta(r), \widehat{g}_\theta(r))$ and involve unemployment. In addition, unemployment is increasing in r. ■

6.2 Proof of Proposition 2

The proof is broken into three parts. In Subsection 6.2.1 we characterize b^q - the lower bound of the equilibrium debt distribution. In Subsection 6.2.2 we prove that debt behaves in a counter-cyclical way. In Subsection 6.5.3 we prove that a non-degenerate stable distribution exists and has full support in $\left[b^q, \overline{b}\right]$.

6.2.1 The lowerbound b^q

Consider problem (12). When the budget constraint is not binding, the mwc will choose a debt level from the set

$$\mathcal{X}(V) = \arg\max_{b' \leq \overline{b}} \left\{ qb' + \beta EV_{\theta'}(b') \right\}.$$

We will show that $\mathcal{X}(V)$ consists of just a single point. We first need:

Lemma A.1. $r_H^q > r_L^q$.

Proof. Recall from Section 3.1 that r_θ^q was defined to be the revenue requirement equal to $R_\theta(\tau_\theta^q, \underline{\omega}) - \underline{\omega} g_\theta^q$. Suppose first that there is full employment at both (τ_H^q, g_H^q) and at (τ_L^q, g_L^q). In this case, we know that $(\tau_\theta^q, g_\theta^q)$ solves the problem

$$x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g + (q-1)\left[R_\theta(\tau, \underline{\omega}) - \underline{\omega} g\right]$$
$$s.t. \quad g = n_w - n_e(1-\tau)(A_\theta - \underline{\omega})/\xi.$$

Now consider the problem for each state θ and any $\lambda \geq 0$

$$x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g + \lambda\left[R_\theta(\tau, \underline{\omega}) - \underline{\omega} g\right]$$
$$s.t. \quad g = n_w - n_e(1-\tau)(A_\theta - \underline{\omega})/\xi \tag{36}$$

Let $(\tau_\theta(\lambda), g_\theta(\lambda))$ denote the solution to this problem and let $R_\theta^*(\lambda) = R_\theta(\tau_\theta(\lambda), \underline{\omega}) - \underline{\omega} g_\theta(\lambda)$ denote net revenues at the solution. We will show that for any $\lambda \geq 0$, $R_H^*(\lambda) > R_L^*(\lambda)$ which implies the result.

Let λ be given. The first order conditions for the above maximization problem are

$$\frac{\gamma}{g} = \lambda \underline{w} + \mu,$$

where μ is the Lagrangian multiplier in (36), and

$$\lambda(1-2\tau)(A_\theta - \underline{w}) = \underline{w} + \tau(A_\theta - \underline{w}) - \mu.$$

Solving the g first order condition for μ and substituting this into the τ first order condition we get

$$\lambda(A_\theta - \underline{w})g - \lambda 2\tau(A_\theta - \underline{w})g - \lambda \underline{w}g = \underline{w}g + \tau g(A_\theta - \underline{w}) - \gamma$$

We also know from the full employment constraint that

$$\tau = \frac{n_e(A_\theta - \underline{w}) - \xi\,(n_w - g)}{n_e(A_\theta - \underline{w})}.$$

Substituting this in and rearranging, we obtain the following quadratic equation in g

$$\xi\,(1+2\lambda)\,g^2 + (n_e A_\theta(1+\lambda) - \xi(1+2\lambda)n_w)\,g - \gamma n_e = 0.$$

This has solution

$$\begin{aligned} g_\theta(\lambda) &= \frac{\sqrt{\begin{array}{c}(n_e A_\theta(1+\lambda) - \xi(1+2\lambda)n_w)^2 \\ +4\gamma n_e \xi\,(1+2\lambda)\end{array}} - (n_e A_\theta(1+\lambda) - \xi(1+2\lambda)n_w)}{2\xi\,(1+2\lambda)} \\ &= \frac{\sqrt{\begin{array}{c}\left(n_e A_\theta(\frac{1+\lambda}{1+2\lambda}) - \xi n_w\right)^2 \\ +4\gamma n_e \xi(\frac{1}{1+2\lambda})\end{array}} - \left(n_e A_\theta(\frac{1+\lambda}{1+2\lambda}) - \xi n_w\right)}{2\xi} \end{aligned}$$

It is straightforward to verify that, for all λ, $g_L(\lambda) > g_H(\lambda)$. Intuitively, labor is more productive in the public sector in the low productivity state. Thus, to show that $R_H^*(\lambda) > R_L^*(\lambda)$ it suffices to demonstrate that $R_H(\tau_H(\lambda), \underline{\omega}) \geq R_L(\tau_L(\lambda), \underline{\omega})$.

We have that

$$\begin{aligned} R_\theta(\tau_\theta(\lambda); \underline{\omega}) &= \tau_\theta(\lambda) n_e (1 - \tau_\theta(\lambda))(A_\theta - \underline{\omega})^2/\xi \\ &= \left(\frac{n_e(A_\theta - \underline{w}) - \xi\,(n_w - g_\theta(\lambda))}{n_e}\right)(n_w - g_\theta(\lambda)) \end{aligned}$$

Given that $g_H(\lambda) < g_L(\lambda)$, it suffices to show that

$$n_e(A_H - \underline{\omega}) - \xi\,(n_w - g_H(\lambda)) > n_e(A_L - \underline{\omega}) - \xi\,(n_w - g_L(\lambda))$$

or, equivalently, that

$$A_H - A_L > \frac{\xi}{n_e}\,(g_L(\lambda) - g_H(\lambda))\,.$$

Note that

$$\frac{\xi}{n_e}\left(g_L(\lambda)-g_H(\lambda)\right) = \frac{\xi}{n_e}\left(\begin{array}{c} \frac{\sqrt{\left(n_e A_L(\frac{1+\lambda}{1+2\lambda})-\xi n_w\right)^2+4\gamma n_e\xi(\frac{1}{1+2\lambda})}-\left(n_e A_L(\frac{1+\lambda}{1+2\lambda})-\xi n_w\right)}{2\xi} \\ -\frac{\sqrt{\left(n_e A_H(\frac{1+\lambda}{1+2\lambda})-\xi n_w\right)^2+4\gamma n_e\xi(\frac{1}{1+2\lambda})}-\left(n_e A_H(\frac{1+\lambda}{1+2\lambda})-\xi n_w\right)}{2\xi} \end{array}\right)$$

$$= \left(\frac{A_H-A_L}{2}\right)\left(\frac{1+\lambda}{1+2\lambda}\right)+\frac{1}{n_e}\left(\begin{array}{c} \frac{\sqrt{\left(n_e A_L(\frac{1+\lambda}{1+2\lambda})-\xi n_w\right)^2+4\gamma n_e\xi(\frac{1}{1+2\lambda})}}{2} \\ -\frac{\sqrt{\left(n_e A_H(\frac{1+\lambda}{1+2\lambda})-\xi n_w\right)^2+4\gamma n_e\xi(\frac{1}{1+2\lambda})}}{2} \end{array}\right)$$

