
G.W.F. Hegel

De orbitis planetarum

by G.W.F. Hegel, Jena, 1801

translated from the Latin original by
David Healan
Berlin and Yokohama, 2006

[Warm thanks to Wolfgang Neuser for the original stimulus to undertake this project.

One word of advice: keep your eye on the term ‘centrifugal force’. It's flagged in the footnotes, but as Neuser explains in greater detail in the introduction to his 1986 German translation in *Hegel: Dissertatio Philosophica de Orbitis Planetarum – Philosophische Erörterung über die Planetenbahnen* (Acta humaniora VCH, Weinheim), the confusion was by no means unique to Hegel. Differing views among physicists as well as philosophers surrounding this concept at the end of the eighteenth century go a long way to explaining Hegel’s most interesting errors in his *Habilitationsschrift* dissertation.

Our solar system as a living organism. Mechanics as the science of dead matter. The spirited advocacy of a philosophical physics (or physical philosophy) overcoming mechanics and mathematics. The first glimpse of the space-time dialectic. A vital reminder of the importance of all these issues in Hegel’s thought. Despite its faults, the dissertation is indeed of great fascination. The author is, after all, not a physicist writing physics, but a philosopher just breaking into one of the most dramatic and momentous teaching careers in the entire history of his field.

The curtain opens…]

Planet Orbits

Hegel's Major Works

- History of Philosophy
- Philosophy of History
- Phenomenology of Spirit
- Science of Logic
- Encyclopedia Logic
- Encyclopedia Nature
- Encyclopedia Spirit
- Philosophy of Right
- Philosophy of Fine Art
- Philosophy of Religion
- Other Writings

Online Courses

- Phenomenology Study
- Introduction to Hegel
- Philosophy of Religion
- Science of Logic
- Science of Logic

Other Sections

- Philosophy of Nature
- Hegel and Personalism
- Illustrated Biography
- Quotations
- Hegel Gallery
- Hegel Bookstore (650+)
- Hegel Bibliography
- Useful Links

Participate

- Become a Subscriber
- Submit an Article
- Join our Mailing Lists
- Live Dialog
- Recommend This Site
- Link To Us

by G.W.F. Hegel

All of nature's earthly creations show their inadequacy in the face of her first force, gravity. Subdued by the pressure of the whole, they perish however perfectly they may, after their own fashion, embody the image of the universe. Like gods, in contrast, the heavenly bodies wander so serenely through the light aether precisely because they bear their centre of gravity fully within themselves and are not bound to the earth. No expression of reason could be purer nor more sublime than that organism we call the solar system and nothing could be worthier of philosophical contemplation. Thus is Cicero's praise of Socrates, that he brought philosophy down from heaven into the lives and hearths of men, to be judged rather meagre unless we understand it as saying that philosophy can bring no benefit to our lives and our homes without, once descended, exerting all its energy to ascend once again back up into heaven.

The brief space of a dissertation is hardly suitable to the treatment of so august an object of enquiry. Indeed, only the elements can be given here. I will attempt to do that by first discussing the concepts usual to the physics contained in the science of astronomy. Then I move on to present what true philosophy has established concerning the structural bonds of the solar system with particular reference to the planets' orbits. Finally, I demonstrate the real strength of philosophy in the mathematical determination of quantitative relations by citing a famous example from ancient philosophy.

I.

Whoever approaches this part of physics soon realises that it is rather a mechanics than a physics of the heavens and that astronomy's laws derive their origin from another science, from mathematics, rather than actually having been teased from nature or constructed by reason. Our great countryman Kepler, blessed with the gift of genius as he was, discovered the laws according to which the planets circulate in their orbits. Later, Newton was celebrated for proving these laws not from physical, but from geometrical grounds, and also, despite that, for integrating astronomy into physics. Now, Newton certainly did not introduce the force of gravity, which he wants to identify with centripetal or attractive force, into this part of physics. All physicists before him regarded the relationship between the planets and the sun as a true one, i.e. as a real and physical force. What Newton did was to compare the magnitude of gravity shown by experience for bodies forming part of our earth with the magnitude of celestial motions; he then proceeded to deal with everything else using mathematical reasoning from geometry and calculus. [1] We must be especially wary of this binding of physics with mathematics; we must beware of confusing pure mathematical grounds with physical ones; namely, of blindly taking lines deployed by geometry as helps to construction in proving its theorems for forces or force directions. We must surely all agree that mathematics as a whole is not merely ideal or formal, but no less real and physical. [2] The causes behind mathematics' magnitudes are grounds, which means they belong to nature and when they are comprehended as such, they are laws of nature. Analysis and explanation, however, both of which retreat from the perfection of nature, must be kept strictly separate from that ground of the whole to which they refer. For when, in mathematics, geometry abstracts from time and is constituted solely on the principle of space, while arithmetic abstracts from space relying solely on the principle of time, then

knowledge connections in the formal whole are clearly quite distinct from the actual relationships of nature, in which space and time are inseparably united. Higher geometry, on the other hand, in the union with analytical calculation, arises precisely from the necessity of moving freely through space and time as a unity. Unfortunately, it deploys the concept of infinity to overcome the separation only negatively without demonstrating any true synthesis of the two. This negation in no way departs from the formal methods of geometry and arithmetic. For these reasons we may not mix that knowledge typical of the secure and formal manner of mathematics with physical relationships by attributing physical existence to what only has reality in mathematics.

Not only was Newton careful to call his famous text, in which he describes the laws of motion and gives examples of them from the world system, 'mathematical principles of natural philosophy;' he also reminds us repeatedly that he uses the expressions 'attraction', 'impulse' and 'propensity towards a centre' indiscriminately and interchangeably taking these forces not in the physical but only in the mathematical sense. [3] The reader must not expect, then, on the basis of such terminology, to find definitions of the types and modes of action, causes, or physical grounds anywhere in Newton's work. Neither may he attribute true and physical forces to the centres, which are only mathematical points, even when Newton speaks of forces strongly attracting to the centre or of these as central forces. [4] Just what concept Newton had of physics is clear alone from his assertion that perhaps in purely physical terms instead of 'attraction' it would have been more correct to say 'impulse'. [5] We, however, maintain that 'impulse' belongs in mechanics and not in the true physics. More will be said about the distinction between these two sciences below. It must be said, however, that if Newton really wants to work with mathematical relations then it is astonishing that he resorts the term 'force' at all, for the study of the magnitudes of phenomena belongs in mathematics, while that of force belongs in physics. Newton believed he had explained the relations of force everywhere, but all he in fact did was erect an edifice from a mixture of physics and mathematics making it hard to determine what belonged to physics and really moved it forward.

