# PROJECTIVE DIFFERENTIAL GEOMETRY OF SINGULARITIES OF PLANE CURVES

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In a recent paper(1) the present author has established some representations for a plane curve $C$ in the neighbourhood of a singular point $O$ characterized by the property that the tangent $t_0$ of $C$ at $O$ has a contact of order $m-1(\geqq 2)$ with the curve. For convenience' sake we have adopted non-homogeneous coordinates $x$, $y$ of a point to represent the expansion of the curve,

$$(1) \qquad y = x^m \sum_{\nu=0}^{\infty} b_\nu x^\nu \qquad (b_0 \neq 0).$$

The coordinates of the invariant point $O_{m+1}$ as well as the $m-3$ conditions for the existence of the invariant point $O_{2m}$ are consequently expressed in terms of the coefficients $b_0$, $b_1$, $\ldots$, $b_m$.

The use of non-homogeneous coordinates $x$, $y$ has the advantage of reducing the expansion (1) to a simple form

$$(2) \qquad y = b_0 x^m + b_{m+1} x^{2m+1} + \cdots,$$

namely, the *semi-canonical expansion* of the curve at its *representable singular point O of order m*, provided that the points $O_{m+1}$ and $O_{2m}$ be taken for the points of infinity of the coordinate axes.

In applying the result to the theory of curves in hyperspace we are frequently led to the consideration of a certain plane curve with a singularity whose expansion is given, instead of (1), by equations of the parametric form

(1) B. Su, An extension of Bompiani's osculants for a plane curve with a singular point, *Tôhoku Math. Journal*, 45 (1939), 239-244.