Thus, the condition that

$$A_H-A_L>\frac{\xi}{n_e}\left(g_L(\lambda)-g_H(\lambda)\right)$$

amounts to

$$n_e\left(A_H-A_L\right)\left[\frac{1+3\lambda}{1+2\lambda}\right]>\left(\begin{array}{c} \sqrt{\left(n_e A_L(\frac{1+\lambda}{1+2\lambda})-\xi n_w\right)^2+4\gamma n_e\xi(\frac{1}{1+2\lambda})} \\ -\sqrt{\left(n_e A_H(\frac{1+\lambda}{1+2\lambda})-\xi n_w\right)^2+4\gamma n_e\xi(\frac{1}{1+2\lambda})} \end{array}\right).$$

Define the function

$$\varphi(A)=\left(\sqrt{\left(n_e A_L(\frac{1+\lambda}{1+2\lambda})-\xi n_w\right)^2+4\gamma n_e\xi(\frac{1}{1+2\lambda})}-\sqrt{\left(n_e A(\frac{1+\lambda}{1+2\lambda})-\xi n_w\right)^2+4\gamma n_e\xi(\frac{1}{1+2\lambda})}\right).$$

Then we need to show that

$$n_e\left(A_H-A_L\right)\left[\frac{1+3\lambda}{1+2\lambda}\right]>\varphi(A_H)$$

Since $\varphi(A_L)=0$, it suffices to show that on the interval $[A_L, A_H]$

$$n_e\left[\frac{1+3\lambda}{1+2\lambda}\right]>\varphi'(A)$$

We have

$$\varphi'(A)=\left(\frac{\left(\xi n_w-n_e A(\frac{1+\lambda}{1+2\lambda})\right)n_e(\frac{1+\lambda}{1+2\lambda})}{\left[\left(\xi n_w-n_e A(\frac{1+\lambda}{1+2\lambda})\right)^2+4\gamma n_e\xi(\frac{1}{1+2\lambda})\right]^{1/2}}\right)$$

So we need that

$$n_e\left[\frac{1+3\lambda}{1+2\lambda}\right]>\left(\frac{\left(\xi n_w-n_e A(\frac{1+\lambda}{1+2\lambda})\right)n_e(\frac{1+\lambda}{1+2\lambda})}{\left[\left(\xi n_w-n_e A(\frac{1+\lambda}{1+2\lambda})\right)^2+4\gamma n_e\xi(\frac{1}{1+2\lambda})\right]^{1/2}}\right)$$

A sufficient condition is that

$$1 > \left(\frac{\left(\xi n_w - n_e A(\frac{1+\lambda}{1+2\lambda})\right)}{\left[\left(\xi n_w - n_e A(\frac{1+\lambda}{1+2\lambda})\right)^2 + 4\gamma n_e \xi(\frac{1}{1+2\lambda})\right]^{1/2}} \right)$$

which is true.

Next suppose that there is unemployment in both the L and the H states. In this case, from the first order conditions for the maximization problem, we know that for each state θ, $\underline{\omega}(q-1) = \frac{\gamma}{g_\theta^q}$ and that

$$q - 1 = \frac{\underline{\omega} + \tau_\theta^q(A_\theta - \underline{\omega})}{(1 - 2\tau_\theta^q)(A_\theta - \underline{\omega})}.$$

From the first condition, we conclude that $g_H^q = g_L^q$. It follows that to establish the result it suffices to show that $R_L(\tau_L^q, \underline{\omega})$ is less than $R_H(\tau_H^q, \underline{\omega})$. To this end, note that the second condition implies that

$$\tau_\theta^q = \frac{(q-1)(A_\theta - \underline{\omega}) - \underline{\omega}}{(1 + 2(q-1))(A_\theta - \underline{\omega})}.$$

Define the function

$$\varphi(A) = \frac{(q-1)(A - \underline{\omega}) - \underline{\omega}}{(1 + 2(q-1))(A - \underline{\omega})}$$

Note that

$$\begin{aligned} \varphi'(A) &= \frac{(1+2(q-1))(A-\underline{\omega})(q-1) - (1+2(q-1))\left[(q-1)(A-\underline{\omega}) - \underline{\omega}\right]}{(1+2(q-1))^2(A-\underline{\omega})^2} \\ &= \frac{(1+2(q-1))\underline{\omega}}{(1+2(q-1))^2(A-\underline{\omega})^2} > 0. \end{aligned}$$

Thus, we have that $\tau_H^q = \varphi(A_H) > \varphi(A_L) = \tau_L^q$. Since $\tau_H^q < 1/2$, this inequality implies that

$$R_H(\tau_H^q, \underline{\omega}) > R_L(\tau_L^q, \underline{\omega}).$$

Finally, suppose that there is unemployment in the L state and full employment in the H state. In this case, from the first order conditions in the L state we have that $\underline{\omega}(q-1) = \frac{\gamma}{g_L^q}$ and that

$$q - 1 = \frac{\underline{\omega} + \tau_L^q(A_L - \underline{\omega})}{(1 - 2\tau_L^q)(A_L - \underline{\omega})}.$$

From the first order conditions for the H state, we have that $\underline{\omega}(q-1) + \mu_H = \frac{\gamma}{g_H^q}$, and that

$$(q-1)(1 - 2\tau_H^q)(A_H - \underline{\omega}) = \underline{\omega} + \tau_H^q(A_H - \underline{\omega}) - \mu_H.$$

The first order conditions for the public good imply that

$$g_H^q = \frac{\gamma}{\underline{\omega}(q-1)+\mu_H} < \frac{\gamma}{\underline{\omega}(q-1)} = g_L^q$$

Thus, again to establish the result it suffices to show that $R_H(\tau_H^q, \underline{\omega})$ is larger than $R_L(\tau_L^q, \underline{\omega})$. The second condition implies

$$\begin{aligned}\tau_H^q &= \frac{(q-1)(A_H-\underline{\omega})-\underline{\omega}+\mu_H}{(1+2(q-1))(A_H-\underline{\omega})} > \frac{(q-1)(A_H-\underline{\omega})-\underline{\omega}}{(1+2(q-1))(A_H-\underline{\omega})} \\ &= \varphi(A_H) > \varphi(A_L) = \tau_L^q\end{aligned}$$

Since in H we are in full employment, we have: $R_H(\tau_H^q, \underline{\omega}) = \tau_H^q(A_H - \underline{\omega})(n_w - g_H^q)$. Thus, we have:

$$R_H(\tau_H^q, \underline{\omega}) = \tau_H^q(A_H-\underline{\omega})(n_w-g_H^q) > \tau_L^q(A_L-\underline{\omega})(n_w-g_L^q) > \tau_L^q(A_L-\underline{\omega})\frac{x_L(\tau_L^q)}{A_L} = R_L(\tau_L^q, \underline{\omega}),$$

where the last inequality follows from the fact that $x_L(\tau_L^q) < A_L\,(n_w - g_L^q)$. ■

We can now show that:

Lemma A.2. *In any equilibrium, the set $\mathcal{X}(V)$ is a singleton.*

Proof. Assume by contradiction that $\mathcal{X}(V)$ is not a singleton. Since V is weakly concave, there must be constants b'_-, b'_+ such that $\mathcal{X}(V) = [b'_-, b'_+]$. Assume first that $r_L^q < \rho b'_+$. Then $r_L^q + b' - (1+\rho)\,b'_+ < 0$ for any $b' \le b'_+$. It follows that there is an $\varepsilon > 0$ such that for all $b \in [b'_+ - \varepsilon, b'_+]$:

$$r_L^q + b' - (1+\rho)\,b < 0$$

for any $b' \le b'_+$. Thus, in state L with initial debt level $b \in [b'_+ - \varepsilon, b'_+]$ it must be the case that $B_L(\tau_L(b), g_L(b), b'_L(b), b, \underline{\omega}) = 0$, and we can write:

$$V_L(b) = \max_{(\tau,g,b')}\left\{\begin{array}{c} b' - (1+\rho)b + x_L(\tau) - n_e\xi\frac{\left(\frac{x_L(\tau)}{A_L n_e}\right)^2}{2} + \gamma\ln g + \beta EV_{\theta'}(b') \\ s.t.\ B_L(\tau, g, b', b, \underline{\omega}) \ge 0,\ g + \frac{x_L(\tau)}{A_L} \le n_w\ \&\ b \le \overline{b}\end{array}\right\}$$

for all $b \in [b'_+ - \varepsilon, b'_+]$. This means that $V_L(b)$ is strictly concave in $[b'_+ - \varepsilon, b'_+]$, and therefore $EV_\theta(b)$ is strictly concave in $[b'_+ - \varepsilon, b'_+]$ as well. This implies that it is not possible that $[b'_+ - \varepsilon, b'_+] \subseteq \mathcal{X}(V)$, a contradiction.

We conclude that $r_L^q \ge \rho b'_+$. In this case for either state θ and any initial debt level $b \in [b'_-, b'_+]$ we have:

$$B_\theta(\tau_\theta^q, g_\theta^q, b, b, \underline{\omega}) = r_\theta^q - \rho b \ge r_L^q - \rho b \ge 0,$$

where the latter inequality follows from Lemma A.1. Thus, it must be the case that $b'_\theta(b) \in \left[b'_-, b'_+\right]$ and that

$$B_\theta(\tau^q_\theta, g^q_\theta, b'_\theta(b), b, \underline{\omega}) = r^q_\theta + b'_\theta(b) - b(1+\rho) \geq 0$$

with $B_H(\tau^q_H, g^q_H, b'_H(b), b, \underline{\omega}) > 0$. For either state θ, we can write:

$$\begin{aligned} V_\theta(b) &= \max_{(\tau,g,b')} \left\{ \begin{array}{c} b' - (1+\rho)b + x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g \\ + (q-1)\, B_\theta(\tau, g, b', b, \underline{\omega}) + \beta E V_{\theta'}(b') \\ s.t.\ B_\theta(\tau, g, b', b, \underline{\omega}) \geq 0,\ g + \frac{x_\theta(\tau)}{A_\theta} \leq n_w \ \&\ b \leq \overline{b} \end{array} \right\} \\ & \quad - (q-1)\, B_\theta(\tau^q_\theta, g^q_\theta, b'_\theta(b), b, \underline{\omega}). \end{aligned} \tag{37}$$

Since V_L and V_H are weakly concave, the previous expression has a left and right derivative at b. Since for all $b \in \left[b'_-, b'_+\right]$, $b \in \mathcal{X}(V)$, the right and left derivatives with respect to b of

$$qb + \beta E V_\theta(b) \tag{38}$$

must be zero, implying that (38) must be differentiable. By the Envelope Theorem the derivative of the first term in (37) is $-(1+\rho)q$ (since the constraint $B_{\theta'}(\tau, g, b', b'_\theta(b), \underline{\omega}) \geq 0$ is not binding). The derivative of the second is $(q-1)\left[(1+\rho) - E db'_\theta(b)/db\right]$, where $db'_\theta(b)/db$ is the derivative of b'_θ at b (it must be differentiable otherwise the second term in (37) would not be differentiable). From (38) we must therefore have the first order necessary condition:

$$q + \beta\left[-(1+\rho)q + (q-1)\left(1 + \rho - E\frac{db'_\theta(b)}{db}\right)\right] = 0 \tag{39}$$

Rewriting (39) and using the fact that $-(1+\rho) = -1/\beta$, we have:

$$q - 1 = \beta\,(q-1)\, E\frac{db'_\theta(b)}{db},$$

which implies

$$E\frac{db'_\theta(b)}{db} = \frac{1}{\beta} > 1 \tag{40}$$

for any $b \in \left[b'_-, b'_+\right]$. This implies a contradiction. To see this, assume first $Edb'_\theta(b'_-)/db < b'_-$, then it must be that there is a θ such that $b'_\theta(b'_-) < b'_-$, but then $b'_\theta(b'_-) \notin \mathcal{X}(V)$, so it is not optimal: a contradiction. It must therefore be $Edb'_\theta(b'_-)/db \geq b'_-$. But then (40) implies that $Edb'_\theta(b'_+)/db \geq b'_+$. This implies that there is a θ such that $b'_\theta(b'_+) > b'_+$: but then, again, $b'_\theta(b'_-) \notin \mathcal{X}(V)$, a contradiction. We conclude that $\mathcal{X}(V)$ is a singleton. ∎

Given Lemma A.2, we define b^q to be the unique element of the set $\mathcal{X}(V)$. We also define b^q_θ to be the value of debt such that the triple $(\tau^q_\theta, g^q_\theta, b^q)$ satisfies the constraint that $B_\theta(\tau^q_\theta, g^q_\theta, b^q, b^q_\theta, \underline{\omega})$ equal 0. This is given by:

$$b^q_\theta = \frac{r^q_\theta + b^q}{1+\rho}. \tag{41}$$

Then, if the debt level b is such that $b \leq b^q_\theta$ the tax-public good-debt triple is $(\tau^q_\theta, g^q_\theta, b^q)$ and the mwc uses the budget surplus $B_\theta(\tau^q_\theta, g^q_\theta, b^q, b^q_\theta, \underline{\omega})$ to finance transfers. If $b > b^q_\theta$ the budget constraint binds so that no transfers are given. Tax revenues net of public good costs strictly exceed r^q_θ and the debt level strictly exceeds b^q. In this case, the policies solve the problem

$$\max_{(\tau,g,b')} \left\{ \begin{array}{c} b' - (1+\rho)b + x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g + \beta E V_{\theta'}(b') \\ s.t.\ B_\theta(\tau, g, b', b, \underline{\omega}) \geq 0,\ g + \frac{x_\theta(\tau)}{A} \leq n_w \ \&\ b \leq \overline{b} \end{array} \right\} \tag{42}$$

Note also that by Lemma A.1., it must be the case that $b^q_H > b^q_L$.