Kepler took gravity to be a common quality of bodies. He discovered the attraction of the moon as the cause of the ocean tides and that the irregularities in moon motion were due to the combination of forces of the sun and the earth. It would certainly have been very easy for him to give pure and mathematical expression to the physical form of the unchangeable laws he discovered; if, that is, endowed with a pure love and sensitivity for philosophy and the sciences as he was, Kepler had been able to tolerate the kind of confusion which arises, as we shall see, from combining gravity, centripetal force and centrifugal force. He could have reformulated the law he established, according to which the areas covered by the vector radii of bodies moving in circles are proportional to the time taken, in the form of a physical law stating that gravity is the relation between the arcs of segments of equal area. And since the areas of the complete circles A and a stand in the same ratio as the squares of their radii R and r, we know that 1/A : 1/a is equivalent to r^2/R^2. Then he could say that gravity or centripetal force stands in inverse ratio to the radii or the distances, because 1/A and 1/a express the quantity of motion or, if it be preferred, the magnitude of the centripetal force. Anyone who accepts what Newton purports to be a proof that the areas covered by the radii of a body moving in a circle about the unmoved force centre are proportional to the times as a true proof is not be envied his trusting nature. Newton's proof in fact shows that both the arcs and the areas are proportional to time whereas the intention was to prove that definitely not the arcs, but only the areas are proportional to time. [6]

I would reckon the famous resolution of forces among the moves most useful to mathematical proof, even if one has to say that it is almost completely destitute of any feeling for nature. The mechanical direction of motion can actually result from the opposing directions of numerous forces. This, however, is not decisive to the derivation of the direction of live force from opposing forces, so it is vital to distinguish clearly this mechanical relation, in which a body is driven by forces external to it, from living force. But when Newton splits the light nature wants to keep whole into parts and decomposes unitary forces willy-nilly, revelling in the proliferation of theorems on their magnitudes using lines he calls forces, then physicists rightly wonder how the mathematical treatment of phenomena can give rise to so many forces of which nature knows nothing. That science comprising mechanics and astronomy depends almost exclusively upon this resolution, on the construction of the parallelogram of forces [7] , and the vast reach of this science, complete in itself and consistent with the appearances of nature, seems to confirm this hypothesis. The result is that the highest confidence is placed in this principle because its wide-ranging utility is clear despite the fact that, considered in its own right, there is no plausible reason to support it. However, we shall see later the true reason why the effects of a force anywhere must be represented by a square, and why all magnitudes referring to it must be represented by relations arising from the construction of a square. Here let it suffice to note that the resolution of a unitary phenomenon represented by a line, straight or curved, into other lines is a mathematical postulate. While its enormous utility in mathematics is certainly a great recommendation, it is nevertheless extremely important to keep in mind that this means that the principle of resolution depends on a different science. [8] Now, a principle may not be judged according to its utility or consequences; neither can it be acceptable to ascribe a physical meaning to the lines into which a line representing a force direction is resolved according to this postulate simply because they prove mathematically convenient.

It seems that centripetal force, when distinguishable from gravity, and centrifugal force have their origin in nothing other than this resolution of the direction of motion into mathematical lines. An infinitely small arc of a circle is enclosed in a parallelogram according to this principle in such a manner as to form its diagonal. The sides are the tangent, which is ultimately equal to the chord or sine, and the versed sine, ultimately equal to the secant. Physical reality is ascribed to both, so that the one is asserted to be effectively the centrifugal force and the other effectively the centripetal force. [9] Let us consider first the reality of the centrifugal force.

One thing is certainly clear. The geometrical necessity of tangential lines in no way implies the necessity of a tangential physical force. Pure geometry definitely does not tamper with the true form of the circle. Neither does it compare and determine the circumference itself with the radius, rather it compares and comprehends lines determined by the relation of circumference to radius. [10] Now, the geometry that tries to subject the circle to calculation and to express the relation of circumference to radius in numbers takes refuge in the hypothesis of a regular polygon with infinitely many sides, but only at the cost of simultaneously abolishing the polygon itself and the straight lines with this concept of infinity and the ultimate ratio. When, however, the geometry that proceeds on the basis of the dissolution of the circle into a multitude of straight lines treats this concept as only a hypothesis, with the straight lines and the parallelogram reduced to infinite smallness and ultimately disappearing, how then can it seriously assume the physical reality of any one of these lines?

Now turning to the physical reality of centrifugal force leaving

geometrical justification aside, shall we not consider the philosophical construction of this force in that experimental philosophy which Newton regards, or rather all Englishmen have always regarded, as far and away the best, indeed as the one and only? They are only able and only want to confirm the hypothesis of this force through experience. Nothing, however, could be sadder than the examples they adduce to that end. Popular with Newton and his followers is the stone in the sling that tends away from the thrower's hand as he swings it, flying off the moment he unleashes it. [11] Then they illustrate centrifugal force with that other example of the lead cannon ball, shot with the explosive force of a cannon at a given velocity on a horizontal line from the summit of some mountain, flying on a curved trajectory before hitting the ground two miles away. By increasing the velocity, the distance to which it is shot can be arbitrarily increased and the curvature of the trajectory it describes reduced so that it ultimately falls at ten or thirty or ninety degrees. Indeed, it need not fall to earth at all and could instead escape into the sky and continue its flight indefinitely. [12] This last offers a concept of rectilinear motion that anyone can imagine even without an example. Both the examples take the concept from flight patterns arrived at soonest by defining centrifugal force as the force that projects the body in a straight line, but neither of the examples shows any trace of a force of this kind in nature.