Further information on the debt level b^q can be obtained by using a first order condition to characterize it. However, before we can do this, we must first establish that the value function is differentiable. We have:

Lemma A.3. *(i) If $q > q^*_\theta$, the equilibrium value function $V_\theta(b)$ is differentiable for all $b \neq b^q_\theta$. Moreover,*

$$-\beta V'_\theta(b) = \begin{cases} 1 & if\ b < b^q_\theta \\ 1 + \frac{\tau_\theta(b)A_\theta + (1-\tau_\theta(b))\underline{\omega}}{(1-2\tau_\theta(b))(A_\theta - \underline{\omega})} & if\ b > b^q_\theta \end{cases}.$$

(ii) If $q < q^*_\theta$, there exists a unique debt level $b^{**}_\theta \in (b^q_\theta, \overline{b}]$ such that the resource constraint is binding if and only if $b \leq b^{**}_\theta$. The equilibrium value function $V_\theta(b)$ is differentiable for all $b \neq b^q_\theta$ and

$$-\beta V'_\theta(b) = \left\{ \begin{array}{cc} 1 & if\ b < b^q_\theta \\ 1 + \frac{\tau_\theta(b)A_\theta + (1-\tau_\theta(b))\underline{\omega} - \frac{\gamma}{g_\theta(b)}}{(1-2\tau_\theta(b))(A_\theta - \underline{\omega}) - \underline{\omega}} & b \in (b^q_\theta, b^{**}_\theta) \\ 1 + \frac{\tau_\theta(b)A_\theta + (1-\tau_\theta(b))\underline{\omega}}{(1-2\tau_\theta(b))(A_\theta - \underline{\omega})} & b \geq b^{**}_\theta \end{array} \right\}.$$

Proof. (i) Suppose first that $q > q^*_\theta$. From the discussion presented above, we know that if the debt level b is such that $b \leq b^q_\theta$ the optimal policies are $(\tau^q_\theta, g^q_\theta, b^q)$, and, if $b > b^q_\theta$, the budget constraint $B_\theta(\tau, g, b', b, \underline{\omega}) \geq 0$ will be binding and the policies solve (42). Moreover, we know from Proposition 1 that the resource constraint will not be binding. Consider some debt level b_o.

Assume first that $b_o < b_\theta^q$. Then, in a neighborhood of b_o it must be the case that

$$V_\theta(b) = b^q - (1+\rho)b + x_\theta(\tau_\theta^q)\left(\frac{A_\theta}{A}\right) - n_e\xi\frac{\left(\frac{x_\theta(\tau_\theta^q)}{An_e}\right)^2}{2} + \gamma\ln g_\theta^q + \beta EV_\theta(b^q).$$

Thus, it is immediate that the value function $V_\theta(b)$ is differentiable at b_o and that

$$V_\theta'(b_o) = -(1+\rho) = -1/\beta.$$

Assume now that $b_o > b_\theta^q$. Consider the function

$$\varphi_\theta(b) = \max_{(\tau,g)}\left\{\begin{array}{c} b_\theta'(b_o) - (1+\rho)b + x_\theta(\tau) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma\ln g + \beta EV_{\theta'}(b_\theta'(b_o)) \\ s.t.\ B_\theta(\tau, g, b_\theta'(b_o), b, \underline{\omega}) = 0. \end{array}\right\} \tag{43}$$

Since the equilibrium policies are such that the budget constraint binds and the resource constraint does not bind, it follows that $V_\theta(b_o) = \varphi_\theta^o(b_o)$ and $V_\theta(b) \geq \varphi_\theta^o(b)$ for all b in a neighborhood of b_o. By the Envelope Theorem, the function $\varphi_\theta(b)$ is differentiable in b and its derivative is equal to $-(1+\rho)\left[1+\lambda_\theta^o(b)\right]$, where $\lambda_\theta^o(b)$ is the Lagrange multiplier on the constraint $B_\theta(\tau, g, b_\theta'(b_o), b, \underline{\omega}) = 0$ in (43) at b. It is also the case that $\varphi_\theta^o(b)$ is concave. To see this note that we may write:

$$\varphi_\theta(b) = \max_{(\tau,g)}\left\{\begin{array}{c} b_\theta'(b_o) - (1+\rho)b + x_\theta(\tau) - n_e\xi\frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma\ln g + \beta EV_{\theta'}(b_\theta'(b_o)) \\ s.t.\ B_\theta(\tau, g, b_\theta'(b_o), b, \underline{\omega}) \geq 0. \end{array}\right\} \tag{44}$$

The objective function in (44) is concave in τ, g and b, and the constraint set is convex in τ, g and b: so by a standard argument $\varphi_\theta(b)$ is concave. It now follows from Theorem 4.10 of Stokey, Lucas and Prescott (1989) that $V_\theta(b)$ is differentiable at b_o with derivative $V_\theta'(b_o) = \varphi_\theta'(b_o) = -(1+\rho)\left[1+\lambda_\theta^o(b_o)\right] = -\left[1+\lambda_\theta^o(b_o)\right]/\beta$. To complete the proof, consider the first order conditions for (43) at b_o:

$$\frac{\gamma}{g_\theta^o(b_o)} = \lambda_\theta^o(b_o)\underline{\omega} \tag{45}$$

and

$$\lambda_\theta^o(b_o)(1-2\tau_\theta^o(b_o))(A_\theta - \underline{\omega}) = \tau_\theta^o(b_o)A_\theta + (1-\tau_\theta^o(b_o))\underline{\omega}. \tag{46}$$

The second condition implies

$$\lambda_\theta^o(b_o) = \frac{\tau_\theta(b_o)A_\theta + (1-\tau_\theta(b_o))\underline{\omega}}{(1-2\tau_\theta(b_o))(A_\theta - \underline{\omega})}$$

where we are using the fact that the solution of (43) at b_o, $(\tau_\theta^o(b_o), g_\theta^o(b_o))$, must equal the equilibrium policies $(\tau_\theta(b_o), g_\theta(b_o))$. We conclude that

$$-\beta V_\theta'(b_o) = 1 + \frac{\tau_\theta(b_o) A_\theta + (1 - \tau_\theta(b_o)) \underline{\omega}}{(1 - 2\tau_\theta(b_o))(A_\theta - \underline{\omega})}. \tag{47}$$

(ii) Suppose now that $q < q_\theta^*$. Then we know from Proposition 1 that the resource constraint is binding for $b \leq b_\theta^q$. Consider the problem

$$\max_{(\tau,g,b')} \left\{ \begin{array}{c} b' - (1+\rho)b + x_\theta(\tau)\left(\frac{A_\theta}{A}\right) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A n_e}\right)^2}{2} + \gamma \ln g + (q-1)B_\theta(\tau, g, b', b, \underline{\omega}) + \beta E V_\theta(b') \\ s.t.\ B_\theta(\tau, g, b', b, \underline{\omega}) \geq 0 \ \&\ b \leq \overline{b} \end{array} \right\}. \tag{48}$$

This is the mwc's problem but ignoring the resource constraint. Let $(\widetilde{\tau}_\theta(b), \widetilde{g}_\theta(b))$ denote the optimal tax and public good levels for this problem. It is easy to show that $\widetilde{\tau}_\theta(b)$ is non-decreasing and $\widetilde{g}_\theta(b)$ is non-increasing in b, implying that $\widetilde{g}_\theta(b) + \frac{x_\theta(\widetilde{\tau}_\theta(b))}{A}$ is non-increasing in b. From Proposition 1, we know that $\widetilde{g}_\theta(b_\theta^q) + \frac{x_\theta(\widetilde{\tau}_\theta(b_\theta^q))}{A_\theta} > n_w$. If $\widetilde{g}_\theta(\overline{b}) + \frac{x_\theta(\widetilde{\tau}_\theta(\overline{b}))}{A_\theta} < n_w$, define b_θ^{**} to be the minimal level of b such that $\widetilde{g}_\theta(b) + \frac{x_\theta(\widetilde{\tau}_\theta(b))}{A_\theta} \leq n_w$. Otherwise let $b_\theta^{**} = \overline{b}$. Then the resource constraint in the mwc's problem is binding if and only if $b \leq b_\theta^{**}$.