Perhaps philosophy itself can deduce a priori what the experimental method, which assumes the name of philosophy, tries to discover with false and fruitless success from experiments, seeking therein with a sort of blind enthusiasm after the shadows of true philosophical concepts in sense perceptions. And for that form of unknowing it must indeed appear as if the opposition between attractive and repulsive force were observed and this motion were an addition to that theory. In truth, however, philosophy ascribes this difference of forces to matter in such a way that it makes gravity or identity itself their precondition. Why a construction of planetary motion from this premise is so sorely lacking becomes clear given that the centrifugal force acts in rectilinear motion; it is not a cause lying deep in the interior of a central body; and, indeed, it is attributed to another body. Hence, also, not even a principle of connection is possible. And since these forces have the character of contradictory opposites, neither is it possible to explain why they cannot be set against each other in a straight line, but only at an angle that splits the straight line of opposition in two. [13] So long as they lack a common principle then, it is undeniable that these forces are merely ideal and not at all physical. This experimental philosophy should not therefore rely on true philosophy's opposition of forces when trying to derive phenomena from forces which obviously have nothing in common and are simply alien to each other. Their relationship is completely different. True philosophy rejects experimental philosophy's principle taken from mechanics, which uses only dead matter for its imitations of nature and effects the synthesis of absolutely distinct forces in some arbitrary body. Anything that serves the imitation of nature, however, must be thoroughly abandoned in the pursuit of knowledge of nature itself, and in physics there can be no place for chance or arbitrariness. If, then, the relationship between centripetal and centrifugal forces is used to explain the motion of sun, planets and comets, clearly this means that these bodies come together without necessity at all through nothing more than some kind of coincidence.

However much the experimental philosophy draws upon geometrical-physical reasoning for its concepts of a force tending towards a centre and a tangential force, the way it constructs the phenomena out of absolute opposites certainly can't be identified with the geometrical method. For geometry does not try to construct a circle or any other kind of curve from lines coming together at right angles or any other angle, but rather assumes a circle or other curve, the object of study, as given and then shows how the relationships between the remaining lines

are determined from this. Physical science must imitate this true method exactly, positing the whole and deriving the relations between the parts from that. It may not under any circumstances compound the whole from opposed forces, which are but parts. How then could astronomical physics arrive at its laws with the help of mathematics without following mathematics faithfully? Thus, even when it believes itself to be talking about centrifugal force, centripetal force or gravity, it is in fact always making assertions about the whole phenomenon. When geometry declares that a certain line is equal to the root of the sum of two squares, it is not speaking about some isolated line of any kind whatever, but of the hypotenuse, that is of a part which is determined by the whole, a right-angled triangle, and distinguished from that whole just as from the other parts. Similarly, one and the same phenomenon of a complete motion is determined by the magnitude of centripetal or centrifugal force or gravity in such a way that it is unimportant which of these three distinct forces is deployed to explain a particular problem, as if they were mere names that were better avoided. All the confusion and lack of clarity in the explanation of the phenomena arises from the vacuity of this distinction. An obvious contradiction emerges here when the versed sine is used to represent the effect of centripetal force and the tangent for centrifugal force, although these forces are said to be equal to one another. This contradiction cannot be overcome by taking refuge in the first ratio of emergence and the ultimate ratio of disappearance, in which the ratio of arc, versed sine and tangent would be equality, so that these lines could be used interchangeably. [14] Like the first, the ultimate ratio is then that of an equality which no longer obtains since there is no more space left for arcs, versed sines, or tangents nor for the differences between their forces discussed above. The centripetal force is only equal to the centrifugal force when the magnitude of the complete motion actually can be correctly expressed by the magnitude of the one or the other force. And the relation between these forces, their difference and their names are all vacuous.

As for the emptiness of the distinction, first it will surely be accepted that centripetal force and gravity are one; Newton's whole concern was to demonstrate their identity. The physical construction of the phenomenon of motion among the heavenly bodies, attributing it all entirely to gravitation, with centripetal and centrifugal forces as gravity's two factors, one of which is set equal to the total force, is thus also void. [15] Then there is the law of centripetal force. It diminishes with distance and Newton wants to account for the total quantity of motion with this law, so it includes the tangential direction imputed to centrifugal force. The assumption is that the circular motion is not effected by means of the propensity to the centre alone, but is composed of one direction to the centre and one on the tangent. [16] Since, however, the total quantity of motion is attributed to centripetal force and is determined by its magnitude, clearly centripetal force is not opposed to centrifugal force; rather it expresses the entire phenomenon. This after all is the reason why in the geometrical construction the effect of centripetal force is represented by the area of a complete triangle, one of its factors being the tangential line, or by a sector. Just how vital it is then in mathematical terms to take one force as equal to the other, or rather as actually the whole, becomes clear from the fact that the total magnitude of the opposing forces is not simply to be measured by the real effect of only the one of them, it must also include the effect that force would have if not hindered by its opponent. In the calculation, to each must be added the effect of the other. Thus, the actual magnitude of centripetal force may not be represented solely by the versed sine, but must also include the tangent or the diagonal line resulting from these two. Similarly, the actual magnitude of centrifugal force cannot be represented by the tangent alone, but must include the versed sine or the diagonal product of these two. This all includes the claim that centrifugal force stands in inverse

ratio to distance. Whether one explains the phenomenon with centripetal or centrifugal force, the solution to all and any problems will always be the same.

From the law stating that the two forces stand in inverse ratio to distance, it is clear that these forces do not constitute the opposition mechanical physics uses to construct the phenomenon of motion here, for while one of the opposing forces grows, the other decreases. We see further that the whole phenomenon with simultaneously increasing and decreasing versed sine and tangent is described and determined now by one or the other of the two forces, now with both depending upon some third force constituting their true principle and identity. What we in fact see is that neither the centripetal nor the centrifugal force can be defined, and neither can the phenomenon be constructed from these factors, but rather that only a magnitude of the total phenomenon of motion is posited.