Turning to differentiability, consider some debt level b_o. If $b_o < b_\theta^q$ or $b_o > b_\theta^{**}$, the argument follows exactly that in part (i). Suppose that $b_o \in (b_\theta^q, b_\theta^{**})$. It follows that both the budget and resource constraints are binding in a neighborhood of b_o. Consider the function

$$\varphi_\theta(b) = \max_{(\tau,g)} \left\{ \begin{array}{c} b_\theta'(b_o) - (1+\rho)b + x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g + \beta E V_{\theta'}(b_\theta'(b_o)) \\ s.t.\ B_\theta(\tau, g, b_\theta'(b_o), b, \underline{\omega}) = 0,\ g + \frac{x_\theta(\tau)}{A_\theta} = n_w. \end{array} \right\} \tag{49}$$

Clearly, $V_\theta(b_o) = \varphi_\theta^o(b_o)$ and $V_\theta(b) \geq \varphi_\theta^o(b)$ for all b in a neighborhood of b_o. The function $\varphi_\theta(b)$ is differentiable in b and its derivative is equal to $-(1+\rho)[1 + \lambda_\theta^o(b)]$, where $\lambda_\theta^o(b)$ is the Lagrangian multiplier of the constraint $B_\theta(\tau, g, b_\theta'(b_o), b, \underline{\omega}) = 0$ in (49) at b. By a similar argument to that just used, $\varphi_\theta(b)$ is concave. It again follows from Theorem 4.10 of Stokey, Lucas and Prescott (1989) that $V_\theta(b)$ is differentiable at b_o with derivative $V_\theta'(b_o) = \varphi_\theta'(b_o) = -(1+\rho)[1 + \lambda_\theta^o(b_o)] = -[1 + \lambda_\theta^o(b_o)]/\beta$. To complete the proof, consider the first order conditions of (49) at b_o:

$$\frac{\gamma}{g_\theta^o(b_o)} = \lambda_\theta^o(b_o)\underline{\omega} + \mu_\theta^o(b_o) \tag{50}$$

and

$$\lambda_\theta^o(b_o)(1-2\tau_\theta^o(b_o))(A_\theta-\underline{\omega}) = \tau_\theta^o(b_o)A_\theta + (1-\tau_\theta^o(b_o))\underline{\omega} - \mu_\theta^o(b_o), \tag{51}$$

where $\mu_\theta^o(b_o)$ is the Lagrange multiplier of the resource constraint. Solving the system, we have:

$$\lambda_\theta(b_o) = \frac{\tau_\theta(b_o)A_\theta + (1-\tau_\theta(b_o))\underline{\omega} - \frac{\gamma}{g_\theta(b_o)}}{(1-2\tau_\theta(b_o))(A_\theta-\underline{\omega}) - \underline{\omega}}$$

where we are using the fact that the solution of (49) at b_o, $(\tau_\theta^o(b_o), g_\theta^o(b_o))$ equals the equilibrium policies $(\tau_\theta(b_o), g_\theta(b_o))$. We therefore conclude that:

$$-\beta V_\theta'(b_o) = 1 + \frac{\tau_\theta(b_o)A_\theta + (1-\tau_\theta(b_o))\underline{\omega} - \frac{\gamma}{g_\theta(b_o)}}{(1-2\tau_\theta(b_o))(A_\theta-\underline{\omega}) - \underline{\omega}}. \tag{52}$$

Finally, suppose that $b = b_\theta^{**}$. It is easy to see that in a neighborhood of b_θ^{**} the function $\varphi_\theta(b)$ defined in (49) satisfies the properties: $V_\theta(b_o) = \varphi_\theta^o(b_o)$ and $V_\theta(b) \geq \varphi_\theta^o(b)$. As noted above, moreover, the function $\varphi_\theta(b)$ is differentiable and concave in b; its derivative is equal to $-(1+\rho)[1+\lambda_\theta^o(b)]$, where $\lambda_\theta^o(b)$ is the Lagrangian multiplier of the constraint $B_\theta(\tau, g, b_\theta'(b_o), b, \underline{\omega}) = 0$ in (49) at b. It again follows from Theorem 4.10 of Stokey, Lucas and Prescott (1989) that $V_\theta(b)$ is differentiable at b_θ^{**} with derivative $V_\theta'(b_\theta^{**}) = \varphi_\theta'(b_\theta^{**}) = -(1+\rho)[1+\lambda_\theta^o(b_\theta^{**})] = -[1+\lambda_\theta^o(b_\theta^{**})]/\beta$. The derivative is the same as above, with the difference that at b_θ^{**} we have $\mu_\theta^o(b_o) = 0$. So (47) is equal to (52) at b_θ^{**}. ■

We can now show:

Lemma A.4. $b^q \in [b_L^q, b_H^q]$.

Proof. From the definition of b^q, we know that if V_L and V_H are differentiable at b^q it must be the case that

$$q = -\beta E V_\theta'(b^q) \tag{53}$$

Assume first that $b^q < b_L^q$. Then, by Lemma A.3, (53) would imply $q = 1$, a contradiction. Assume next that $b^q > b_H^q$. This would imply that for each state θ, we have that $\tau_\theta(b^q) > \tau_\theta^q$. Using the first order conditions for τ_θ^q and the expressions in Lemma A.3, we can show that this implies $\beta V_\theta'(b^q) < -q$. This implies: $-\beta E V_\theta'(b^q) > q$: again a contradiction. We conclude that $b^q \in [b_L^q, b_H^q]$ as claimed. ■

We can use this result to establish the assertion in the proposition that $b^q > r_L^q/\rho$. We have from Lemma A.4 that

$$b^q \geq b_L^q = \frac{r_L^q + b^q}{1+\rho}$$

Multiplying this inequality through by $1 + \rho$ yields the result.

6.2.2 Proof of countercyclical behavior

We begin with the following useful result.

Lemma A.5. *For all $b \in [b_L^q, \overline{b}]$ it is the case that $\lambda_L(b) > \lambda_H(b)$, where $\lambda_\theta(b)$ is the Lagrange multiplier on the budget constraint for the problem*

$$\max \left\{ \begin{array}{c} b' - b(1+\rho) + x_\theta(\tau) - n_e \xi \frac{\left(\frac{x_\theta(\tau)}{A_\theta n_e}\right)^2}{2} + \gamma \ln g + \beta E V_{\theta'}(b') \\ s.t.\ B_\theta(\tau, g, b', b, \underline{\omega}) \geq 0,\ g + \frac{x_\theta(\tau)}{A_\theta} \leq n_w \ \&\ b' \leq \overline{b} \end{array} \right\}.$$

Proof. Let $b \in [b_L^q, \overline{b}]$. Suppose, contrary to the claim, that $\lambda_H(b) \geq \lambda_L(b)$. Then, by the concavity of the value function, we know that $b'_H(b) \geq b'_L(b)$. Following the same basic steps as in the proof of Lemma A.1, we can show that:

$$R_H(\tau_H(b), \underline{\omega}) - \underline{\omega} g_H(b) > R_L(\tau_L(b), \underline{\omega}) - \underline{\omega} g_L(b). \tag{54}$$