Just how barren of any true meaning the opposition of centripetal and centrifugal forces and their representation by versed sine and tangent is becomes clearest with the velocity variation that has to be accounted for in the motion of one and the same body on an ellipse. In this case, the relation between focal ray, representing centripetal force, and tangent, for centrifugal force, is not the same everywhere, so the difference in velocities must be explained by disruption of the equilibrium of forces. [17] Now, at both of the two points of mean displacement, the same relation obtains between focal ray and tangent and the body has the same velocity, but, in contrast, the relation between focal ray and tangent is the same at aphelion and perihelion, while the velocities at these points is completely different. Considering all this, it is extremely curious that, although everything is based on mathematical demonstration, many also claim, as we have seen above, that the centrifugal force stands in inverse relation to the square of displacement, while some even favour the cube.

In this method, which accounts for velocity variation in individual planets in the same way it does that of all circulating bodies, the single and eternal justification of empiricism reveals itself to have made a full circle. For the various velocities of the planets are explained by differences in force magnitude and variation in force magnitude by the different velocities.

Let us turn now to the other celebrated application featuring centrifugal force, namely the phenomenon of the pendulum swinging slower in lower geographical latitudes, for the experimental philosophy attempts to show that gravity there is less. [18] The explanation uses the reduced gravity at the equator and its increase with the square of the latitude's sine by claiming that at the equator centripetal force is not equal to gravity but diminished by 1/289th, ascribed to centrifugal force. [19] This fraction, however, is arrived at in the following manner. If a body lying at a distance of 19,695,539 ft from the centre of the earth on a day with 23 hours, 56', 4'' moves uniformly in a circle, then the arc described in the very small time interval of only one second would be 1,436.2 feet and its versed sine, 0.0523 feet or 7.54 lines. [20] Now, since on our earth a fall of only one second at the latitude of Paris would be almost 15 ½ ft or 2,174 lines and since the centripetal force is determined by the distance fallen in a given time and is expressed by the versed sine, the difference that would exist between the former and latter versed sines would be such that that (centrifugal force) would be just 1/289th of gravity at the equator. [21] This factor is assigned to the centrifugal force, which we have elsewhere seen represented by a tangential line. [22] Since we have seen that the one force is replaced by the other

arbitrarily and that each can be used alternately for the other quite freely without altering any laws, there is nothing to stop us taking that minute versed sine for the effect of centripetal force and adding it to gravity. Then we could say that the increase and not the decrease in gravity causes the slower motion of the pendulum at the equator and that the weight of the bob increases instead of decreasing at low latitudes, thus enabling both measurement and explanation of the phenomenon. Experience shows that the clock pendulum moves slower at lower latitudes and the oscillation is effectively derived from the gravity causing the fall of the bob. Now, because of the slower motion of a pendulum of given length and weight, the experimental philosophers want to regard gravity as reduced. The motion of the pendulum bob, however, is no simple fall. The bob is in fact immediately hindered from following a straight line of fall because it is not released from the suspension point but at an elongation, from the side, turning the vertical line into a curve through centrifugal and centripetal forces, if you wish, which we say induce the resulting horizontal or tangential direction. Why then should we not explain the slowing down in oscillation at the equator by saying that the deviation from the vertical fall line, just as from the horizontal motion, is caused, if you wish, by a greater restraint against centrifugal force at the equator? That is, through nothing other than a supposed greater propensity to the vertical line, itself due to greater centripetal force at lower latitudes, drawing the vertical more strongly to oscillation (through the null point) and quickly overcoming its opposing direction. From this we can conclude that it all agrees brilliantly with the shape of the earth, given its elevation at the equator, whose diameter is less than the axis. [23] This is why a pendulum suspended in lower latitudes is nearer a greater mass and is more strongly attracted, so that because of the greater weight it tends more strongly towards the earth and to the vertical line. [24] The body cannot deviate from this line at an elongation as easily as a body at higher parallels, because there it is attracted by a smaller mass and describes a lateral motion.

It would be tedious to discuss the distinction Newton draws between motive and accelerative force. [25] The way he uses them interchangeably, he seems to be concealing the fact that in the famous application of the law of centripetal force to the motion of the moon and to the planets with their satellites, there is no reference to any relation between the masses. Clearly this gravitation law is a law merely of the phenomenon of motion and not a force law at all. The effect of a force necessarily depends not on the law of force alone, but also on the mass; neither can the appearances agree solely with the force law. Others certainly include the mass relations of moon and earth in their explanations of the agreement between this law and lunar motion; but they surely suspect that the different planet masses cause no modification in this law, which is supposed to involve force only, because these are extremely low compared with that of the sun. [26] They claim that this relation obtains also for comparable satellites and the planets about which they orbit. However, from the velocity of the satellites and its relation to distance they estimate the density of the planet, just as they do the density of the sun from the same relation of the planets.

We have shown that centripetal and centrifugal forces can be interchangeably deployed to explain the phenomena, that a decrease in gravity can be replaced by an increase, and that those phenomena explained from decrease in the force of gravity can be derived from its increase. Similarly, that law can be inverted which says that the gravitation force stands in inverse ratio to the square of the distances, so we can say instead that it stands in direct ratio to the square of distances. For when it is claimed that at greater distances gravity is reduced this brings in a completely different factor of the gravity to be determined, namely velocity. Because at greater distance the velocity

is lower, one says that gravity is lower. But we must also gauge the magnitude of the force from the value of the distance at which it works and expect a four-fold magnitude for a force operating at twice the distance. When, thus, the law of gravity is commonly expressed as that this force increases or decreases solely on the basis of the given magnitude of velocity, and the distance is in no way drawn upon to determine the relation of increase or decrease, and certainly not to predict anything else about increase or decrease itself, then, if the law of gravity is expressed in this way, we could with the same justice neglect velocity when predicting the magnitude and call the force that is effective at greater distance 'greater' and claim that it varies directly with the distance. As with the lever, whose two factors, distance and weight, stand in inverse ratio, one can say that gravity becomes arbitrarily greater or smaller as distance increases. A greater distance means, since equilibrium should prevail, a lower weight, which Newton calls 'motive force', and consequently gravity is less. Or the greater distance means greater gravity, for at greater distance the same weight means a greater force.