Since $b \geq b_L^q$, moreover, we know that $\lambda_L(b) > 0$ and hence that the budget constraint is binding in the high cost state. Thus, if (54) holds, we have

$$R_H(\tau_H(b), \underline{\omega}) - \underline{\omega} g_H(b) + b'_H(b) - (1+\rho)b > R_L(\tau_L(b), \underline{\omega}) - \underline{\omega} g_L(b) + b'_L(b) - (1+\rho)b = 0.$$

This implies $\lambda_H(b) = 0$. So we have $\lambda_H(b) < \lambda_L(b)$, a contradiction. ■

We now prove:

Lemma A.6. *In equilibrium: (i) $b'_L(b) > b$ for all $b < \overline{b}$, and, (ii) $b'_H(b) > b$ for all $b \leq b^q$ and $b'_H(b) < b$ for all $b \in (b^q, \overline{b}]$.*

Proof (i) We need to show that $b'_L(b) > b$ for all $b < \overline{b}$. Let $b < \overline{b}$. Suppose first that $b \leq b_L^q$. Then, we have that $b'_L(b) = b^q \geq b_L^q > b$. Suppose next that $b > b_L^q$. We know that $b'_L(b) > b^q$ and that $b'_L(b)$ satisfies the first order condition:

$$1 + \lambda_L(b) \geq -\beta E V'_\theta(b'_L(b)) \ (= \text{ if } b'_L(b) < \overline{b})$$

where $\lambda_L(b)$ is the Lagrangian multiplier on the budget constraint on the maximization problem (42). We also know from the proof of Lemma A.3 that

$$-\beta V'_\theta(b) = \left\{ \begin{array}{ll} 1 + \lambda_\theta(b) & \text{if } b > b_\theta^q \\ 1 & \text{if } b < b_\theta^q \end{array} \right. \tag{55}$$

Suppose that $b_L'(b) \leq b$. Then if $b \geq b_H^q$, we have that

$$1 + \lambda_L(b) = -\beta EV_\theta'(b_L'(b)) \leq -\beta EV_\theta'(b) = (1-\alpha)(1+\lambda_L(b)) + \alpha(1+\lambda_H(b)) < 1 + \lambda_L(b)$$

since $\lambda_L(b) > \lambda_H(b)$ for all $b \geq b_L^q$ by Lemma A.5. If $b < b_H^q$, we have that

$$1 + \lambda_L(b) = -\beta EV_\theta'(b_L'(b)) \leq -\beta EV_\theta'(b) = (1-\alpha)(1+\lambda_L(b)) + \alpha < 1 + \lambda_L(b).$$

(ii) We first show that $b_H'(b) > b$ for all $b \leq b^q$. Let $b \leq b^q$. Then since $b^q < b_H^q$, we know that $b_H'(b) = b^q > b$. We next show that $b_H'(b) < b$ for all $b \in (b^q, \overline{b}]$. Let $b \in (b^q, \overline{b}]$. Suppose first that $b \leq b_H^q$. Then we know that $b_H'(b) = b^q < b$. Now suppose that $b > b_H^q$. We know that $b_H'(b)$ satisfies the first order condition:

$$1 + \lambda_H(b) \geq -\beta EV_\theta'(b_H'(b)) \ \ (= \text{ if } b_H'(b) < \overline{b})$$

Suppose that $b_H'(b) \geq b$. Then since $b > b_H^q$ we have that

$$1 + \lambda_H(b) \geq -\beta EV_\theta'(b_H'(b)) \geq -\beta EV_\theta'(b) = (1-\alpha)(1+\lambda_L(b)) + \alpha(1+\lambda_H(b)) > 1 + \lambda_H(b),$$

where the last step relies on (55) and the fact that by Lemma A.5 $\lambda_L(b) > \lambda_H(b)$. This is a contradiction. ■

6.2.3 The stable distribution

Let $\psi_t(b)$ denote the distribution function of the current level of debt at the beginning of period t. The distribution function $\psi_0(b)$ is exogenous and is determined by the economy's initial level of debt b_0. The transition function implied by the equilibrium is given by

$$H(b,b') = \begin{cases} \Pr\{\theta' \text{ s.t. } b_{\theta'}'(b) \leq b'\} & \text{if } \exists\, \theta' \ \ s.t.\ b_{\theta'}'(b) \leq b' \\ 0 & otherwise \end{cases},$$

for any $b' \in [b^q, \overline{b}]$. $H(b,b')$ is the probability that in the next period the initial level of debt will be less than or equal to $b' \in [b^q, \overline{b}]$ if the current level of debt is b. Using this notation, the distribution of debt at the beginning of any period $t \geq 1$ is defined inductively by $\psi_t(b) = \int_z H(z,b) d\psi_{t-1}(z)$. The sequence of distributions $\langle \psi_t(b) \rangle$ converges to the distribution $\psi(b)$ if we have that $\lim_{t\to\infty} \psi_t(b) = \psi(b)$ for all $b' \in [b^q, \overline{b}]$. Moreover, $\psi^*(b)$ is an invariant distribution if

$$\psi^*(b) = \int_z H(z,b) d\psi^*(z).$$

We now establish that any sequence of equilibrium debt distributions $\langle \psi_t(b) \rangle$ converges to a unique invariant distribution $\psi^*(b)$.

It is easy to prove that the transition function $H(b, b')$ has the Feller Property and that it is monotonic in b (see Ch. 8.1 in Stokey, Lucas and Prescott (1989) for definitions). Define the function $H^m(b, b')$ inductively by $H^0(b, b') = H(b, b')$ and $H^m(b, b') = \int_z H(z, b') dH^{m-1}(b, z)$. By Theorem 12.12 in Stokey, Lucas and Prescott (1989), therefore, the result follows if the following "mixing condition" is satisfied:

Mixing Condition: *There exists an $\epsilon > 0$ and $m \geq 0$, such that $H^m(\bar{b}, b^q) \geq \epsilon$ and $H^m(b^q, b^q) \leq 1 - \epsilon$.*

We proceed in two steps.

Step 1. We first show that there exists an $\epsilon > 0$ and $m \geq 0$, such that $H^m(\bar{b}, b^q) \geq \epsilon$. Assume by contradiction that $H^m(\bar{b}, b^q_H) = 0$ for any m. Then the political equilibrium involves no transfers since with probability one: $B_\theta(\tau'_\theta(b), g'_\theta(b), b'_\theta(b), b, \underline{\omega}) = 0$. The equilibrium choices must then coincide with those that would be made by a benevolent planner. As noted in Section 3.4, in the planner's solution there are no distortions in the long run. This implies that in the long run the Lagrange multiplier on the budget constraint is zero: $\Pr\left(\lim_{n\to\infty} \lambda_{\theta_n}(b_n) = 0\right) = 1$. In order for this to be the case, it must be that the debt level cannot exceed r^1_L/ρ where r^1_L is the critical revenue requirement necessary to achieve the first best policies in state L (see Section 3.2). Formally, $\Pr\left(\lim_{n\to\infty} b'_{\theta_n}(b_n) > r^1_L/\rho\right) = 0$, but then $\Pr\left(\lim_{n\to\infty} b'_{\theta_n}(b_n) > b^q_H\right) = 0$, a contradiction. So there must be an $\varepsilon > 0$ and $m \geq 0$, such that $H^{m-1}(\bar{b}, b^q_H) > \varepsilon$. This implies $H^m(\bar{b}, b^q) \geq \alpha H^{m-1}(\bar{b}, b^q_H) > 0$.

Step 2. We now show that there exists an $\varepsilon > 0$ and $m \geq 0$, such that $1 - H^m(b^q, b^q) \geq \varepsilon$. With probability $H^{m-1}(b^q, b^q_H)$ the level of debt chosen in period $m-1$ is b^q_H when the initial level of debt is b^q. Given this, the probability that the level of debt is larger than b^q in period m is at least $H^{m-1}(b^q, b^q_H)\left[1 - H(b^q_H, b^q)\right]$. By the previous step and the monotonicity of $H(b, b')$ we have that there is a $\varepsilon > 0$ such that $H^{m-1}(b^q, b^q_H) > \varepsilon$. Since $b^q_H > b^q_L$, we have $b'_L(b^q_H) \geq b^q$: it follows that $\left[1 - H(b^q_H, b^q)\right] H^{m-1}(b^q, b^q_H) \geq (1-\alpha)\varepsilon > 0$.

To prove that the stable distribution has full support in $[b^q, \bar{b}]$ we now show that for any $b \in [b^q, \bar{b}]$, $\psi^*(b) \in (0, 1)$. The fact that $\psi^*(b) > 0$, follows from Step 1 presented above. We therefore only need to show that $\psi^*(b) < 1$. Let $b_0 \in \left[b^q_L, \bar{b}\right]$. Define recursively a sequence b_t

such that $b_t = b_L(b_{t-1})$. This sequence is monotonically increasing and bounded, so it converges. Assume that $\lim_{t\to\infty} b_t = b_\infty < \bar{b}$. We have that $b'_L(b_t) \leq b_t + \varepsilon_t$, where $\varepsilon_t > 0$ and $\varepsilon_t \to 0$ as $t \to \infty$. We therefore have:

$$\begin{aligned} 1 + \lambda_L(b_t) &\leq -\beta E V'_\theta(b_t + \varepsilon_t) \\ &= (1-\alpha)(1 + \lambda_L(b_t + \varepsilon_t)) + \alpha(1 + \lambda_H(b_t + \varepsilon_t)) \\ &< 1 + \lambda_L(b_t + \varepsilon_t) - \alpha\Delta^*, \end{aligned} \tag{56}$$

where the last inequality follows from the fact that by Lemma A.5 $\lambda_L(b) > \lambda_H(b)$ for all $b \in [b^q, b_\infty]$, so there is a $\Delta^* > 0$ so that $\lambda_L(b) - \Delta^* > \lambda_H(b)$ in a left neighborhood of b_∞. But since $\lambda_H(b_t)$ is continuous in b, (56) implies $\lambda_L(b_\infty) < \lambda_L(b_\infty) - \alpha\Delta^*$, a contradiction.

The argument above implies that for any $b < \bar{b}$, there is a finite T such that starting from any $b_0 \geq b^q$, we have $b_T > b$ with strictly positive probability. This implies that $\psi^*(b) < 1$. ■

6.3 Proof of Proposition 3

We begin by proving the following properties: (i) that $r_\theta(b)$ is increasing in b for each state θ, and (ii) that $r_L(b^q) > r^q_L$, and $r_H(b^q) \leq r^q_H$. To see the first property, assume first that $b \leq b^q_\theta$. In this case $b'_\theta(b) = b^q$, so $r_\theta(b) = (1+\rho)\,b - b'_\theta(b)$ is increasing in b. Assume now that $b > b^q_\theta$. Then we know that $B_\theta(\tau_\theta(b), g_\theta(b), b'_\theta(b), b, \underline{\omega}) = 0$, implying that $b(1+\rho) - b'_\theta(b) = R_\theta(\tau_\theta(b), \underline{\omega}) - \underline{\omega} g_\theta(b)$. An increase in b implies that $\lambda_\theta(b)$ increases, implying that $R_\theta(\tau_\theta(b), \underline{\omega}) - \underline{\omega} g_\theta(b)$ increases in b. The fact that $r_L(b^q) \geq r^q_L$, and $r_H(b^q) \leq r^q_H$ follows from Lemma A.4. We also note that, as it is easy to verify, depending on parameters, $r_H(\bar{b})$ may or may not exceed r^*_H.

To prove the Proposition, recall from Proposition 1 that when q is less than q^*_θ, there will be full employment with a distorted output mix if the revenue requirement is less than r^*_θ and unemployment if r exceeds r^*_θ. This unemployment will be increasing in the revenue requirement. When q exceeds q^*_θ, there will always be unemployment. This unemployment will be constant in the revenue requirement when r is less than r^q_θ and increasing when r exceeds r^q_θ. Since $r_L(\bar{b})$ exceeds r^*_L, when q is less than q^*_L, there will be unemployment in the low productivity state for sufficiently large debt levels. Since $r_L(b^q)$ exceeds r^q_L, when q exceeds q^*_L, unemployment will be increasing in r. The facts that $r_H(b^q)$ is less than r^q_H and that $r_H(\bar{b})$ may or may not exceed r^*_H, imply that when q is less than q^*_H, there will be full employment in the high productivity state for low debt levels and there may or may not be unemployment for sufficiently large debt levels.

When q exceeds q_H^*, unemployment in the high productivity state will be constant in r for low debt levels and increasing for sufficiently high debt levels. Pulling all this information together, yields the Proposition. ■

6.4 Proof of the claim in Section 3.4

In Section 3.4, we asserted that when there is full employment the output mix is distorted towards public production when

$$n_e < \frac{1 - 2\gamma/A_\theta}{1 + A_\theta/2\xi}. \tag{57}$$

Otherwise, it is distorted towards the private good.

Let $(\tau_\theta^1, g_\theta^1)$ solve Problem (15) when $q = 1$ and the budget constraint is not binding. The public good level g_θ^1 is the first best level and the first best output of the private good is just

$$n_e A_\theta (1 - \tau_\theta^1)(A_\theta - \underline{\omega})/\xi = A_\theta (n_w - g_\theta^1).$$

From (26), we have that

$$(\tau_\theta^1, g_\theta^1) = \left(1 - \frac{\xi \left(n_w - g_\theta^1 \right)}{n_e (A_\theta - \underline{\omega})}, \frac{\sqrt{\left(A_\theta n_e - \xi n_w \right)^2 + 4\xi n_e \gamma} - (A_\theta n_e - \xi n_w)}{2\xi} \right).$$

From Proposition 1, there will be full employment in political equilibrium when $q < q_\theta^*$ and $r \leq r_\theta^*$. When $r \leq r_\theta^q$, the equilibrium policies will be $(\tau_\theta^q, g_\theta^q)$. When $r \in (r_\theta^q, r_\theta^*]$, the equilibrium policies will be $(\tau_\theta^-(r), g_\theta^-(r))$ if $\tau_\theta^q < \tau_\theta^-(r)$ and $(\tau_\theta^+(r), g_\theta^+(r))$ if $\tau_\theta^q > \tau_\theta^+(r)$. We need to show that when (57) is satisfied, the equilibrium public good level will be above the first best level, and when the reverse inequality is satisfied, the equilibrium will be below the first best.