From all this, the first consequence is that the distinction between centrifugal and centripetal force is vacuous and that the apparent laws of centripetal force and centrifugal force in fact are mathematical laws of motion adulterated with the name and aura of physical force. Thus are increase and decrease wrongly attributed to gravity, while neither a quantity nor any other kind of quantitative relation to anything else whatsoever, including space and time, falls to gravity itself. Gravity must be seen as one and constant existing in the form of two factors: space and time; or as I would say, as space at rest and space generated by motion in time. All quantitative differences and relations refer to these factors, of which the one increases when the other is decreased. Nor does there exist any relation or proportion between them unless it be asserted in one and the same thing. Moreover, their absolute identity cannot be altered, increased or decreased.

So much purer were Kepler's insight and genius! He posited nothing beside the relation of factors which can really increase and decrease. He did not sully their pure relation and their truly empyrean expression with quantities of gravity, which has no magnitude. But the enormous accumulation of mathematical results and applications especially in astronomy, where mathematics struts magnificent and serene, has strongly recommended to scholars that mixture of physics and mathematics introduced by Newton. The general public, however, still like what they know as gravity, less because they understand it as a universal world force, as simply one and constant, the conception of Kepler and other philosophers; rather as a common or garden force by which stones fall to the earth, and now by which the heavenly bodies move in their orbits. The common people are now thoroughly well-informed about all that from the pathetic tale of the apple falling before Newton's eyes. The people prefer security to heaven, forgetting that an apple was there at the origin of the misfortune of the entire human race and again at Troy's – a bad omen for the philosophical sciences.

One must acknowledge that the science of astronomy now, or at least the mathematics in it, is due largely to Newton. It remains crucial, however, that the mathematical relations be separated from the physical raiment with which he clad them and that philosophy determine what is true in the latter. I want to introduce an example here from the experimental philosophy, which only they could have produced from that essential English character as expressed by Newton, Locke and the others in their writings. According to a theorem of Descartes, of Aristotle and others, the weight of a body depends on the form of its material. [27] In order to refute this and prove that weight is not determined by form, but by the quantity of material, Newton preformed the following experiment. He made pairs of pendulums all equal as to

length, weight, shape and air resistance by putting equally heavy weights of gold, silver, sand, corn, etc. in pairs of equal boxes, designed to eliminate deviations due to air resistance. [28] What is clarified by means of pendulums of the same shape, length and air resistance? The equality or difference of the weights! He started by making the weights of the pendulum bodies the same and happily discovered that body weights were equal, believing that by means of this kind of experimentation and philosophising he could refute those philosophers who assume such differences of form in one and the same material. From this single example we can see that the experimental philosophy is profoundly ignorant of what principle true philosophy seeks for itself. The true origin of centripetal and centrifugal force will be explained from that very principle.

Alienated from the life of nature, the science of mechanics can offer no other first concept of matter than death, which it calls “inertia” in the sense of indifference to both rest and motion. This version of matter is nothing other than the most abstract concept of object, absolute opposition. They add alll the diversity they perceive in the material together with that which only becomes visible through motion, even if it comes from elsewhere. That weight is a universal property of matter they determine by experiment and induction. According to Newton’s second rule of philosophising, causes of the same natural effects are the same, for instance, for falling stones in Europe and America. In the third rule we find that properties common to bodies on which one can perform experiments must be regarded as universal properties of all bodies. [29] Now, experience teaches that matter has weight. Clearly, weight in the case of the stone which falls to earth is different from that in the stars and especially from that in the bodies which belong to our solar system, which do not fall to earth, so they propose another force as the cause of these phenomena: centrifugal force. A philosophy that sees the nature of gravity, as well as that of impulse, in an infinite horizontal line, passing this off as centrifugal force, and thereby completely missing the real cause, must be allowed to ascribe everything to God. That said, it is then legitimate to demand that it philosophise correctly about God and his rational action and, while it misconstrues nature, that it at least apprehend God truly.

II

God’s actions are not external nor mechanical nor arbitrary nor coincidental. One thing must be clear: the forces they claim God put into matter truly dwell therein; indeed, they constitute the essence of matter in the principle of opposed forces, internal and immanent to it. Mechanics avoids this concept with its claim that inertial matter is always moved by an external impulse or, what amounts to the same thing, by forces alien to matter. It recognises neither God nor true force effectively, nor that which is internal and necessary. Mechanics only accepts external causes and does not comprehend nature rationally, so it is incapable of advancing to the principle of an identity that asserts difference within itself. Once restored to us, this principle went on to revivify philosophy, separated mechanics from physics, which, distinguished from mechanics by more than the name ‘dynamics’, it finally gave back to philosophy. We shall now present the elements of the planetary system and develop them briefly.

Gravity constitutes matter such that matter is objective gravity. It is one and the same matter dividing itself into poles and thereby creating a line of cohesion, generating diverse shapes in a series of evolutions with

different relations between the factors. This is gravity's real difference, from which we distinguish the other, ideal difference, that of the potentials of time and space. One double thus implies another: one of poles, the other of potentials; and that makes four regions.

Let us first consider the cohesion line. Gravity draws this line by asserting itself at all points, each of which is distinct in itself due to the reciprocal relations of factors, producing a series of nodes and centres for itself. In each of these points there is no lack of that multiplicity of relations to the others, now drawn together under the law and organisation of each, bundled by the power of its own principle. The solar system draws a line so much greater than the rest, which makes it so much more powerful, for where the cohesion line is broken, the body at that point carries its centre of gravity within itself; not with an absolute power certainly, but with greater force than that of the other bodies. No body, no matter that it is a whole in itself, is completely independent of the others and each is part and organ of the larger system. Still, the heavenly bodies enjoy, if not perfect, surely the greatest possible freedom and independence from gravity. The planets were not wandering aimlessly through infinite space on rectilinear paths, when they just happened to be flying in the neighbourhood of the sun and were forced under its law onto their orbits. And that hypothetical centrifugal force is not what holds them back from the sun. Rather, because they form an original system with the sun, the true cohesion force holds them firmly in place and keeps them apart.