Note first that

$$(\tau_\theta^q, g_\theta^q) = \begin{cases} (\tau_\theta^-(r_\theta^q), g_\theta^-(r_\theta^q)) \text{ if } n_e < \frac{1-2\gamma/A_\theta}{1+A_\theta/2\xi} \\ (\tau_\theta^+(r_\theta^q), g_\theta^+(r_\theta^q)) \text{ if } n_e > \frac{1-2\gamma/A_\theta}{1+A_\theta/2\xi} \end{cases}. \tag{58}$$

To see this recall that, by definition,

$$R_\theta(\tau_\theta^q, \underline{\omega}) - \underline{\omega} g_\theta^q = r_\theta^q, \tag{59}$$

and that there is full employment at $(\tau_\theta^q, g_\theta^q)$ which means that

$$g_\theta^q + \frac{x_\theta(\tau_\theta^q)}{A} = n_w. \tag{60}$$

Combining these equations, we obtain

$$R_\theta(\tau_\theta^q, \underline{\omega}) - \underline{\omega}\left(n_w - \frac{x_\theta(\tau_\theta^q)}{A}\right) = r_\theta^q. \tag{61}$$

As discussed in Proposition 1, this has two solutions $\tau_\theta^-(r_\theta^q)$ and $\tau_\theta^+(r_\theta^q)$ and so it follows that either τ_θ^q is equal to $\tau_\theta^-(r_\theta^q)$ or $\tau_\theta^+(r_\theta^q)$. As in the proof of Proposition 1, let τ_θ^* denote the tax rate at which the slope of the budget line is equal to the slope of the full employment line. It is easy to verify diagrammatically that τ_θ^q will equal $\tau_\theta^-(r_\theta^q)$ if $\tau_\theta^q < \tau_\theta^*$ and equal $\tau_\theta^+(r_\theta^q)$ if $\tau_\theta^q > \tau_\theta^*$. From (16) and (19), we see that τ_θ^* satisfies

$$\frac{\partial R_\theta(\tau_\theta^*, \underline{\omega})/\partial\tau}{\underline{\omega}} = \frac{n_e(A_\theta - \underline{\omega})}{\xi}.$$

Using (9) this implies that

$$\tau_\theta^* = \frac{(A_\theta - 2\underline{\omega})}{2(A_\theta - \underline{\omega})}.$$

From (26), we have that

$$\tau_\theta^q = 1 - \frac{\xi\,(n_w - g_\theta^q)}{n_e(A_\theta - \underline{\omega})}.$$

Thus,

$$\tau_\theta^q \lesseqgtr \tau_\theta^* \Leftrightarrow g_\theta^q \lesseqgtr \frac{2\xi n_w - n_e A_\theta}{2\xi}$$

Using the expression for g_θ^q in (26), we have that

$$g_\theta^q \lesseqgtr \frac{2\xi n_w - n_e A_\theta}{2\xi} \Leftrightarrow (qA_\theta n_e - (2q-1)\xi n_w)^2 + (2q-1)4\xi n_e\gamma \lesseqgtr (n_e A_\theta - (qA_\theta n_e - (2q-1)\xi n_w))^2.$$

Further manipulation reveals that

$$g_\theta^q \lesseqgtr \frac{2\xi n_w - n_e A_\theta}{2\xi} \Leftrightarrow 4\xi\gamma \lesseqgtr A_\theta\,(2\xi n_w - n_e A_\theta).$$

Using the fact that $n_w = 1 - n_e$, we conclude that

$$\tau_\theta^q \lesseqgtr \tau_\theta^* \Leftrightarrow n_e \lesseqgtr \frac{1 - 2\gamma/A_\theta}{1 + A_\theta/2\xi},$$

as required.

It follows from this that when (57) is satisfied, the equilibrium policies will be $(\tau_\theta^-(r_\theta^q), g_\theta^-(r_\theta^q))$ when $r \leq r_\theta^q$ and $(\tau_\theta^-(r), g_\theta^-(r))$ when $r \in (r_\theta^q, r_\theta^*]$. When (57) is not satisfied, the equilibrium policies will be $(\tau_\theta^+(r_\theta^q), g_\theta^+(r_\theta^q))$ when $r \leq r_\theta^q$ and $(\tau_\theta^+(r), g_\theta^+(r))$ when $r \in (r_\theta^q, r_\theta^*]$. Given that the tax rate determines the size of the private sector and there is full employment, to prove the result it suffices to show that $\tau_\theta^1 < \tau_\theta^-(r_\theta^q)$ when (57) is satisfied and $\tau_\theta^1 > \tau_\theta^+(r_\theta^q)$ when (57) is not satisfied. It is clear that $r_\theta^q > r_\theta^1$. Thus, since $\tau_\theta^-(r)$ is increasing in r and $\tau_\theta^+(r)$ is decreasing, it suffices to show that $\tau_\theta^1 = \tau_\theta^-(r_\theta^1)$ when $n_e < \frac{1-2\gamma/A_\theta}{1+A_\theta/2\xi}$ and $\tau_\theta^1 = \tau_\theta^+(r_\theta^1)$ when $n_e > \frac{1-2\gamma/A_\theta}{1+A_\theta/2\xi}$. But this follows from (58) by setting $q = 1$. $\blacksquare$

Empirical relationship between the debt/GDP ratio and unemployment

-OECD countries, 2006-2010-

VARIABLES	(1) unem	(2) unem	(3) unem	(4) absunem	(5) absunem	(6) absunem
debt	0.0762***	0.0751***	0.0787***	0.0269***	0.0243***	0.0258***
	(0.0179)	(0.0182)	(0.0161)	(0.00799)	(0.00825)	(0.00784)
debt_plus_90			-0.00712			-0.00565
			(0.00884)			(0.00497)
l_dependency	0.0804	0.0819	0.0925	0.217	0.208	0.218
	(0.358)	(0.356)	(0.364)	(0.245)	(0.232)	(0.230)
l_popgrowth	-1.612*	-1.566*	-1.668*	0.842*	0.775*	0.784*
	(0.900)	(0.883)	(0.958)	(0.494)	(0.429)	(0.428)
l_open	-0.0315	-0.0326	-0.0293	-0.0299	-0.0317	-0.0306
	(0.0293)	(0.0289)	(0.0263)	(0.0208)	(0.0230)	(0.0228)
l_bond		0.287	0.287		0.471**	0.508**
		(0.279)	(0.293)		(0.214)	(0.207)
Constant	1.811	1.775	0.929	-9.394	-10.35	-11.51
	(16.77)	(16.17)	(16.60)	(11.88)	(11.31)	(11.22)
Observations	150	150	150	150	149	149
R-squared	0.611	0.615	0.618	0.387	0.410	0.414
Number of countries	30	30	30	30	30	30

Notes: OLS estimation results. Columns 1-3: u(t) = a + b*debt(t) + c*controls(t-1) + error. Columns 4-6: |(u(t)-u(t-1)|= a + b*debt(t-1) + c*controls(t-2) + error. Country and year fixed effects are included. Standard errors (in parentheses) are clustered by country. *** p<0.01, ** p<0.05, * p<0.1. Data sources: Debt/GDP, A Historical Public Debt Database (IMF); Unemployment rate, World Economic Outlook (IMF); Interest rate, OECD stats; Rest of controls, World Development Indicators (World Bank).