The indifference point, always expressed just as in the magnet and then in the lever, which imitates the natural line of magnetism in dead matter, forms a midpoint distinct from the centre of forces. Since indifference is neutral, it exerts no force subject to the condition of difference itself. Centres of forces are thus established within this definite line, but not in the middle; and they are bodies. A body is, after all, nothing other than the phenomenal manifestation of a force of physics or of a true idea. Since its position would vary slightly due to the attraction of the planets, Newton thought the centre of gravitation, or of indifference, was not to be located in the sun. [30] He assumes nothing other than mutual attraction between bodies to explain planetary motion, and this hypothesis does not immediately imply a centre, nor does it enable him to demonstrate his propositions on curvilinear motion. This he can only achieve by assuming a centre of orbits. In Book I, Section XI of the *Principia*, where Newton describes the motion of bodies mutually attracting each other by centripetal force, he requires that action between attracting and attracted bodies should be reciprocal such that neither can remain at rest [31] , and further that both would be moved through that mutual attraction as if around a common centre of gravity. Here he appeals to the fourth corollary of the laws [32] , which in fact only says that the state of motion or rest of the common gravity centre of two or more bodies is not affected by reciprocal action between them. But there is nothing there that suggests the necessity of a true and actual centre or of a central body. That common gravity centre is thus merely a mathematical point and the fact that the sun is the force centre, or in its neighbourhood, is not down to necessity, but to mere chance, which has given it the greatest mass. Now, the enormous mass of the sun, whose concept includes density, is determined once again from the hypothesis that all force depends on mass. Physical philosophy, however, teaches us that the true force centre is necessarily the light source and that this is what constitutes the true force and power of the sun.

We have stated that this force centre is not expressed in the middle; for just as the cohesion line generates two outer poles, so also two inner force centres. Thus we have duality in the culmination points of the magnet and in the foci of the ellipse [33] , whose main axis is the true

line of magnetism. These culmination points are so ordered that each is nearer the opposite pole to the one on which its force works; thus the inner pole +M lies between the indifference point and the outer pole –M, and likewise the inner pole –M lies between the middle and the outer pole +M. Since however the planetary system has a broken cohesion line and does not form a continuous body and, as we shall see below, since both poles act in one and the same body, there exits only one real culminating force point: the sun in one focus of the ellipse, the other being dark and purely mathematical. The line of natural magnetism thus takes on the form of a natural pendulum, just as the mechanical pendulum is effectively an incomplete lever that has lost its other pole – this cannot be generated by a hanging body subject to gravity. Thus, the bodies mutually relate to each other in this rectilinear and powerful, but not rigid series, which is established as the foundation of the entire system. Nature wants the force existing here as a line to take on bodily form, so we can see that such a system, clearly not formed by one single body, does not do justice to her.

Now that we have explained the cohesion and the real difference between the poles, we can proceed to the other, ideal difference: that of the potentials of subject and object. Thinking of matter as filling space leaves it devoid of form; space and matter are then nothing other than the abstract concept of the objective. Matter's physical or real concept can only be grasped in the form of subjectivity. The point is an expression of space – admittedly an abstraction from space – in space, such that the bond between point and space remains firm. While the concept of matter as filled and, as I would say, dense space, for that reason also at rest, clearly includes the concept of resistance against other matter moving into its location, this concept of resistance is purely negative and empty. For if the space is full up, that eliminates every principle of change and resistance, which must thus be sought elsewhere. In order to comprehend real matter, to the abstract concept of space must be added matter's contrary or subjective form, which in the Latin language we designate with the word *mens* (German Geist, English 'mind, spirit') and in terms of space: 'point'. Thus the point or time, a form with the same characteristic difference, and space generate the elements of matter, which is certainly not forged from them, but is their principle. From this inner and original identity and difference of the opposed potentials of coming to be and passing away – for the poles are at rest – the necessity of change and motion becomes intelligible. Change is thus nothing other than the eternal restoration of identity out of difference and the production of new difference: contraction and expansion. The other potential, spirit/mind (*Geist/mens*), as perpetually generating itself after complete abstraction from space, is time, which, when it relates its own production back to space, generates the line. The line is spirit as it generates itself – albeit in subjective form – and reveals itself in itself assuming complete and natural form by transiting into its opposite, space, generating the plane, which lacks all other differences, since we have not asserted any besides extension and mind, and is a square.

Looking away from the thing itself, supposedly comparing its numbers and measures, but not time and space themselves, considered incommensurable, mathematical reflection seems very far removed from this transition of time into space. Geometry and calculus forget the things themselves, handling only those lines and numbers found by calculus operations or geometrical demonstrations, but only to the extent that they possess a semantic reference to the things. It is clear, then, that not only the magnitudes, but also the things themselves are being compared here. Moreover, mathematics uses mutual transitions of the incommensurables in other ways, such as line into plane and plane into body by extension. It usually hides this identity of incommensurables under the name of infinity, such as the claim that the plane is composed of innumerable lines. Furthermore, expressing relations among many numbers as infinite series amounts to

an admission that it has gone beyond reflection's absolute difference and is comparing incommensurables. In particular, however, the so-called higher geometry reduces the plane to the line and both to the infinitely small, that is to the point, while analysis forms the line from points, the infinite line. But how the line arises from the points and the plane from the line and so on, that is not comprehended in any other way than by resort to the extraneous concept of motion, i.e. after the identity of space and time has been asserted. Now we have seen that the line is *mens/Geist*/spirit/mind generating itself in the subjective form appropriate to it; also that its transition into objective form is in fact the square, which is why on the other hand its product in *natura naturata* is the cube. After all the abstractions of mind are done, when space generates itself there are three dimensions and the body in emergence is the square, while the body in existence is the cube. Since the relation between bodies separated from each other is the line, a subjective relation lacking objective form, as one falls onto the other they turn the line into the square by abolishing the difference and regrouping into one body. Hence the law of falling bodies features the square of the displacement or the square into which the line mutates.

There is another difference here: the difference between the two bodies is either actually overcome or it remains; one body results, either real or ideal. The former arises from free fall, the latter from circular motion. In the fall, the element of the square is simply represented by a sum of time units or as a line, divided by a fixed but arbitrary measure and expressed in numbers. For the ideal body produced by circular motion in contrast, the difference between the bodies remains and consequently also in a certain respect that between time and space. The former generates the time period while the latter is responsible for the displacement between the bodies. Periodic time, however, must be correlated with the space traversed by the body, which makes an angle with the space of displacement. The resulting synthesis, which determines the magnitude of motion, is the square itself. From this two elements emerge of what is called the matter of motion, which express the whole relation of two bodies moving around each other: the line of displacement and the square of motion. Thus the magnitude of the whole composed from these two elements will be the cube or the body. And since gravity is always one and constant, my claim is that this cube is the same for all planets. Kepler's famous law is easily shown from this.

The philosophical lemmas for mathematics must be taken from what we have presented above. From that also must be deduced the proofs for those theorems lying at the foundation of almost all of applied mathematics. For their true proofs, which can never be arrived at by mathematics alone, have been lacking right up into our own time. We would now like to approach that path using concepts we have already presented. The common resolution of forces depends upon that exposition of the synthesis of time and space and the transition of spirit/mind, or the line, into the square. Its mathematical truth and necessity are merely postulated, while it is cheated of physical truth. An easy path then lies open from that resolution to the mechanical laws which transpose physics onto dead matter. The laws themselves, however, must be derived from nature, not from a mechanics that only imitates nature. We now return to our topic.

The heavenly bodies' relation of displacement, which we shall look at later, is determined by the cohesion line. Separated from each other, their masses form centres of density opposed to the diffuseness of the aether, points of the greatest contraction opposed to the greatest expansion. Thus the physicists ascribe absolute elasticity and the force of repulsion to the aether, but the force of attraction to the bodies, to which alone the force of gravity refers, none of this going to the aether at all. The original identity of nature strives to overcome this opposition of greatest density and greatest diffuseness along with the manifestation

of this opposition, namely the separation of the bodies; while the virtual line strives to turn into the square, to assume form and body. This striving is the phenomenon of motion. Nature, however, does not want the system of heavenly bodies to coagulate into a single mass, nor that it fall into that sad state of *natura naturata*, there to share the fate of bodies, but that it be a living expression of reason and its likeness. Curvilinear motion does not produce a real body, but an ideal one, i.e. a square, so the body generated by their line is nothing other than the space enclosed by the planets' orbits. Thus, if we want to define orbital motion in terms of its opposite, we have to say that it is the overcoming of the body, the reduction of the body or the cube by the square, and this expresses Kepler's sublime law.

In the formal circle the concept of equal distance from a point generates the circumference: and its original characteristic is that none of its diameters and none of the locations on the circumference predominate over any of their infinity of fellows. Motion cannot therefore arise from the original cohesion line if we assume only the difference between the bodies and not nature's striving to unite them into one body. If, however, a circle can really be mechanically constructed from the attractive force of the central body and the centrifugal force of the orbiting body, how then are we to distinguish any diameter, or the cohesion line of a culmination point, or arrive at the ellipse?

No matter how the bodies of the solar system are separated and the fixed cohesion line is overcome turning into motion, it is absurd to suggest that this line's force would be dissipated in the indifference of all diameters of a formal circle. On the contrary, the line demonstrates its force precisely by asserting itself as the orbital axis, creating a polarity in the variation of motion; one pole reducing and the other accelerating it. Motion slows to the aphelion, where the force of the culmination point, the sun, is greatest; while to the perihelion, with that force minimal and the planet's immanent force maximal, it accelerates. Disturbances in planetary motion must be referred to this as they are effects of a weak and brief cohesion readily subjugated by the primary cohesion.

Finally, we have contrasted an ideal difference of potentials with the real magnetic difference and now we must observe briefly that the real difference itself exists in the form of a double difference, that namely the real east-west line is formed as well as the line of bodies we call comets, circulating in orbits with immeasurable apsides, precisely because east and west stand under the law of potential difference.

III

It remains to add some observations on the relations of planetary displacements, which appear to be a matter of experience alone. In truth, they cannot be measures or numbers of nature alien to reason. For our pursuit of the laws of nature, and our knowledge of them, is founded on nothing other than the belief that nature is shaped by reason, and that we are convinced of the identity of all natural laws. Whenever those who seek laws through experience and induction happen upon something that looks like a law, they rejoice at their find and the identity of nature and reason therein, and when other appearances are difficult to accommodate with that they feel some doubt in the earlier experiments and try in every way to establish harmony between the findings. Our topic, the planets' orbits, offers a case in point: While the displacements of the planets suggest an arithmetic progression in which?unfortunately, no planet in nature

corresponds to the fifth member in the series, it is supposed that there really does exist between Mars and Jupiter, unbeknown to us, a planet moving through outer space. It is now being eagerly looked for.

Since this progression is arithmetic and does not follow a number series that generates them itself, i.e. not by powers, it is of no interest to philosophy. The extensive work of the Pythagoreans on the relations of philosophical numbers is well-known; so I will now, if I may, consider the traditional number series presented in the two *Timaeus* texts. For although Timaeus does not refer to the planets, he thinks the demiurge formed the universe according to this series. The number series is: 1, 2, 3, 4, 9, 16, 27, if I may take 16 instead of 8, which we find in the *Timaeus*. If this series really does give the true order of nature as an arithmetic series, then there is a great space between the fourth and fifth places where no planet appears to be missing.

To state the remaining briefly: it turns out that the cubic root of these numbers gives the displacements of the planets as the square of the squares:

1.4 – 2.56 – 4.37 – 6.34 – 18.75 – 40.34 – 81
(not to omit unity, we take it as $^{3}v3$).

It is also clear, however, that the satellites of Jupiter stand in a relation to each other in a series in which the first four planets precede them, although the fourth satellite exceeds its number slightly. [34]

For the satellites of Saturn there is a somewhat irregular but highly remarkable relation: the orbit periods of the first four stand in the relation of the square roots of 1, 2, 4, and 8, while their displacements form the series of cubic roots of the same numbers. [35] If the number of the orbit times is desired, the result is: $v(-2^{9}, 2^{10}, 2^{11}, 2^{12}$, 22, 32, 45, 64). [36] The fifth satellite reverses the formal progression just like the fifth planet; and since of course the displacements of the first four planets were in the relation of the cubic roots of 1, 2, 4, 8, that means 1 – 1.26 – 1.63 – 2, and $^{3}v8$ would thus belong to the fourth, v8 or $^{3}v(16 : 32)$ belong to the fifth and the series of cubes whose roots would express the relation of the displacements is:

$1, 2, 2^{2}, 2^{3}, (2^{4} : 2^{5}), 2^{8}, (2^{12} : 2^{13})$
or .....$2^{9/7}$.....$2^{25/2}$.

Footnotes

Hegel quotes freely from, and refers frequently to, Newton's *Principia* (here: *Philosophiae naturalis principia mathematica* in the English translation of 1729 by Andrew Motte: *Isaac Newton, Mathematical Principles of natural philosophy and his system of the world*, translation revised and annotated by Florian Cajori, Berkeley 1960). All references to this edition of the English translation of the *Principia* given below are derived from Neuser's references to the German translation by J.P Wolfers *Mathematische Prinzipien der Naturlehre*, Berlin 1872 and Darmstadt 1963 in his Commentary.

1. *Principia*, Book I. Section XI. p.164, “The motions of bodies tending to each other with centripetal forces.”
2. ibid.
3. *Principia*, Book I. Definition VIII, p. 5.
4. ibid.
5. *Principia*, Book I. Section XI. p.164, “The motions of bodies tending to each other with centripetal forces.”
6. *Principia*, Book I, Section II, Proposition I, p. 40.
7. *Principia*, Book I, Section II, p. 40.
8. *Principia*, p. 14.
9. Like B. Martin in his *Philosophia britanica oder Lehrbegriffe der newtonischen Weltweisheit, Sternkunde etc*. Leipzig 1778, Hegel confuses the momentum of Newton’s 'dynamic' theory with the centrifugal force of D’Alembert’s 'static' theory of planetary motion. Neuser, Commentary, Note 9,10.
10. *Principia*, p. 47.
11. *Principia*, Book I, Definition V, pp. 2-3
12. *Principia*, Book I, Definition V, pp. 2-3
13. *Principia*, p. 14; Martin, vol. I, p. 168
14. *Principia*, Book I, Section I, “The method of first and last ratios of quantities, by the help of which we demonstrate the definitions that follow”.
15. *Principia*, Book III, Propositions, Proposition V, Scholium, p. 409.
16. *Principia*, Axioms, or Laws of Motion, Corollary 1, p. 14.
17. Here and in the following Hegel’s arguments are based on Laplace, *Exposition du Système du Monde*, Paris 1796. Hegel used the German translation *Darstellung des Weltsystems*, by J.K.F. Hauff, Frankfurt 1797 and here Neuser gives the reference vol. 1, pp. 295-6. Laplace decomposes the effective force at a given point on an elliptical planet orbit into a normal component (direction of the ‘radius of curvature’) and a tangential component. “The first holds the centrifugal force in equilibrium; the other increases or decreases the velocity of the bodies. ” This must be Hegel’s source for confusing normal and tangential components with centripetal and centrifugal forces. Neuser, Commentary, Note 15,20.
18. *Principia*, Book III, Proposition XIX, Problem III, p. 424f.
19. *Principia*, Book I, Section X, p. 158; Book II, Section VI, p. 303.
20. *Principia*, Book III, Section I, p. 427; Book III, Section III, pp. 482-3.
21. *Principia* p. 429.

22. Hegel's description here is false. Neuser, Commentary, Note 16,29.

23. Hegel shares Descartes opinion at this point, that the earth is flattened at the equator. This statement stands in contradiction to the rest of Hegel's argumentation. The claim that the earth is flattened at the equator is not found anywhere else in Hegel's writings. Neuser, Commentary, Note 17,31.

24. *Principia*, Book III, Section I, p. 424, p. 491.

25. *Principia* p. 4, Definitions VII and VIII.

26. Compare *Principia*, Book III, Propositions, Proposition XII, p. 419.

27. *Principia*, "Cotes's Preface to the Second Edition," p. xxi; Book III, Propositions, Proposition VI, p. 411; Book I, Axioms, or Laws of Motion, Corollary II, p. 14.

28. *Principia*, Book II, Section VI, The motion and resistance of pendulous bodies, p. 303ff, and p. 411.

29. *Principia* p. 398 "Rules of Reasoning in Philosophy".

30. *Principia*, Book I, Section XI, Proposition LVIII, Theorem XXI, p. 165.

31. *Principia*, Book I, Section XI, p. 164f.

32. *Principia*, Book I, Axioms, or Laws of Motion, Corollary IV, p. 19.

33. *Principia*, p. 569.

B. Martin, ibid., vol. 3, p. 153ff. Hegel's values for the displacements of Jupiter's moons are thus:

34.
1. Io 1.4
2. Europa 2.56
3. Ganymede 4.37
4. Callisto 6.34

Neuser, Commentary, Note 16,29.

B. Martin, ibid., vol. 3, p. 155 and Laplace, ibid. vol. 1, p. 261.

The orbit periods of Saturn's moons in days, agreeing well with Laplace's values, thus amount to:

35.
1. Mimas v1 = 1
2. Enceladus v2 = 1.414
3. Tethys v4 = 2
4. Dione v8 = 2.828

Neuser, Commentary, Note 16,29.

Hegel gives the displacements of Saturn's moons as follows:

36.
1. Mimas $^{3}v1 = 1$
2. Enceladus $^{3}v2 = 1.26$
3. Tethys $^{3}v2^{2} = 1.63$
4. Dione $^{3}v2^{3} = 2$
5. Rhea $^{3}v2^{3} \sim {}^{3}v2^{9/2} = 2.828$
6. Titan $^{3}v2^{8} = 4.226$
7. Iapetus $^{3}v2^{25/2} = 17.959$

Neuser, Commentary, Note 16,29. The following is a direct translation from Neuser's last Commentary note.
The division given in the formula for the moons Rhea and Iapetus must be replaced by a multiplication – as shown by the recalculation in the last line from the last but one line of the dissertation. Furthermore, a square root must also be drawn, if the recalculation is to be taken as correct. Interpreting these numbers of Hegel's in units of (107) Paris feet results in displacements for all Saturn's moons twice as great as the real values. Laplace gives the correct values. Due to conditions of observation, the astronomers often express the moon displacements in terms of Saturn diameters or Saturn ring diameters. It is not inconceivable that Hegel has confused diameter with radius here.

mailto: daheal3@yahoo.de